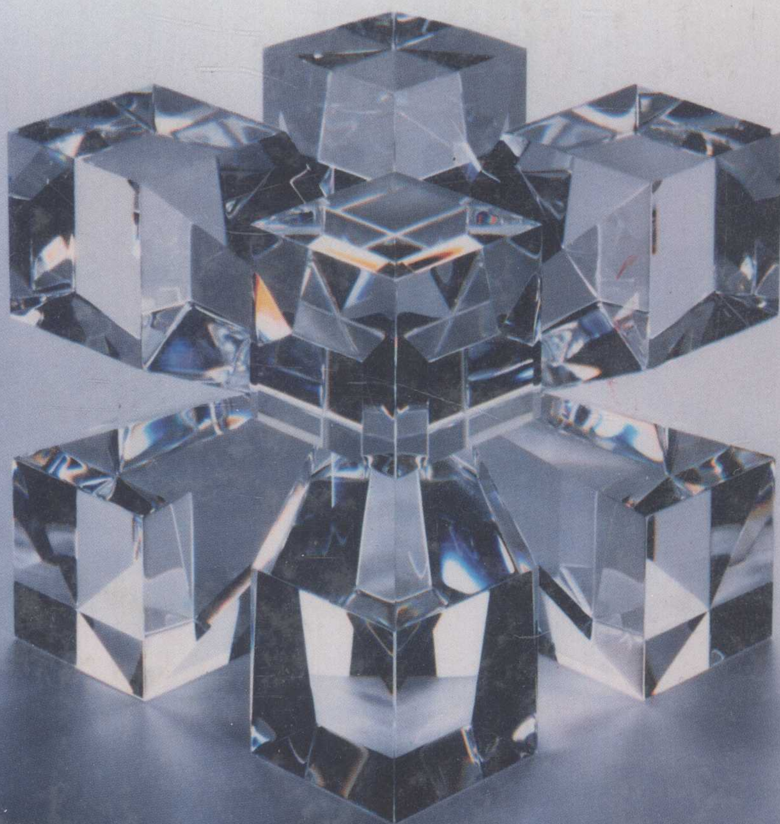
ALTERNATE SECOND EDITION

CALCULUS and Analytic Geometry



Edwards and Penney

Calculus and Analytic Geometry

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C. H. Edwards, Jr. and David E. Penney

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Preface

Calculus is one of the supreme accomplishments of the human intellect. Many of the scientific discoveries that have shaped our civilization during the past three centuries would have been impossible without the use of calculus. Today this body of computational technique continues to serve as the principal quantitative language of science and technology.

We prepared this revision with the goal of making the riches of calculus more attractive and understandable to the increasing number of men and women who take the standard calculus course for science, mathematics, and engineering students. This edition (like its predecessors) was written with five related objectives in constant view: *concreteness*, *readability*, *motivation*, *applicability*, and *accuracy*.

CONCRETENESS

The power of calculus is impressive in its precise answers to realistic questions and problems. In the necessary development of the theory, we keep in mind the central question: How does one actually *compute* it? We place special emphasis on concrete examples, applications, and problems that serve both to highlight the development of the theory and to demonstrate the remarkable versatility of calculus in the investigation of important scientific questions.

READABILITY

Difficulties in learning mathematics often are complicated by language difficulties. Our writing style stems from the belief that crisp exposition, both intuitive and precise, makes mathematics more accessible—and hence more readily learned—with no loss of rigor. We hope our language is clear and attractive to students and that they can and actually will read it, thereby enabling the instructor to concentrate class time on the less routine aspects of teaching calculus.

MOTIVATION

Our exposition is centered around examples of the use of calculus to solve real problems of interest to real people. In selecting such problems for our examples and exercises, we took the view that stimulating interest and motivating effective study go hand in hand. We attempt to make it clear to students how the knowledge gained with each new concept or technique will be worth the effort expended. In theoretical discussions, especially, we try to provide an intuitive picture of the goal before we set off in pursuit of it.

APPLICATIONS

Its diverse applications are what attract many students to calculus, and realistic applications provide valuable motivation and reinforcement for all students. Section 1-1 contains a list of twenty *sample* applications that the student can anticipate for later study. This list illustrates the unusually broad range of applications that we include, but it is neither necessary nor desirable that the course cover all the applications in the book. Each section or subsection that may be omitted without loss of continuity is marked with an asterisk. This provides flexibility for each instructor to steer his or her own path between theory and applications.

