TRIGONOMETRY REVIEW MANUAL

HAUCK

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PREFACE

The text is a *review* of the topics of traditional trigonometry found in high school and college texts. Trigonometric functions of any acute angle, a concept basic to all trigonometry, are treated in detail. The functions of special angles—30°, 45°, 60°, and quadrantal angles, noting restrictions—are reviewed. Interpolation by proportional parts, a method too often only "touched on," is treated thoroughly. Since the solution of many triangles depends upon interpolation of logarithm tables, mastery of this material will result in a useful skill. The final chapter on circular functions provides a foundation for further work in calculus by emphasizing the interrelationship of trigonometric and circular functions.

Although not intended for use as a full course text, since it does provide the essentials for further study of trigonometry and of those courses in mathematics dependent upon trigonometry, the text is of particular value in preparing students to study modern trigonometry, calculus, general college mathematics, etc. It may be used at the twelfth grade or at the college freshman level as a review of the fundamentals of trigonometry. The *prerequisites* for success with the text are two years of high school algebra and some work in geometry.

TO THE STUDENT

Program content is presented in a question and answer form, with the learner reading each unit of information and *responding* to it before preceeding to the next one. Each item produces the correct answer with a high degree of frequency for most students. The student *and* the teacher should always keep in mind the importance of the following sequence. *Read and answer each frame carefully. Compare that answer with the answer in the text.* If students *write* their answers to the question frames, active responding is more closely guaranteed.

The program's structure enhances its systematic use by the teacher and by the student. Generally, new information is presented in a set of concept frames. Each set of concept frames is followed by one or more criterion frames which test whether or not the learner has acquired the concept. The criterion frames are designated by rules set above and below the frame number, for example, 47. If a student misses a criterion frame, he is advised to reread the set of concept frames which precede it, but if he responds correctly in the criterion frame, he proceeds to the next set of frames. A pre-test is provided at the beginning of each section to indicate individual weaknesses. A post-test is provided after each section to indicate the extent to which the material for that section has been learned. If the student misses more than the maximum number of items indicated in the post-test instructions, he should review. The review should be carried out systematically as follows:

(*a*) Read and answer each criterion frame in the part of the program for which review is needed.

(*b*) When the response to a criterion frame is incorrect, reread the concept frames preceding it.

If the student uses the program properly and reviews the material as advised, he will find it to be a very efficient way to learn. A final examination with answers is provided at the back of the book. If the student has difficulty answering these questions, he should return to the program for further review.

EVALUATION of a PROGRAMMED TEXT

In some respects, the evaluation of programmed texts in mathematics is much the same as the evaluation of a standard textbook; however, there are some important differences. The following suggestions may aid in this evaluation.

Teaching objectives and level and quality of the mathematics For this program, an estimate of the teaching objectives and of the level and quality of mathematics can be made most effectively through a thorough examination of (*a*) the table of contents, (*b*) the criterion frames, (*c*) the self-test, and (*d*) the final achievement test. As a result of the systematic structuring and sequencing of the material, evaluating the book on the basis of these four factors will be nearly as accurate as reading the entire work. *Quality and effectiveness of the program* Although there is some controversy concerning the criteria of and effective learning program, achievement test data seem reasonable evidence of what students have learned. The average achievement test scores and the average percent of erroneous responses made by 65 students who worked through the third revision of this program are given below:

	Pre-test mean	*Post-test mean*	*Mean error rate*
Section A	41%	82%	8.2%
Section B	32.6%	79.8%	6.4%

Section A was composed on 31 first-year students enrolled in a community college. Both groups had had some work in trigonometry in high school and both groups were enrolled in "basic" college mathematics. The program was used as a review of material prerequisite to further work in college mathematics.

The estimated time for completion of the program by the average student exclusive of the self-test and the final test is 11 to 15 hours. The average time for students in Section A was 13.7 hours and for students in Section B 12.4 hours.

ACKNOWLEDGMENTS

Mary Haupt Smith aided in the preparation and the editing of the final manuscript. In addition she prepared most of the chapter on circular functions with the help of Myra McFadden. The author and program editors gratefully acknowledge their significant contribution to the final product.

William Hauck

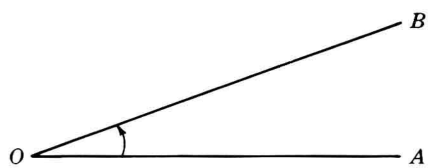
CONTENTS

ONE
Trigonometric Functions

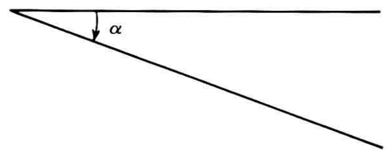
The field of trigonometry deals primarily with the measurement of angles and distances by utilizing geometric figures. Although we assume that you have studied geometry, we shall remind you of some simple concepts related to angles and triangles before you read the programmed material.

