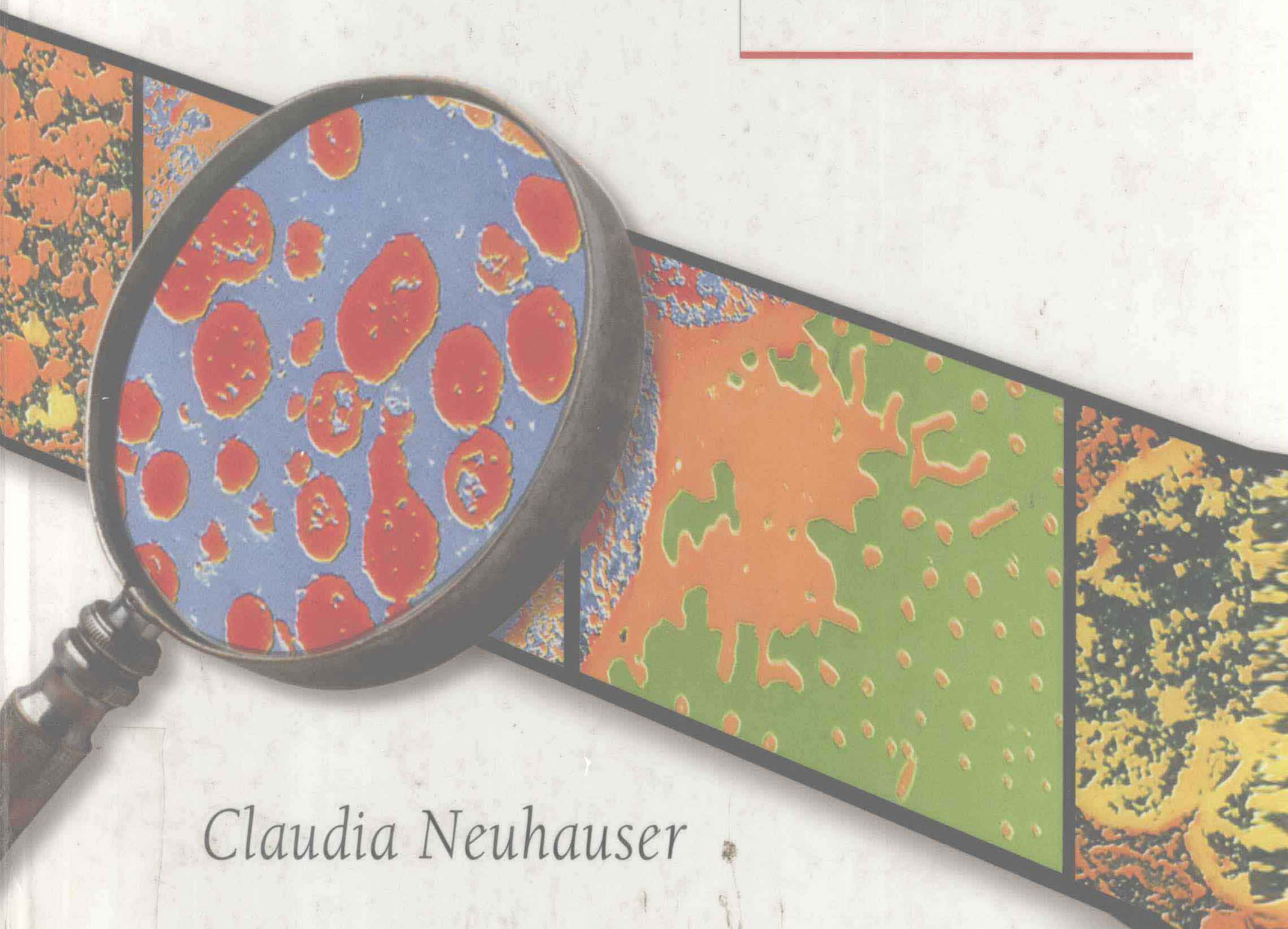
CALCULUS for BIOLOGY and MEDICINE



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Calculus for Biology and Medicine

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Preface

For modeling and analysis of phenomena in the life sciences, calculus is an indispensable tool. This is not obvious in a traditional calculus course, where biology students rarely see how the material is relevant to their training. This text is written exclusively for students in the biological and medical sciences. It makes an effort to demonstrate from the beginning how calculus can help them to understand phenomena in nature. Students find it easier to understand concepts in mathematics if they are related to their field, and this makes mathematics more interesting.