ACCURACY

To help ensure authoritative and complete coverage of calculus, both this edition and its predecessors were subjected to a comprehensive reviewing process. With regard to the selection, sequence, and treatment of mathematical topics, our approach is traditional. With regard to the level of rigor, we favor an intuitive and conceptual treatment that is careful and precise in the formulation of definitions and the statements of theorems. Some proofs that may be omitted at the discretion of the instructor are placed at the ends of sections. Others (such as the proofs of the intermediate value theorem and of the integrability of continuous functions) are deferred to the book's appendices. In this way we leave ample room for variation in seeking the proper balance between rigor and intuition.

SPECIAL FEATURES OF THIS EDITION

Substantial portions of this alternate edition are based on our *Calculus and Analytic Geometry*, second edition (1986). The present edition is intended for use in those courses where a later introduction of trigonometric functions is desired. Here the calculus of trigonometric functions is delayed until Chapter 8. Other changes for this revision include an earlier introduction to the chain rule (in Section 3-3) for use in differentiating algebraic functions, and a more intuitive introduction to exponential and logarithmic functions (in Section 7-1).

Trigonometric Functions A review of the elementary trigonometry needed for calculus has been inserted in Section 8-1, preceding the first appearance of trigonometric limits in Section 8-2. Derivatives and integrals of trigonometric functions first appear in Sections 8-3 and 8-4.

The paragraphs that follow describe features that this alternate edition shares with the second edition.

Additional Problems This edition contains over 6000 problems. Most of the new problems are drill or practice exercises. They have been inserted mainly at the beginnings of problem sets to insure that students gain sufficient confidence and computational skill before moving on to less routine problems.

New Examples and Computational Details In many sections throughout the book, we have inserted a simpler first example as an initial illustration of the main ideas of the section. Moreover, we have inserted an additional line or two of computational detail in many of the worked-out examples to make them easier for student readers to follow.

Split Sections We divided a number of the longer sections in the first edition into two sections for this revision. For instance, each of the following *pairs* of sections corresponds to a single original section: Sections 1-2 and 1-3 (real numbers and functions), Sections 2-1 and 2-5 (limits), Sections 3-5 and 3-6 (maxima and minima), Sections 4-4 and 4-5 (the first derivative test and graphs of polynomials), Sections 5-4 and 5-5 (evaluation of integrals and the fundamental theorem of calculus), Sections 11-1 and 11-2 (indeterminate forms and l'Hopital's rule), Sections 14-4 and 14-5 (space curves and curvature), Sections 15-2 and 15-3 (functions of several variables and limits), Sections 16-1 and 16-2 (double integrals), and Sections 17-2 and 17-3 (line integrals). In each case the separation of sections enabled us to add more explanatory discussions for the benefit of the student.

Optional Computer Applications We have included twenty-one optional programming notes for supplementary reading by those students who might be motivated by computer applications. Each of these notes appears at the end of a section (following the problems) and applies very simple BASIC programming to illustrate the ideas of the section. These programming notes are completely optional—we never assume that any have been included in the calculus course, and we never refer to them in the text proper. Their purpose is to stimulate interest in calculus in the rapidly increasing population of students who are already interested in computers. Those who would like to explore this topic further may consult Edwards: *Calculus and the Personal Computer* (Englewood Cliffs, N.J.: Prentice-Hall, 1986) for self-study or as a computer calculus laboratory text.

Introductory Chapters The initial chapter of the first edition has been divided into two shorter chapters for this edition. This permits the inclusion of more review material and a slightly slower pace at the beginning of the course. But we still retain the objective of a quick start on calculus itself. Section 1-6 gives a first look at the derivative, and it serves to motivate the formal treatment of limits in Chapter 2.

Differentiation Chapters We have substantially reordered the sequence of topics on differentiation in Chapters 3 and 4. Our objective is to build student confidence by introducing topics more nearly in order of increasing difficulty. We cover the basic techniques for differentiating algebraic functions in Sections 3-2 through 3-4 before discussing maxima and minima in Sections 3-5 and 3-6. Section 3-9 on Newton's method has been simplified. The mean value theorem and its applications are deferred to Chapter 4. All curve-sketching techniques now appear consecutively in Sections 4-5 through 4-7. In Section 4-8 we have all but eliminated the alternative D^{-1} notation for antiderivatives that appeared in the first edition.