ANGLES

In geometry an angle is usually defined as a figure consisting of two straight lines drawn from the same point. A more general concept of angle involves the rotation of a line about a point:

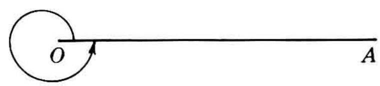


The angle above is generated by rotating line OA counterclockwise about point O to position OB as indicated. The angle formed has O, the point about which the rotation occurred, as a vertex. When angles are named with three letters, the middle letter is always the vertex; thus the angle generated above is named angle AOB. Often an angle is named by a single Greek letter:



Notice from angles AOB and α that an angle may be generated by rotation in either the clockwise or the counterclockwise direction.

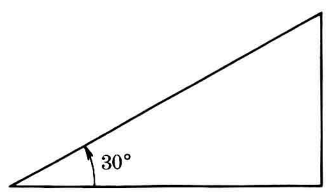
An angle formed by one complete revolution looks like this:



An angle of one degree is formed by rotation through $\frac{1}{360}$ of a complete revolution:

1 degree = $\frac{1}{360}$ revolution

The degree is a unit of measurement of angles and is indicated by the sign °. For example, an angle of thirty degrees is written 30° and is thirty times the size of the angle of 1°, or $\frac{1}{12}$ of a complete revolution:



The degree may be subdivided into minutes and seconds:

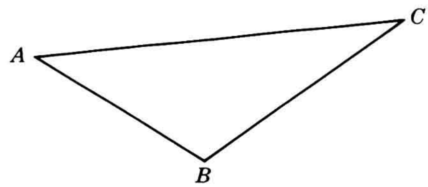
1 degree = 60 minutes

1 minute = 60 seconds

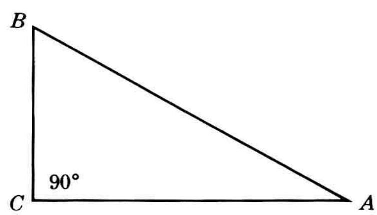
An angle of fifty degrees, four minutes, ten seconds is written 50°4'10''. A right angle is one which contains 90°.

TRIANGLES

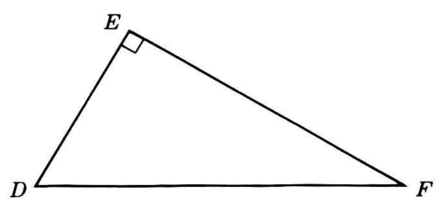
A triangle may be defined as a plane closed figure formed by exactly three straight lines:



A triangle may have sides of any length. The letters used to name a triangle are usually written in alphabetical order; thus the one above is called triangle *ABC*. A right triangle is one which contains one right angle:



A right angle may be indicated by the symbol ⦜. Thus angle *E* is a right angle:



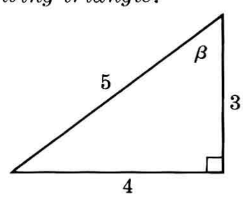
Triangle *DEF* is a right triangle.

Angles and triangles are the elements basic to the study of trigonometric functions, the topic of this section. Every angle has six trigonometric functions. Upon completion of Section One you should be able to write the six functions and to show how they are related.

PRE-TEST

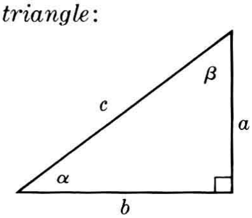
Before beginning the programmed material, take this test to determine what you already know about trigonometric functions. After you complete the test, check your answers with those given in Appendix A. Each item of the test is related to the frames of the programmed material by the frame numbers given with the answers in Appendix A. As you read the text, be sure to concentrate on the frames corresponding to the items you have missed.

1. *Given the following triangle:*



Write all the trigonometric functions of angle β in fraction form.

2. *Consider this triangle:*



Answer in terms of a, b, and c.

a. $\tan \beta \cdot \cot \alpha =$ ________

b. $\tan^2 \alpha =$ ________

c. $\dfrac{1}{\sec \beta} =$ ________

PART A

Two frequently used trigonometric functions of an angle are the sine and the cosine. Learn to compute these two functions and the other four are easily mastered. No matter what the length of the sides of an angle, the trigonometric functions of that angle are the same. The Pythagorean theorem is useful in computing these functions.

