

RESISTANCE *of* MATERIALS

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THIRD EDITION

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BY

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RESISTANCE *of*
MATERIALS

WORKS OF FRED B. SEELY, M.S.

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RESISTANCE OF MATERIALS

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PREFACE TO THE THIRD EDITION

In the third edition of this book, as in the previous editions, the main problem dealt with consists of determining the relationships between the loads acting on a member and the resulting stresses and deformations in the member. Throughout the book the aim has been to give strong emphasis to the engineering significance of the subject, and not merely to give emphasis to mathematical methods of analyses.

This book is concerned primarily with the elementary or basic knowledge of the subject of mechanics of materials or resistance of materials, but the nature of basic knowledge in the subject gradually changes with the development of new methods and materials resulting from applications to new needs and service conditions. Changes in the treatment of the subject in this third edition resulting from relatively recent trends, however, have been introduced only to the extent that seemed feasible. It may be well to mention several of the trends that have gained in importance during the twelve years that have elapsed since the second edition of this book appeared, even though they can be introduced in an elementary treatment only to a minor degree.

Lightweight metals, such as aluminum and magnesium, and plastics have become more widely used in load-resisting members, especially in members in which the strength-weight ratio must be large. This trend accompanied by the increased use of members having thin sections has emphasized the importance of buckling as a mode of failure of members.

Likewise the wider use of high-strength metals such as alloy steels has raised the question of the extent to which higher working stresses should be employed, and especially the question of the optimum advantage that can be taken of such materials in relation to the properties of the material such as yield point, ultimate strength, endurance limit, ductility, etc.

The more extensive use of welded connections and of such practices as shot peening and cold rolling of highly stressed surfaces has introduced residual stresses and multiaxial states of

stress as important factors in determining the strength of various members. The increasing use of members under repeated loading as in airplanes, automobiles, railroad rolling stock, and also in railroad tracks and bridges as well as in innumerable types of high-speed equipment has made the problem of stress concentration of extreme importance.

Similarly the use of metals at very low temperatures as occurs in airplanes at great altitudes, in oil-cracking equipment and the like, and the use of metals at very high temperatures as occurs in steam turbines, supercharges, jet-propulsion equipment introduce quite different properties of materials and quite different methods of determining significant stresses from those commonly used in connection with room temperatures.

In the usual problem in resistance of materials, the material is assumed to act only within its elastic range. For certain uses of material, even under ordinary temperatures and static loads, some plastic deformation may occur without damaging the load-carrying capacity of the member, and such plastic deformations may influence the method of calculating the significant stresses. Or to express the idea in other words, some knowledge is needed of the subject of plasticity as well as of elasticity of materials.

The properties of materials which have been employed successfully to indicate satisfactory materials for use at room temperature under static loads and uniformly distributed uniaxial stress may be quite unreliable for indicating whether material is satisfactory for use in another set of conditions involving quite different combinations of temperature, state of stress, distribution of stress, speed of straining, and strain-aging properties of the materials.

Obviously, the foregoing developments and changes in emphasis in the types of problems involving load-resisting members cannot be reflected adequately in an elementary textbook on the mechanics of materials or resistance of materials. In this third edition, however, some attention has been given to a number of these trends with the thought that the instructor or reader can amplify the topics to suit his purposes. This emphasis nevertheless has not changed materially the main topics treated nor the character of the book.

In the third edition, as in the second edition, the book is divided into two parts. Part I consists of the more elementary parts of the subject, and Part II consists of special topics which lie just beyond several of those discussed in Part I. Furthermore, each

topic or chapter in Part II is independent of other topics in Part II so that any chapter in this part may be studied, after the main portion of Part I has been completed, without referring to any other chapter in Part II.

The book has been reset completely; a large number of new problems and new figures have been added. Many minor changes in wording, in arrangement, and in content have been made throughout the book in response to suggestions offered by teachers, students, and practicing engineers who have used the second edition of the book during the past twelve years.

