

**Third Edition**

**James M. Henderson  
Richard E. Quandt**

**Micro-  
economic  
Theory**

**A Mathematical  
Approach**

# MICROECONOMIC THEORY

A Mathematical Approach

---

Third Edition

**James M. Henderson**

*Professor of Economics  
University of Minnesota*

**Richard E. Quandt**

*Professor of Economics  
Princeton University*

**McGraw-Hill Book Company**

New York St. Louis San Francisco Auckland Bogotá Hamburg  
Johannesburg London Madrid Mexico Montreal New Delhi  
Panama Paris São Paulo Singapore Sydney Tokyo Toronto

## **MICROECONOMIC THEORY**

### **A Mathematical Approach**

Copyright © 1980, 1971, 1958 by McGraw-Hill, Inc. All rights reserved.  
Printed in the United States of America. No part of this publication  
may be reproduced, stored in a retrieval system, or transmitted, in any  
form or by any means, electronic, mechanical, photocopying, recording, or  
otherwise, without the prior written permission of the publisher.

1 2 3 4 5 6 7 8 9 0 FGFG 8 9 8 7 6 5 4 3 2 1 0

This book was set in Times Roman. The editors were Bonnie E. Lieberman  
and Frances A. Neal; the production supervisor was Leroy A. Young.  
New drawings were done by J & R Services, Inc.  
Fairfield Graphics was printer and binder.

#### **Library of Congress Cataloging in Publication Data**

Henderson, James Mitchell, date  
Microeconomic theory.

Includes index.

1. Microeconomics 2. Economics, Mathematical.

I. Quandt, Richard E., joint author. II. Title.

HB171.5.H424 1980 330'.01'51 79-22064

ISBN 0-07-028101-7.

---

# PREFACE TO THE THIRD EDITION

---

The rapid growth of knowledge in microeconomics has made it desirable to revise *Microeconomic Theory* a second time. This has also provided an opportunity for making a number of improvements that were suggested by colleagues and students over the years.

The level of mathematics remains basically unchanged. An attempt has been made to improve the exposition of optimization in the Appendix, and to make the mathematical treatment of various topics conform more closely to modern practice. This has led, for example, to more emphasis on the concept of quasi-concavity in the treatment of utility functions, and to less emphasis on difference equations in the treatment of dynamic problems. As before, calculus is the basic mathematical tool. Readers are urged to refresh their memories about the mathematics by reading the Appendix and working its exercises before beginning Chapter 2.

The changed importance of some topics has led to a reorganization of chapters. The theory of the consumer, the theory of the firm, and the analysis of multimarket equilibrium each now occupies two chapters. Chapters 2 and 4 deal with the basic theories of the consumer and the firm, respectively, and Chapters 3 and 5 are devoted to extensions of each. The elements of multimarket equilibrium are covered in Chapter 9, with questions of existence and stability of equilibrium the subject of Chapter 10. The second edition's chapter on linear models has been eliminated and some of its material introduced into other chapters on the basis of its logical relation to the topics of those chapters. Thus, linear-programming approaches to the theory of the firm are now covered in Chapter 5, the theory of games in Chapter 8, and

input-output analysis in Chapter 10. A number of new exercises have been added, and solutions for even-numbered exercises are now provided at the end of the volume.

Among the new topics for the third edition are the duality theory of the consumer (Chapter 3) and producer (Chapter 5), the concept of risk aversion (Chapter 3), production under uncertainty (Chapter 5), futures markets (Chapter 6), cooperative games and the Nash bargaining solution (Chapter 8), the introduction of money into the utility function (Chapter 9), Lindahl equilibrium and equity (Chapter 11), and exhaustible resources and human capital (Chapter 12). A number of other extensions are included where appropriate.

Suggestions for improvements have been made by many colleagues and students who have often helped us to a better understanding of difficult points in microeconomic theory. In particular we remember the suggestions or assistance of Harvey S. Rosen, Hugo Sonnenschein, and Michael Spence; Meir Barnea read through the entire manuscript and offered many valuable suggestions. To them and many others whom we have not explicitly mentioned we express our deep thanks.

We hope that few new errors have been introduced in this edition. As before, our work is thoroughly intermingled and we take equal responsibility for the final result.