Integration Chapters The proof of the fundamental theorem of calculus in Section 5-5 is preceded by an intuitive treatment in Section 5-4. We have also inserted a number of additional and simpler examples in Chapters 5 and 6, as well as in Chapter 9 (techniques of integration).

Infinite Series and Taylor's Formula Taylor's formula and polynomial approximations appear in Section 11-3. The extension to Taylor series is now delayed until Section 12-7 in the chapter on infinite series.

Analytic Geometry and Vectors Vectors in the plane and vectors in space now appear in the consecutive Chapters 13 and 14; these are now easy to combine as a single unit if the instructor so wishes. We have augmented substantially the discussion of vector fields in Section 17-1.

Differential Equations Sections 7-6 and 7-8 introduce the very simplest separable differential equations and their impressive applications. Nevertheless, these are optional sections, and the instructor may delay them until Chapter 18 (on differential equations) is covered. Chapter 18 has been revised substantially—it now ends with Section 18-7 on elementary power series methods and Section 18-8 on elementary numerical methods. Some of the lengthier applications have been deleted but are now included in Edwards and Penney, *Elementary Differential Equations with Applications* (Englewood Cliffs, N.J.; Prentice-Hall, 1985).

Computer Graphics The ability of students to visualize surfaces and graphs of functions of two variables should be enhanced by the IBM-PC graphics that appear in Chapters 14 and 15. For these excellent computer graphics, fully integrated with text discussions, we are indebted to John K. Edwards. He developed and programmed them using the APL*PLUS/PC System from STSC, Inc. (with the exception of Figures 15.22–15.33, for which he used the PLOT-CALL System from Golden Software, Inc.).

ANSWERS AND MANUALS

Answers to most of the odd-numbered problems appear in the back of the book. Solutions to most problems (other than those odd-numbered ones for which an answer alone is sufficient) are available in an Instructor's Manual. A subset of this manual, containing solutions to problems numbered 1, 4, 7, 10, . . . is available as a Student Manual. The statements of Problems 1, 4, 7, 10, . . . , in the first twelve chapters, followed by their solutions and renumbered conventionally, are available as *Worked Problems in Calculus*. A collection of some 1400 additional problems suitable for use as test questions, *Calculus Test Item File*, is available for use by instructors.

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Many of our best improvements must be credited to colleagues and users of the first edition throughout the country (and abroad). We are grateful to all those, especially students, who have written to us, and hope that they will continue to do so. We also believe that the quality of the finished book itself is adequate testimony to the skill, diligence, and talent of an exceptional staff at Prentice Hall; our special thanks go to David Ostrow, mathematics editor; Maria McColligan production editor; Walter A. Behnke and Anne T. Bonanno, designers; and Eric G. Hieber, illustrator. Finally, we cannot adequately thank Alice Fitzgerald Edwards and Carol Wilson Penney for their continued assistance, encouragement, support, and patience.

Athens, Georgia

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1

Prelude to Calculus

1-1

Introduction

We live in a world of ceaseless change, filled with bodies in motion and with phenomena of ebb and flow. The principal object of the body of computational methods known as **calculus** is the analysis of problems of change and motion. This mathematical discipline stems from the seventeenth-century investigations of Isaac Newton (1642–1727) and Gottfried Wilhelm Leibniz (1646–1716). Many (if not most) of the scientific discoveries that have shaped our civilization during the past three centuries would have been impossible without the use of calculus, and today it continues to serve as the principal quantitative language of science and technology. We list below some of the problems that you will learn to solve as you study this book. The first two problems will be discussed in this chapter, and the others will be covered in later chapters.

1 *The Fence Problem.* What is the maximum rectangular area that can be enclosed with a fence of perimeter 140 m?

2 *The Refrigerator Problem.* The manager of an appliance store buys refrigerators at a wholesale price of \$250 each. On the basis of past experience, the manager knows she can sell 20 refrigerators each month at \$400 each and an additional refrigerator each month for each \$3 reduction in selling price. What selling price will maximize the monthly profit of the store?