In addition to these minor changes important modifications that have been made may be listed as follows: A decrease in the use made of the double-integration method of determining deflection of beams and a corresponding increase in the use of the area-moment method; an increase in emphasis on the use of Mohr's circle in determining the relation of stresses at a point on different planes passing through the point; a considerable change in the chapter on columns to bring the treatment of this topic more nearly in line with recent reports of technical societies and with present practice; some additional emphasis on stress concentration; and the addition of a brief chapter on the vibrational characteristics on load-resisting members.

The author wishes to express his appreciation of the interest of teachers and others in the second edition of the book as shown by the many comments and helpful criticisms offered.

F. B. SEELY

Urbana, Illinois
November 1946

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Part I. Elementary Topics

CHAPTER I

STRESS AND STRAIN

1. Introduction. Resistance of materials is that branch of mechanics which treats of the internal forces in a physical body and of the changes of shape and size of the body, particularly in their relation to the external forces that act on the body and to the physical properties of the material of the body. The external forces that act on the body are called *loads*; the internal forces, which resist the external forces, are called *stresses*, and the accompanying changes in the dimensions of the body are called *deformations* or *strains*.

The *total stress*¹ on a section through a body is the total internal force acting on the section; that is, if the body is thought of as being divided in two parts by the section, the total stress at the section is the force exerted by one part of the body on the other part in order to hold in equilibrium the loads acting on the part considered. The component of the internal force normal to the area is called *normal stress*, and the component tangent to (or in) the area is called *shearing stress*. Further, a normal stress may be a tensile stress or a compressive stress according to whether the body is stretched or shortened.

Intensity of stress or *unit stress*¹ is defined to be internal force per unit of area. In general the unit stress varies from point to point over a section, its value at any point being considered to be the internal force on an elementary or differential part of the area, including the point, divided by the elementary area; but when the stress is distributed uniformly on an area the unit stress at all points in the section is equal to the total force on the area divided by the whole area. It is important to note that, in general,

¹ In technical literature the term stress sometimes is used to denote what is here defined as intensity of stress or unit stress, and the term internal force is used to denote what is here called total stress.

a unit stress occurs *at a point* in a body *on some plane* passing through the point, and that the most significant unit stresses at the point on the plane considered are the normal and shearing stresses, since these stresses are most closely associated with damage to the material.

A unit stress may be expressed in various units, such as pounds per square inch (lb. per sq. in. or lb./in.²; the abbreviation psi. also is used), kilograms per square centimeter (kg. per sq. cm.), tons per square inch (ton per sq. in.), etc.

The *total deformation* or *total strain* in any direction of a body subjected to loads is the total change in the dimension of the body in that direction, and the *unit deformation* or *unit strain* in any direction is the deformation or strain per unit of length in that direction.

The Problem Defined. A body when subjected to loads is stressed and deformed, and the values of the stresses and the deformations in the body are of great importance in many engineering problems. These stresses are found by use of the general principles of mechanics (mainly of statics) and of the experimental laws that have been found to govern the action of the material. The main objectives, then, in the study of resistance of materials are:

(1) To determine the relation between the external forces (loads) acting on a body and the unit stress at some point in the body on some plane passing through the point, so that the unit stresses caused by known loads may be calculated. The point and plane selected, in general, will be the ones that give the stress which would first cause damage to the material if the loads were increased.

(2) To determine the relation between loads acting on a body and the resulting strains produced in the body so that strains may be determined from known loads, or vice versa.

(3) To obtain a knowledge of the physical properties, such as strength, stiffness, ductility, toughness, resilience, of the various structural materials, since physical properties of the materials are involved in the relations under (1) and (2).