*James M. Henderson*  
*Richard E. Quandt*

---

# PREFACE

## TO THE SECOND EDITION

---

The experience of using *Microeconomic Theory* in graduate and advanced undergraduate courses over a number of years, the many helpful suggestions of colleagues and students, and recent developments in economics have all provided incentives for a major revision. The second (enlarged) edition differs from the first in three major respects. There have also been some minor changes in the mathematical tools used in the text.

First, an attempt has been made to include a fair amount of new material that either appeared in the economic literature since the publication of the first edition or was considered too new or difficult for inclusion at the earlier time. Examples are the constant-elasticity-of-substitution production function (Chapter 3), a proof for the existence of equilibrium in a competitive economy (Chapter 5), the case of the revenue-maximizing monopolist (Chapter 6), and the theory of second best (Chapter 7). Many other extensions are included in the appropriate chapters.

A second change of major significance is the addition of a new chapter on linear models. It replaces the fragmentary treatments of linear programming, input-output analysis, and game theory which are in three different chapters of the first edition. These topics are now covered in an extended and unified manner in Chapter 9. Unlike the first eight chapters, this new chapter is method oriented rather than subject oriented.

The inclusion of exercises at the end of each chapter is the third major change. Their inclusion was the most frequent recommendation for change received from users of the first edition. The exercises contain both concrete illustrations and extensions of the materials in the text. The ability to work the exercises is an important aspect of gaining a working knowledge of microeconomic theory.

The level of mathematics has remained substantially unchanged. The emphasis remains upon methods and applications rather than detailed proofs. The calculus is the basic mathematical tool. A treatment of simple differential equations has been added. The concepts of convex and concave functions, which have gained increased importance in the economic literature, are introduced. Elementary convex-set theory is employed in Chapters 5 and 9. In only one instance is significantly more advanced mathematics, namely Brouwer's fixed-point theorem, employed. It is used to prove the existence of competitive equilibrium in Chapter 5. As before, readers are urged to refresh their memories about the mathematics and fill whatever gaps may exist by reading the Appendix (and working its exercises) before beginning Chapter 2.

Suggestions for improvements in the first edition have been made by many colleagues and students to whom profound thanks are hereby expressed. It is hoped that few new errors have been introduced. As before, the authors' work is thoroughly intermingled and they take equal responsibility for the final result.

*James M. Henderson*  
*Richard E. Quandt*

---

# PREFACE TO THE FIRST EDITION

---

The last two decades have witnessed an increasing application of mathematical methods to nearly every branch of economics. The theories of individual optimizing units and market equilibrium which are included within the microeconomics branch are no exception. Traditional theory has been formulated in mathematical terms, and the classical results proved or disproved. The use of mathematics has also allowed the derivation of many new results. Mathematical methods are particularly useful in this field since the underlying premises of utility and profit maximization are basically mathematical in character.

In the early stages of this development economists were rather sharply divided into two groups: the mathematical economists and the literary, or nonmathematical, economists. Fortunately, this sharp division is breaking down with the passage of time. More and more economists and students of economics are becoming acquainted with at least elementary mathematics and are learning to appreciate the advantages of its use in economics. On the other side, many mathematically inclined economists are becoming more aware of the limitations of mathematics. It seems a safe prediction that before too many more years have passed the question of the use of mathematics in microeconomic theory will be only a matter of degree.

As the number of economists and students of economics with mathematical training increases, the basic problem shifts from that of teaching mathematics to economists to that of teaching them economics in mathematical terms. The present volume is intended for economists and students of economics who have some mathematical training but do not possess a high degree of mathematical sophistication. It is not intended as a textbook on mathematics for economists. The basic concepts of microeconomic theory are



developed with the aid of intermediate mathematics. The selection of topics and the order of presentation are indicated by economic, rather than mathematical, content.

This volume is intended for readers who possess some knowledge, though not necessarily a great deal, of both economics and mathematics. The audience at which it is aimed includes advanced undergraduate and graduate students in economics and professional economists who desire to see how intermediate mathematics contributes to the understanding of some familiar concepts. Advanced knowledge in one of these fields can partially compensate for a lack of training in the other. The reader with a weak background in microeconomics will not fully appreciate its problems or the limitations of the mathematical methods unless he consults some of the purely literary works in this area. A limited number of these are contained in the lists of selected references at the end of each chapter.