3 A cork ball of specific gravity $\frac{1}{4}$ is thrown into water. How deep will it sink? (Section 3-9)

4 What is the maximum possible radius of a "black hole" with the same mass as the sun? (Section 4-9)

5 If you have enough spring in your legs to jump straight up 4 ft on the earth, could you blast off under your own power from an asteroid of diameter 3 mi? (Section 4-9)

6 How much power must a rocket engine produce in order to put a satellite into orbit around the earth? (Section 6-5)

7 If the population of the earth continues to grow at its present rate, when will there be standing room only? (Section 7-5)

8 Suppose that you deposit \$100 each month in a savings account that pays 7.5% interest compounded continuously. How much will you have in the bank after 10 years? (Section 7-5)

9 The factories polluting a certain lake are ordered to cease immediately. How long will it take for natural processes to restore the lake to an acceptable level of purity? (Section 7-6)

10 According to newspaper accounts, it is possible to survive a free fall (without parachute) from a height of 20,000 ft. Can this be true? (Section 7-6)

11 How can a pendulum clock be used to determine the altitude of a mountain peak? (Section 8-6)

12 What is the best shape for the reflector in a solar heater? (Section 10-4)

13 How often can a fixed dose of a drug be administered without producing a dangerous level of the drug in the patient's bloodstream? (Section 12-3)

14 How do we know that $\pi = 3.14159265\ldots$? (Section 12-7)

15 At what angle should the curve on a race track be banked to best accommodate cars traveling at 150 mi/h? (Section 13-5)

16 How does a satellite of the earth use its thrusters to transfer from one circular orbit to another? (Section 13-7)

17 Does a baseball pitch actually curve, or is it some sort of optical illusion? (Section 14-4)

18 What temperature can a mercury thermometer withstand before its bulb bursts? (Section 15-7)

19 How can two companies that make the same product conspire to maximize their total profits? (Section 15-11)

20 A coin, a hoop, and a baseball roll down a hill. Which will reach the bottom first? (Section 16-5)

This first chapter contains some review material and material preliminary to your study of calculus: real numbers and inequalities, functions and their graphs, straight lines and slopes, and equations of circles and of parabolas. In Section 1-6 we introduce (quite informally) *limits* of functions and the problem of finding tangent lines to curves. This leads to the key concept of the *derivative* of a function, which can be used to solve problems such as the first two just given. This preliminary discussion is intended to motivate the more detailed and formal treatment of limits in Chapter 2 and of derivatives in Chapter 3.

1-2 Real Numbers

The **real numbers** are already familiar to you; they are just those numbers ordinarily used in most measurements. The mass, speed, temperature, and charge of a body are measured by real numbers. Real numbers can be represented by **terminating** or **nonterminating** decimal expansions. Any terminating decimal can be written in nonterminating form by adding zeros:

$$\tfrac{3}{8} = 0.375 = 0.375000000\ldots.$$

Any **repeating** nonterminating decimal, such as

$$\tfrac{7}{22} = 0.31818181818\ldots,$$

represents a **rational** number, one that is the quotient of two integers. Conversely, every rational number is represented by a repeating decimal expansion (as displayed above). The decimal expansion of an **irrational** number (one that is not rational), such as

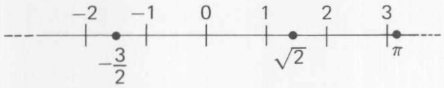
$$\sqrt{2} = 1.414213562\ldots$$

or

$$\pi = 3.141592653589793\ldots,$$

is both nonterminating and nonrepeating.

The geometric representation of real numbers as points on the **real line** $\mathscr{R}$ should also be familiar to you. Each real number is represented by precisely one point of $\mathscr{R}$, and each point of $\mathscr{R}$ represents precisely one real



1.1 The real line $\mathscr{R}$

number. By convention, the positive numbers lie to the right of zero and the negative numbers to its left, as indicated in Fig. 1.1.

If the real number a lies (on $\mathscr{R}$) to the left of the number b, then we write $a < b$ (or $b > a$) and say that a is *less than* b and that b is *greater than* a. The number a is *positive* if $a > 0$; if $a < 0$, then a is said to be *negative*. The following properties of inequalities of real numbers are fundamental and are often used.

$$\begin{aligned} &\text{If } a < b \text{ and } b < c, \text{ then } a < c. \\ &\text{If } a < b \text{ then } a + c < b + c. \\ &\text{If } a < b \text{ and } c > 0, \text{ then } ac < bc. \\ &\text{If } a < b \text{ and } c < 0, \text{ then } ac > bc. \end{aligned} \tag{1}$$

The last two statements mean that an inequality is preserved when its members are multiplied by a *positive* number but *reversed* when they are multiplied by a *negative* number.