As suggested by the statements under (1) and (2), in some problems of design the stress produced in the body by the load is the governing factor in the design, whereas in other problems the strain (or elastic deflection) produced is the more important factor. For example, the chain of a hoist fulfills its function of lifting loads regardless of the amount of stretch of the chain that occurs, provided that the load does not produce a stress large

enough to cause excessive permanent distortion or to rupture the chain. That is, the stress developed in the chain by the maximum load to be applied is the governing factor in the design of the chain rather than the elastic deformation of the chain. Likewise, stress is the governing factor in the design of many of the parts of buildings, cranes, ships, etc. On the other hand, the maximum load that some members of structures and machines can resist satisfactorily is limited by the permissible elastic deflection of the member rather than by the permissible maximum stress in the member. For example, machine tools such as lathes, drill presses, and boring mills will not produce work of sufficient accuracy if the elastic deformations of the parts are too large.

2. Types of Loading. With reference to the manner in which loads are applied or transmitted to a structure or machine, the loads will be considered under three distinct headings: namely, static loads, repeated loads, and impact and energy loads.

1. *Static, steady, or dead* loads are forces that are applied slowly and not repeated, and remain nearly constant after being applied to the body, or are repeated relatively few times, such as the loads on most buildings or the load applied to a bar in the usual type of testing machine (see Fig. 3).

2. *Repeated* loads are forces that are applied a very large number of times causing a stress in the material that is continually changing, usually through some definite range. The loads applied to the connecting rod of an engine when the engine is running and the wheel loads on a railroad rail as a train passes over the rail are repeated loads. Repeated loads are discussed in Chapter X.

3. *Impact* loads are forces that are applied to the resisting body in a relatively short period of time. An impact load, in general, is applied by a body that is in motion when it comes in contact with the resisting body, and the force exerted by the moving body and the period during which it acts in general cannot be determined. For this reason in some problems it is more satisfactory to calculate the stress and strain produced by an impact load from the energy delivered to the resisting body by the moving body. When this is done the energy delivered to the resisting body is called an *energy load* and is expressed in foot-pounds (not in pounds). Impact and energy loads are considered in Chapter XI.

Other Classifications of Loads. Loads may be classified as *distributed* loads and *concentrated* loads. A distributed load may be uniformly distributed or non-uniformly distributed. Thus, if sand

is spread on a floor so that its depth is constant, the floor is subjected to a uniformly distributed load, whereas, if the sand is distributed so that its depth is not constant, the floor is said to carry a non-uniformly distributed load. A concentrated load is one whose area of contact with the resisting body is negligible in comparison with the area of the resisting body.

The more common types of loads to which structural and machine members are subjected may be classified as *central* loads, *torsional* loads, and *bending* loads. A body may be subjected simultaneously to loads of any two or to all three of these types, however.

The stresses and strains caused by static central loads are discussed in this chapter. The stresses and strains caused by static torsional loads are considered in Chapter III, and those produced by static bending loads in Chapter IV.

Static Central Loads

3. Stresses Due to Central Loads. A central load is a concentrated load whose action line passes through the centroid of the area on which the stresses are to be considered, or a distributed load whose resultant passes through the centroid of the area. The stress produced by a central load may be any one of the three types, tensile stress, compressive stress, or shearing stress, but the distinguishing feature of a central load that acts on a straight bar of constant cross section as compared to torsional and bending loads is that the stress in the bar may be assumed to be uniformly distributed over the cross-sectional area; that is, the intensity of stress (unit stress) may be assumed to be constant. If a central load acts in a direction normal to the area on which the stress is to be found, it is called an *axial* load and it may be a tensile or a compressive axial load according to whether it produces tensile or compressive stress; if the central load lies in the plane of the area it is called a shearing central load. The compressive stress exerted by one body on another (different) body at the area of contact usually is called a *bearing* stress.

Stresses due to central loads may be found as follows: Fig. 1a represents a straight bar AB subjected to an axial tensile load P causing tensile stress on any section of the bar perpendicular to the loads; Fig. 1c represents a bar in compression under the action of an axial load P causing compressive stress in the bar; and