A one-year college course in calculus, or its equivalent, is sufficient mathematical preparation for the present volume.<sup>1</sup> A review of the mathematical concepts employed in the text is contained in the Appendix. The Appendix is not adequate for a reader who has never been exposed to calculus, but it should serve the dual purpose of refreshing the reader's memory on topics with which he has some familiarity and of introducing him to the few concepts that are employed in the text but are not usually covered in a first course in calculus—specifically, Cramer's rule, Lagrange multipliers, and simple difference equations. The reader interested in extending his knowledge of specific mathematical concepts will find a list of references at the end of the Appendix.

In order to simplify the reader's introduction to the use of mathematical methods in microeconomic theory, two- and three-variable cases are emphasized in Chapters 2 and 3. The more general cases are emphasized in the later chapters. The analysis is frequently accompanied by diagrams, in order to provide a geometric interpretation of the formal results. The formal analysis is also illustrated with specific numerical examples. The reader may test his comprehension by working through the examples and working out the proofs and extensions of the analysis that are occasionally left as exercises.

The authors have both served as senior partners in the preparation of this volume, with each contributing approximately one-half of the material. Henderson is primarily responsible for Chapters 3, 5, 6, and 8, and Quandt is primarily responsible for Chapters 2, 4, 7, and the Appendix. However, the manuscript was prepared in very close collaboration, and each author helped plan, review, and revise the work of the other. Therefore, all errors and defects are the responsibility of both.

The authors are indebted to many of their teachers, colleagues, and students for direct and indirect aid in the production of this volume. Their

<sup>1</sup> The reader without this background is referred to the first fifteen chapters of R. G. D. Allen, *Mathematical Analysis for Economists* (London: Macmillan, 1938).

greatest debt is to their former teacher, Wassily W. Leontief. His general outlook is in evidence throughout the volume, and he is responsible for much of the authors' affection for microeconomic theory. The authors gratefully acknowledge the advice and criticism of William J. Baumol, who read the entire manuscript in an intermediate stage and offered numerous suggestions for its improvement. Others who deserve specific mention are Robert Dorfman, W. Eric Gustafson, Franklin M. Fisher, Carl Kaysen, and Seymour E. Harris. The marginal productivities of the inputs of the authors' above-mentioned friends are strictly positive in all cases.

The authors also owe a very significant debt to the economists who pioneered the application of mathematical methods to microeconomic theory. Their written works provide the framework for this book. The outstanding pioneers are J. R. Hicks and Paul A. Samuelson, but there are many others. The names and works of many of the pioneers can be found in the lists of selected references at the end of each chapter.

*James M. Henderson*  
*Richard E. Quandt*

---

# CONTENTS

---

	Preface to the Third Edition	xiii
	Preface to the Second Edition	xv
	Preface to the First Edition	xvii
<b>Chapter 1</b>	<b>Introduction</b>	<b>1</b>
	1-1 The Role of Theory	1
	1-2 Microeconomics	2
	1-3 The Role of Mathematics	4
<b>Chapter 2</b>	<b>The Theory of Consumer Behavior</b>	<b>5</b>
	2-1 Basic Concepts	8
	The Nature of the Utility Function / Indifference Curves / The Rate of Commodity Substitution / Existence of the Utility Function	
	2-2 The Maximization of Utility	13
	The First- and Second-Order Conditions / The Choice of a Utility Index / Two Special Cases	
	2-3 Demand Functions	18
	Ordinary Demand Functions / Compensated Demand Functions / Demand Curves / Price and Income Elasticities of Demand	
	2-4 Income and Leisure	24
	2-5 Substitution and Income Effects	25
	The Slutsky Equation / Direct Effects / Cross Effects / Substitutes and Complements	
	2-6 Generalization to $n$ Variables	32

2-7	Summary	34
	Exercises	35
	Selected References	36
<b>Chapter 3</b>	<b>Topics in Consumer Behavior</b>	<b>37</b>
3-1	A Linear Expenditure System	37
3-2	Separable and Additive Utility Functions	39
3-3	Homogeneous and Homothetic Utility Functions	40
3-4	Indirect Utility Functions and Duality in Consumption	41
	Indirect Utility Functions / Duality Theorems / An Example / Utility-Expenditure Duality	
3-5	The Theory of Revealed Preference	45
	Weak Axiom of Revealed Preference / Strong Axiom of Revealed Preference / The Substitution Effect	
3-6	Composite Commodities	48
3-7	Consumer's Surplus	49
3-8	The Problem of Choice in Situations Involving Risk	52
	The Axioms / Expected Utility	
3-9	Behavior under Uncertainty	56
	Attitudes toward Risk / Risk and Insurance	
3-10	Summary	60
	Exercises	61
	Selected References	62
<b>Chapter 4</b>	<b>The Theory of the Firm</b>	<b>64</b>
4-1	Basic Concepts	65
	The Production Function / Product Curves / Isoquants / Shape of the Production Function / Elasticity of Substitution	
4-2	Optimizing Behavior	74
	Constrained Output Maximization / Constrained Cost Minimization / Profit Maximization	
4-3	Input Demands	80
	Input Demand Functions / An Application of the Le Chatelier Principle	
4-4	Cost Functions	83
	Short-Run Cost Functions / Long-Run Cost Functions	
4-5	Joint Products	92
	Basic Concepts / Constrained Revenue Maximization / Profit Maximization	
4-6	Generalization to $m$ Variables	98