In the problems at the end of this section, we ask you to deduce from the properties in (1) that the sum of two positive numbers is positive, while the sum of two negative numbers is negative; the product of two positive numbers *or* of two negative numbers is positive, while the product of a positive number and a negative number is negative. Moreover,

$$a > b \quad \text{if and only if} \quad a - b > 0. \tag{2}$$

ABSOLUTE VALUE

The (nonnegative) distance along the real line between zero and the real number a is the **absolute value** of a, written $|a|$. Equivalently,

$$|a| = \begin{cases} a & \text{if } a \geqq 0; \\ -a & \text{if } a < 0. \end{cases} \tag{3}$$

The notation $a \geqq 0$ means that a is *either* greater than zero *or* equal to zero. Note that (3) implies that $|a| \geqq 0$ for every real number a, while $|a| = 0$ if and only if $a = 0$. For example,

$$|4| = 4, \qquad |-3| = 3, \qquad |0| = 0, \quad \text{and} \quad |\sqrt{2} - 2| = 2 - \sqrt{2},$$

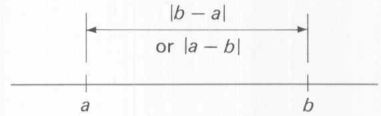
the latter being true because $2 > \sqrt{2}$. Thus $\sqrt{2} - 2 < 0$, and hence

$$|\sqrt{2} - 2| = -(\sqrt{2} - 2) = 2 - \sqrt{2}.$$

The following properties of absolute values are frequently used.

$$\begin{aligned} &|a| = |-a| = \sqrt{a^2} \geqq 0, \\ &|ab| = |a|\,|b|, \\ &-|a| \leqq a \leqq |a|, \\ &|a| < b \quad \text{if and only if} \quad -b < a < b, \\ &|a| = b \geqq 0 \quad \text{if and only if} \quad a = b \text{ or } a = -b, \qquad \text{and} \\ &|a| > b \geqq 0 \quad \text{if and only if} \quad a < -b \text{ or } a > b. \end{aligned} \tag{4}$$

The **distance** between the real numbers a and b is defined to be $|a - b|$ — or, if you prefer, $|b - a|$, which is equal to $|a - b|$. This distance is simply the length of the line segment of the real line $\mathscr{R}$ with endpoints a and b, as indicated in Fig. 1.2.



1.2 The distance between a and b

The properties of inequalities and of absolute values listed above imply the following important fact.

Triangle Inequality $\quad |a + b| \leqq |a| + |b|. \qquad (5)$

Proof We add the inequalities $-|a| \leqq a \leqq |a|$ and $-|b| \leqq b \leqq |b|$. This gives us

$$-(|a| + |b|) \leqq a + b \leqq |a| + |b|. \tag{6}$$

But the properties in (4) imply that

$$|c| \leqq d \quad \text{if and only if} \quad -d \leqq c \leqq d. \tag{7}$$

We set $c = a + b$ and $d = |a| + |b|$, and it follows that (5) and (6) are equivalent. ∎

INTERVALS

Suppose that S is a set (collection) of real numbers. Then we write $x \in S$ if and only if x is an element of S. The set S may be described by means of the notation

$$S = \{x : \cdots\},$$

where the ellipsis represents a condition that the real number x satisfies exactly when x belongs to S. The most important sets of real numbers in calculus are **intervals.** If $a < b$, then the **open interval** (a, b) is defined to be the set

$$(a, b) = \{x : a < x < b\}$$

of real numbers, and the **closed interval** $[a, b]$ is

$$[a, b] = \{x : a \leqq x \leqq b\}.$$

Thus a closed interval contains its endpoints, while an open interval does not. We also use the **half-open intervals**

$$[a, b) = \{x : a \leqq x < b\}$$

and

$$(a, b] = \{x : a < x \leqq b\}.$$

Thus the open interval $(1, 3)$ is the set of those real numbers x such that $1 < x < 3$; the closed interval $[-1, 2]$ is the set of those real numbers x such that $-1 \leqq x \leqq 2$; and the half-open interval $(-1, 2]$ is the set of those real numbers x such that $-1 < x \leqq 2$. In Fig. 1.3 we show examples of such intervals, as well as some **unbounded intervals,** which have forms such as

$$[a, \infty) = \{x : x \geqq a\},$$
$$(-\infty, a] = \{x : x \leqq a\},$$
$$(a, \infty) = \{x : x > a\}, \qquad \text{and}$$
$$(-\infty, a) = \{x : x < a\}.$$

1.3 Some examples of intervals of real numbers