NOTE: *Refer to Panel A to answer frames* 1 *to* 7.

Answer	Frame
BG	**1** Consider triangle *ABG*. The side opposite the 30° angle is side _______.
AG	**2** The hypotenuse of triangle *ABG* is side _____.
AB	**3** In triangle *ABG* the side which is not the hypotenuse but which is adjacent to the 30° angle is side _______.
(1) *AC* (2) *AF* (3) *CF*	**4** Give the following sides of triangle *ACF*: (1) The side adjacent to the 30° angle (2) The hypotenuse (3) The side opposite the 30° angle

5 Consider triangle ADE.

(1) Side AD is the side ____________ to the 30° angle.

(2) Side AE is the ____________ of the triangle.

(3) Side DE is the side ____________ the 30° angle.

(1) adjacent
(2) hypotenuse
(3) opposite

6 Consider the triangle ABG and the ratio

$$\frac{\text{side opposite } 30^\circ}{\text{hypotenuse}}$$

The ratio could be computed in this manner:

$$\frac{\text{side opposite } 30^\circ}{\text{hypotenuse}} = \frac{15 \text{ units}}{30 \text{ units}} = \frac{1}{2}, \text{ or } .5$$

Compute the same ratio for the following triangles:

(1) ACF (2) ADE

(1) $\frac{1}{2}$ or .5
(2) $\frac{1}{2}$ or .5

7 No matter what the length of the sides of the right triangle, the ratio

$$\frac{\text{side opposite } 30^\circ}{\text{hypotenuse}}$$

is equal to .5. This ratio is defined as the sine (*pronounced* sīne) of a 30° angle.

The ________ of any 30° angle is .5.

sine (*abbreviated* sin)

right

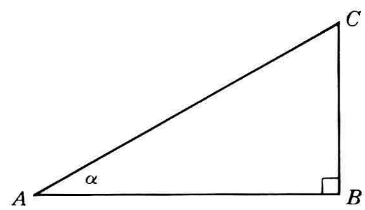
8

$$\sin 30^\circ = \frac{\text{side opposite } 30^\circ}{\text{hypotenuse}}$$

The side opposite the 30° angle and the hypotenuse are sides of a __________ triangle.

(1) BC
(2) AC

9 The sine of any angle is a constant.



$$\sin \alpha = \frac{\text{side opposite } \alpha}{\text{hypotenuse}} = \frac{(1)\ \_\_\_\_\_\_\_\_}{(2)\ \_\_\_\_\_\_\_\_}$$

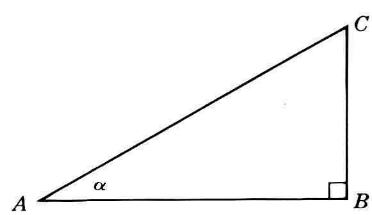
Answer by naming the sides.

.57

10 Given: $AB = 10$

$BC = 7$

$AC = 12.2$



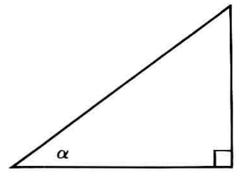
$$\sin \alpha = \frac{\text{side opposite } \alpha}{\text{hypotenuse}}$$

Compute the sine of α to the nearest hundredth.

$\sin \alpha$ ________

side opposite α

11

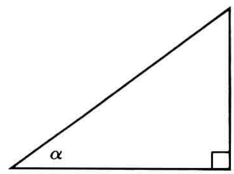


$\sin \alpha = \dfrac{\_\_\_\_\_\_\_\_}{\text{hypotenuse}}$

---

hypotenuse

12

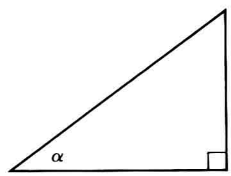


$\sin \alpha = \dfrac{\text{side opposite } \alpha}{\_\_\_\_\_\_\_\_}$

---

sin α

13

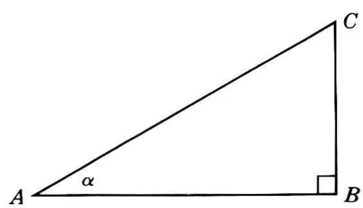


$\_\_\_\_\_\_\_\_ = \dfrac{\text{side opposite } \alpha}{\text{hypotenuse}}$

---

right

14



$\sin \alpha = \dfrac{\text{side opposite } \alpha}{\text{hypotenuse}}$

Triangle *ABC* is a ________ triangle.