	Profit Maximization / Substitution Effects	
4-7	Summary	101
	Exercises	103
	Selected References	104
<b>Chapter 5</b>	<b>Topics in the Theory of the Firm</b>	<b>105</b>
5-1	Homogeneous Production Functions	105
	Properties / Euler's Theorem and Distribution	
	Long-Run Cost Functions	
5-2	CES Production Functions	111
	Properties / Isoquants / The Equilibrium	
	Condition / A Generalized CES Production	
	Function	
5-3	The Kuhn-Tucker Conditions	115
	An Input Option / A Discontinuous Labor	
	Contract	
5-4	Duality in Production	117
5-5	Production under Uncertainty	119
5-6	Linear Production Functions	120
	The One-Output Case / Multiple-Output Cases	
5-7	Linear Programming	124
	The Feasible Point Set / Optimal Solutions /	
	Duality	
5-8	Summary	131
	Exercises	132
	Selected References	133
<b>Chapter 6</b>	<b>Market Equilibrium</b>	<b>135</b>
6-1	The Assumptions of Perfect Competition	136
6-2	Demand Functions	137
	Market Demand / Producer Demand	
6-3	Supply Functions	139
	The Very Short Period / The Short Run / The	
	Long Run / External Economies and	
	Diseconomies	
6-4	Commodity-Market Equilibrium	145
	Short-Run Equilibrium / Long-Run	
	Equilibrium / Differential Cost Conditions and	
	Rent	
6-5	An Application to Taxation	152
6-6	Factor-Market Equilibrium	154
	Demand Functions / Supply Functions /	
	Market Equilibrium	
6-7	The Existence and Uniqueness of Equilibrium	157
	Existence / Uniqueness	
6-8	The Stability of Equilibrium	159

	Static Stability / Dynamic Stability: Lagged Adjustment / Dynamic Stability: Continuous Adjustment	
6-9	Dynamic Equilibrium with Lagged Adjustment	166
6-10	A Futures Market Hedging / Risk Assumption	169
6-11	Summary	171
	Exercises	172
	Selected References	173
<b>Chapter 7</b>	<b>Monopoly, Monopsony, and Monopolistic Competition</b>	<b>175</b>
7-1	Monopoly: Basic Theory Average and Marginal Revenue / Profit Maximization: Cost Function / Profit Maximization: Production Function	176
7-2	Monopoly: Price Discrimination Market Discrimination / Perfect Discrimination	181
7-3	Monopoly: Applications The Multiple-Plant Monopolist / The Multiple-Product Monopolist / Taxation and Monopoly Output / The Revenue-Maximizing Monopolist	185
7-4	Monopsony	190
7-5	Monopolistic Competition	193
7-6	Summary	196
	Exercises	197
	Selected References	198
<b>Chapter 8</b>	<b>Duopoly, Oligopoly, and Bilateral Monopoly</b>	<b>199</b>
8-1	Duopoly and Oligopoly: Homogeneous Product The Quasi-Competitive Solution / The Collusion Solution / The Cournot Solution / The Stackelberg Solution	200
8-2	Duopoly and Oligopoly: Differentiated Products Product Differentiation / The Market-Shares Solution / The Kinked-Demand-Curve Solution	207
8-3	Duopsony and Oligopsony	212
8-4	Theory of Games Two-Person, Zero-Sum Games / Mixed Strategies / Linear-Programming Equivalence / Cooperative Games / The Nash Bargaining Solution	213
8-5	Bilateral Monopoly Reference Solutions / Collusion and Bargaining	222
8-6	Summary	226
	Exercises	227
	Selected References	228

