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QUICK FINITE ELEMENTS FOR ELECTROMAGNETIC WAVES

SECOND EDITION

CD-ROM INCLUDED

Quick Finite Elements for Electromagnetic Waves

Second Edition

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Preface

Ten years have passed since the publication of the first edition of this book. The enormous growth in computing power and the full establishment of high-level mathematical computing languages has revolutionized the world of numerical electromagnetics, together with the presence on the market of very high level finite element-based commercial software.

There is, nevertheless, the clear need for an educational approach to finite elements, valid both at an undergraduate and a graduate student level, allowing for a rapid and effective hands-on approach. This was the aim of the first edition of this book, whose philosophy, sketched in the preface of the 1998 edition, has not changed.

What has changed are the expectations of students and researchers. FORTRAN, which is still a leading language for numerical computation has lost, due to its intrinsic architecture, some of its appeal, especially in undergraduate and graduate student courses, with The MathWorks Matlab, just to cite one of the currently available high-level mathematical languages, taking its place.

Furthermore, if ten years ago a two-dimensional working FEM code was appealing as common personal computers were not able to provide the sheer amount of memory and computing power needed to three-dimensional finite elements implementations, nowadays this computing power is commonly available, so three-dimensional codes are appealing also in an educational framework.

For the reasons expressed above, while maintaining the fundamental philosophy of the first edition, this second edition provides the reader with Part I, which is an upgrade of the 1998 edition, containing the original bidimensional codes in FORTRAN and their newly developed Matlab translation. A complete new set of tools for preprocessing and postprocessing exploiting Matlab graphical capabilities has also been developed.

Furthermore, a brand-new Part II, presenting three chapters devoted to three-dimensional FEM problems has been included. This part presents codes developed in FORTRAN, for speed and efficiency reasons, and presents a three-dimensional tool library as well as two codes, one for resonant cavity problems and one for solving waveguide devices. The development of this second part was carried out with the substantial help of Dr. Giacomo Guarnieri, former Ph.D. student at the University of Florence and now with Galileo Avionica Co., Campi Bisenzio (Florence, Italy).

Finally Part III includes an up-to-date annotated bibliography of the main books and articles published on finite elements for electromagnetics.

The CD-ROM bounded to this book contains the complete software described in the text, in FORTRAN 77 for the whole book and in Matlab for Part I. The software has been designed to be as platform independent as possible and the

Matlab code to be as version independent as possible. While Matlab is of course a commercial code, the codes provided make use, when possible, of some freely available mathematical, mesh generators, and graphical software packages that can be retrieved from public repositories accessible through the Internet. For the convenience of the readers, those packages for which permission was granted are included in the CD-ROM.

This book, as it was already in the first edition, is dedicated to the memory of P. P. Silvester, to whose teachings and expertise is due a great portion of the growth of the scientific activity in the field of numerical analysis at the University of Florence in the nineties. The authors are also indebted to many colleagues for useful advices which lead to noticeable improvements for this second edition.

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May 2009

Preface to the First Edition

The first complete and clear application of the finite element method (FEM) to applied electromagnetism dates back to 30[1] years ago. At the beginning of 1969, a paper from P. P. Silvester was published in the Italian technical journal *Alta Frequenza*, in which a detailed finite element formulation of the classic problem of guided propagation was presented. Although the problem treated in that paper was rather simple, the author clearly showed and pointed out the promising potentialities of the method. Since then FEM evolved quickly: many improvements in the formulation have widely expanded its capability and flexibility so that applications to electromagnetic problems spawned largely. Currently, FEM has gained a fundamental role for the analysis of electromagnetic problems and is one of the most widely used CAD tools for microwave components ranging from simple guiding structures to circuits, antennas, and scattering problems.

The implementation of finite element methods is not simple though. In comparison to other popular numerical techniques, most notably the finite difference method, it requires a finer analytical development of the formulation before going to the implementation, a deeper knowledge of linear algebra methods, as well as a more involved preprocessing phase. Probably for these reasons, the usage and full knowledge of FEM is still limited to a relatively small number of researchers and microwave engineers despite its enormous possibilities and versatility.

Textbooks on finite element methods are still few in number, even if new ones are published constantly, and the method still lacks a full citizenship in university courses, both at undergraduate and graduate levels. Most of the books on finite elements cover the theory behind the method, rarely providing the reader with codes to experiment with, even very simple ones indeed. This implies that, after having studied the book, the reader needs some time to develop his first working FEM code and, most often, this time is too long with respect to that allocated to finite elements in university courses on numerical methods, or available to practitioners. Consequently, it is very difficult for those approaching the method for the first time to fully appreciate its power without quickly getting some numerical results from an FEM code.

The main goal of this book, as suggested by its title, is that of shortening the learning curve of finite element methods as well as to cut down the implementation time. *Quick FEM* is aimed to provide the user with the essential theory behind FEM, limiting the analytical development to that strictly needed to get started, and to guide the reader to implement the method by describing in detail some applications and the related codes. This allows understanding FEM and writing the very first working FEM code just after having read only the first chapter only. The interested reader may further investigate the theoretical aspects of the method by referring to other available books, such as the classic and well-known text

1. This preface was written in 1998. Now it dates back 40 years.

written by P. P. Silvester and R. L. Ferrari, now in its third revised and expanded edition.