This text differs in a number of ways from traditional calculus texts. First, it is written in a life-science context; concepts are motivated with biological examples, emphasizing that calculus is an important tool in the life sciences. Second, differential equations, one of the most important modeling tools in the life sciences, are introduced very early, immediately after the formal definition of derivatives. Third, biological applications of differentiation and integration are included throughout the text. Fourth, two chapters deal exclusively with differential equations and systems of differential equations; both chapters contain numerous up-to-date applications. Fifth, multivariable calculus is taught in the first year, recognizing that most students in the life sciences will not take the second year of calculus, and that multivariable calculus is needed to analyze systems of differential equations, which they will encounter in their science courses.

This text does not teach modeling; the objective is to teach calculus. Modeling is an art that should be taught in a separate course. However, throughout the text, students encounter mathematical models for biological phenomena; this will facilitate the transition to actual modeling.

Examples Each topic is motivated with biological examples, followed by a thorough discussion outside of the life-science context, to enable students to become familiar with both the meaning and the mechanics of the topic. Examples in the text are completely worked out; steps in calculations are frequently explained in words.

Problems Calculus cannot be learned by watching someone do it. Recognizing this, students are provided with both drill and word problems. Word problems are an integral part of teaching calculus in a life-science context. The word problems are new and up-to-date; they are adapted from either standard biology texts or original research. Because this text is written for college freshmen, the examples were chosen so that no formal training in biology is needed.

Technology This book takes advantage of graphing calculators, which allows students to develop a much better visual understanding of the concepts in calculus. Beyond that, no special software is required.

Chapter Summary

Chapter 1 Basic tools from algebra and trigonometry are summarized in Section 1.1. Section 1.2 contains the basic functions used in text, including exponential and logarithmic functions; their graphical properties and biological relevance are emphasized. Section 1.3 covers log-log and semi-log plots; these are graphical tools that are frequently used in the life sciences.

Chapter 2 Limits and continuity are key concepts for understanding the conceptual parts of calculus. Visual intuition is emphasized before the theory is discussed.

Chapter 3 The geometric definition of a derivative as the slope of a tangent line is given before the formal treatment. After the formal definition of the derivative, differential equations are introduced as models for biological phenomena. Differentiation rules are discussed. These sections give students time to acquaint themselves with the basic rules of differentiation before applications are discussed. Related rates and error propagation, in addition to differential equations, are the main applications.

Chapter 4 This chapter presents biological and more traditional applications of differentiation. Many of the applications are consequences of the mean value theorem. Many of the word problems are adapted from either biology textbooks or original research articles; this puts the traditional applications (such as extrema, monotonicity, and concavity) in a biological context.

Chapter 5 Integration is motivated geometrically. The fundamental theorem of calculus and its consequences are discussed in depth. Both biological and traditional applications of integration are provided before integration techniques are covered.

Chapter 6 This chapter contains integration techniques. However, only the most important techniques are covered. Tables of integrals are used to integrate more complicated integrals. The use of computer software is not covered in the text, though their usefulness in evaluating integrals is acknowledged.

Chapter 7 This chapter provides an introduction to differential equations. The treatment is not complete, but it will equip students with both analytical and graphical skills for analysis. Eigenvalues are introduced early, to facilitate the analytical treatment of systems of differential equations in Chapter 11. Many of the differential equations discussed in the text are important models in biology. Though this text is not a modeling text, students will see how differential equations are used to model biological phenomena, and will be able to interpret differential equations. Chapter 7 contains a large number of up-to-date applications of differential equations in biology.

Chapter 8 This chapter contains additional applications of integration, which help students to understand the importance of integrals. Unless Chapter 12 is included, some or all sections of this chapter can be omitted. (Sections 8.2–4 should be discussed before Chapter 12.)

Chapter 9 Matrix algebra is an indispensible tool for every life scientist. The material in this chapter covers the most basic concepts, tailored to Chapters 10 and 11, where matrix algebra is frequently used. Special emphasis is given to the treatment of eigenvalues and eigenvectors because of their importance in analyzing systems of differential equations.

Chapter 10 This is an introduction to multidimensional calculus. The treatment is brief and tailored to Chapter 11, where systems of differential equations are discussed. The main topics are partial derivatives and linearization of vector-valued functions. The discussion of gradient and diffusion are not required for Chapter 11.