<b>Chapter 9</b>	<b>Multimarket Equilibrium</b>	<b>230</b>
9-1	Pure Exchange	231
	Equilibrium of the $i$ th Consumer / Market Equilibrium / Multimarket Equilibrium	
9-2	Two-Commodity Exchange	237
	A Calculus Example / The Edgeworth Box	
9-3	Production and Exchange	240
	Equilibrium of the $i$ th Consumer / Equilibrium of the $h$ th Firm in the $j$ th Industry / Market Equilibrium / Walras' Law / Multimarket Equilibrium	
9-4	The Numéraire and Money	246
	The Numéraire / Monetary Equilibrium / Money in the Utility Function	
9-5	Summary	252
	Exercises	253
	Selected References	254
<b>Chapter 10</b>	<b>Topics in Multimarket Equilibrium</b>	<b>255</b>
10-1	Existence of Equilibrium	256
	Solutions for Particular Systems / Brouwer's Fixed-Point Theorem / Existence of Excess Demand Functions / Existence of Equilibrium Prices / Advanced Existence Proofs	
10-2	Stability of Equilibrium	267
	Static Stability / Dynamic Stability	
10-3	Uniqueness of Equilibrium	274
10-4	The Input-Output Model	276
	Output Determination / Decomposability / Existence / Price and Income Determination / The Substitution Theorem	
10-5	Summary	282
	Exercises	283
	Selected References	284
<b>Chapter 11</b>	<b>Welfare Economics</b>	<b>285</b>
11-1	Pareto Optimality	286
	Pareto Optimality for Consumption / Pareto Optimality for Production / Pareto Optimality in General	
11-2	The Efficiency of Perfect Competition	292
11-3	The Efficiency of Imperfect Competition	293
	Imperfect Competition in Consumption / Imperfect Competition in Commodity Markets / Imperfect Competition in Factor Markets / The Efficiency of Bilateral Monopoly	

11-4	External Effects in Consumption and Production Interdependent Utility Functions / Public Goods / Lindahl Equilibrium / External Economies and Diseconomies	296
11-5	Taxes and Subsidies External Effects in Production / Monopoly	304
11-6	Social Welfare Functions Determination of a Welfare Optimum / Social Preference and Indifference / The Arrow Impossibility Theorem / Income Distribution and Equity	308
11-7	The Theory of Second Best	315
11-8	Summary	317
	Exercises	319
	Selected References	320

<b>Chapter 12</b>	<b>Optimization over Time</b>	<b>322</b>
12-1	Basic Concepts The Bond Market / Market Rates of Return / Discount Rates and Present Values	323
12-2	Multiperiod Consumption Multiperiod Utility Functions / The Budget Constraint / The Consumption Plan / Substitution and Income Effects	326
12-3	Investment Theory of the Firm The Multiperiod Production Function / The Investment-Opportunities Function / The Investment Plan / Point-Input-Point-Output	333
12-4	Interest-Rate Determination	339
12-5	Investment Theory and the Role of Time Continuous Compounding and Discounting / Point and Flow Values / Point-Input-Point- Output / Continuous-Input-Point-Output / Point-Input-Continuous-Output	341
12-6	Retirement and Replacement of Durable Equipment Assumptions / The Quasi-Rent Function / Retirement of a Single Machine / Replacement for a Chain of Machines	346
12-7	Exhaustible Resources	349
12-8	Human Capital Investment in Education / Investment in Training / Earnings-Cycle Investment	350
12-9	Summary	354
	Exercises	356
	Selected References	357



<b>Appendix: Mathematical Review</b>	<b>358</b>
A-1 Simultaneous Equations, Matrices, and Determinants	358
A-2 Calculus	363
Functions, Limits, Continuity / Derivatives for Functions of One Variable / Techniques of Differentiation / Partial Derivatives for Functions of Many Variables / The Total Differential / Envelopes / Implicit-Function Theorem and Jacobians	
A-3 Maxima and Minima	357
Unconstrained Maxima and Minima / Maxima and Minima with Equality Constraints / Constrained Optima and Quasi-Concavity (Quasi-Convexity) / Maxima and Minima with Inequality Constraints	
A-4 Integrals	386
A-5 Difference Equations	388
The Nature of the Solution / Homogeneous Equations / Nonhomogeneous Equations	
A-6 Differential Equations	391
Exercises	393
Selected References	394
<b>Answers for Even-Numbered Exercises</b>	<b>396</b>
<b>Index</b>	<b>409</b>