Chapter 11 This material is most relevant for students in the life sciences. Both graphical and analytical tools are developed to enable students to analyze systems of differential equations. The material is divided into linear and nonlinear systems. Understanding the stability of linear systems in terms of vector fields, eigenvectors, and eigenvalues helps students to master the more difficult analysis of nonlinear systems. Theory is explained before applications are given—this allows students to become familiar with the mechanics before delving into applications. An extensive problem set allows students to experience the power of this modeling tool in a biological context.

Chapter 12 This chapter contains basic probabilistic and statistical tools. It cannot replace a full semester course in probability and statistics, but it allows students to see some of the concepts needed in population genetics and experimental design.

How to Use This Book

This book contains more material than can be covered in one year. This was deliberate, and allows for more flexibility in the choice of material. Sections that are noted by asterisks in the table of contents can be omitted; their material is not needed in subsequent sections.

The material can be arranged to suit a one-semester, two-quarter, two-semester, or four-quarter course. Chapters 1–3 must be covered in that order before any of the other sections are covered. In addition to Chapters 1–3, the following arrangements can be chosen:

One semester, emphasis on integration 4.1, 4.2, 4.3, 4.4, 4.5, 4.7, 5.1, 5.2, 5.3 (without 5.3.4), 6.1

One semester, emphasis on differential equations 4.1, 4.2, 4.3, 4.4, 4.5, 4.7, 5.1, 5.2, 7.1 (without 7.1.2, 7.1.3), 7.2

One semester, emphasis on probability 4.1, 4.2, 4.3, 4.4, 4.5, 4.7, 5.1, 5.2, 12.1, 12.2, 12.3, 12.4

Two quarters 4.1, 4.2, 4.3, 4.4, 4.5, 4.7, 5.1, 5.2, 5.3, 6.1, 6.2, 6.3, 6.4, 6.5, 6.7, 7.1, 7.2, 7.3 (one of the subsections), 8.1, 8.2

Two semesters 4.1, 4.2, 4.3, 4.4, 4.5, 4.7, 5.1, 5.2, 5.3, 6.1, 6.2, 6.3, 6.4, 6.5, 6.7, 7.1, 7.2, 7.3 (one of the subsections), 9.1, 9.2 (without 9.2.4), 9.3, 9.4, 10.1, 10.2, 10.3, 10.4, 11.1, 11.2, 11.3, 11.4 (two of the subsections)

Four quarters All sections that are not labeled optional; optional sections should be chosen as time permits

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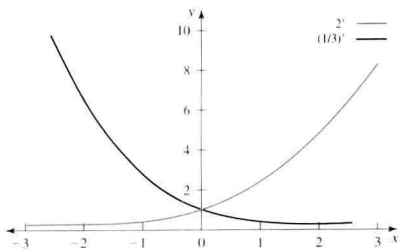

CHAPTER

Preview and Review

Isaac Newton (1642–1727) and Gottfried Wilhelm Leibniz (1646–1716) are typically credited with the invention of calculus. They were not the sole inventors but rather the first to systematically develop it.

Calculus is divided into two parts, differential and integral calculus. Historically, the problems that led to the development of differential calculus were concerned with finding tangent lines to curves, and extrema (i.e., maxima and minima) of curves. Integral calculus, on the other hand, has its roots in attempts to determine the areas of regions bounded by curves, or the volumes of solids. It turns out that the two parts of calculus are closely related; in fact, the basic operation of one can be considered the inverse of the other. This result is known as the Fundamental Theorem of Calculus and goes back to Newton and Leibniz. They were the first to understand the meaning of this inverse relationship, and to put this relationship to use in solving difficult problems.

Finding tangents, extrema, and areas are very basic problems, which led to the development of methods that are useful in solving a wide range of scientific problems. For this reason, calculus has been one of the most powerful tools in the mathematical formulation of scientific concepts. For example, many physical laws and many phenomena in biology are formulated in the language of calculus.

In addition to developing the theory of differential and integral calculus, we will consider many examples in which calculus is used to describe or model situations in the biological sciences. The use of mathematics is becoming increasingly more important in biology, for instance, when modeling interactions between species in a community, describing neuron activities, explaining genetic diversity in populations, predicting the impact of global warming on vegetation, and so on.

1.1 PRELIMINARIES

We provide a brief review of some of the concepts and techniques from precalculus that are frequently used in calculus. (The problems at the end of this section will help you to reacquaint yourself with this material.)