The CD-ROM bounded to *Quick FEM* contains the complete software described in the text and uses the practical, implementation-oriented attitude of the book. The source codes have been written in FORTRAN 77 and are designed to be as platform independent as possible. The class of problems that they can solve ranges from waveguide characterization to scattering and radiation, and can be easily expanded and customized by the reader introducing new modules to treat the problem at hand. The codes provided make use of some freely available mathematical and graphical software packages that can be retrieved from public repositories accessible through the Internet. For the convenience of those who cannot easily access the Internet, those packages for which permission was granted are included in the CD-ROM.

This book is dedicated to P. P. Silvester, to whose teachings and expertise is due a great portion of the growth of the scientific activity in the field of numerical analysis at the University of Florence in the past few years. The authors are also indebted to many colleagues for their useful advice.

Giuseppe Pelosi
Florence, Italy
Roberto Coccioli
Los Angeles, California
Stefano Selleri
Nice, France
1998

How to Use Quick FEM

This book has been organized so that each reader can hopefully benefit the most in the shortest possible time by choosing the chapters to read on the basis of his or her own interests and knowledge.

The book is divided into three parts. The first devoted to two-dimensional problems, the second dealing with three-dimensional problems, the last containing a bibliography referencing key papers and books on finite elements.

In Part I the first chapter describes an extremely simple application to a guided propagation problem, using the quasistatic approach. It gives a brief introduction to FEM and to point out the various phases of the implementation of an FEM code, thus it can be skipped by those already acquainted with the method. Chapter 2, on the other hand, describes a set of software tools and libraries employed by all codes described in the remaining chapters, thus it should be of interest for all those wishing to understand the codes and to modify them. In particular, it describes the various preprocessing and postprocessing tools, as well as some utilities and a library of subroutines for generations of the elemental matrices.

In this second edition all software in Part I is provided both in FORTRAN 77 and in Matlab languages, with an almost 1 to 1 correspondence between implemented functions. It is important to note how Matlab is excellent for prototyping and class exercises, due to its high level and comparable simplicity, whereas FORTRAN codes are more interesting in a research and production environment, due to their general faster execution times and the possibility of handling larger problems on a given machine.

Chapters 3 to 6 describe the finite element formulation and the relative implementation to solve fairly general problems ranging from guided propagation in homogeneous and inhomogeneous waveguides (Chapter 3), scattering from rectangular waveguide devices (Chapter 4), scattering from infinitely extending periodic structures (Chapter 5), and both scattering from isolated objects and radiation from apertures (Chapter 6). All these chapters are self-contained and can be read in any order depending on the interest of the reader. The codes and the examples described cover topics of current research interest, such as complex modes in lossless waveguides and scattering from photonic bandgap structures.

Part II is articulated into three chapters and deals with three-dimensional problems. Chapter 7 is analogous to Chapter 2 inasmuch as it presents the basic elements and tools for three-dimensional finite elements in electromagnetics. In this part, for efficiency reasons, only FORTRAN codes are provided even if some Matlab pre- and postprocessing tools are given. Chapters 8 and 9 then deal with typical waveguide problems; in particular Chapter 8 presents the FEM solution for resonant cavities, while Chapter 9 presents a full three-dimensional code for microwave passive devices in rectangular waveguide, greatly extending the simpler code of Chapter 4.

Finally, Part III contains just a single Chapter 10, presenting a wide and structured bibliography where material to further investigate specific topics of interest, or to read about recent development in FEM applications to electromagnetic engineering can be found.

Some words are due also on the content and usage of the enclosed CD-ROM, as well as on how to retrieve from the Internet the software packages not developed by the authors. The software referenced in this book is subdivided into four categories:

- Tools;
- Electromagnetic analysis;
- Linear algebra solvers;
- Graphical rendering.

The codes belonging to the second category, most of those belonging to the first and to the fourth has been developed expressly for this book and are described in the text. The other essential codes, when permission was granted, are also included into the accompanying CD-ROM (Figure 1). In any case, these freely available codes are available from the network and the reader is encouraged to seek the latest versions.

Except for the first code described in Chapter 1, which is self-contained, all the other codes in this book need linear algebra solvers, sometimes pretty sophisticated ones, as in the case of Chapter 3. For what concerns Matlab codes, Matlab internal solvers are used. For FORTRAN codes any mathematical library is appropriate, and the reader should not encounter any problems in customizing the codes for his own use. The linear algebra package (LAPACK), which is a very widespread mathematical library usually natively available on any LINUX system, has been used.

The main Web site from which the LAPACK library can be downloaded is *http://www.netlib.org*, but European users may find the site *http://www.netlib.no/* more convenient. LAPACK routines are self-explanatory, and the way they are called within the codes here provided should be clear enough. A manual edited by LAPACK authors is available from SIAM. Ordering instructions are on the above WWW addresses.

LAPACK subroutines make ample use of basic linear algebra subprograms (BLAS), a collection of subroutines able to perform standard linear algebra operations. This package is included in the CD-ROM too, and it can also be downloaded from the same sites as the LAPACK. However, BLAS routines vendor optimized for the platform used should be employed whenever available.

Graphical visualization of computed data can be achieved exploiting a great variety of software packages. The commercial packages are numberless but, of course, Matlab is used throughout this book. Among the various freely available packages, for the readers unwilling to use Matlab, we prefer GNUPLOT, which again can be freely downloaded from the site *http://www.gnuplot.info/*.

Mesh visualization is a demanding task, which is well managed via the Matlab codes provided in this book. Otherwise, a visualization program has been devel-

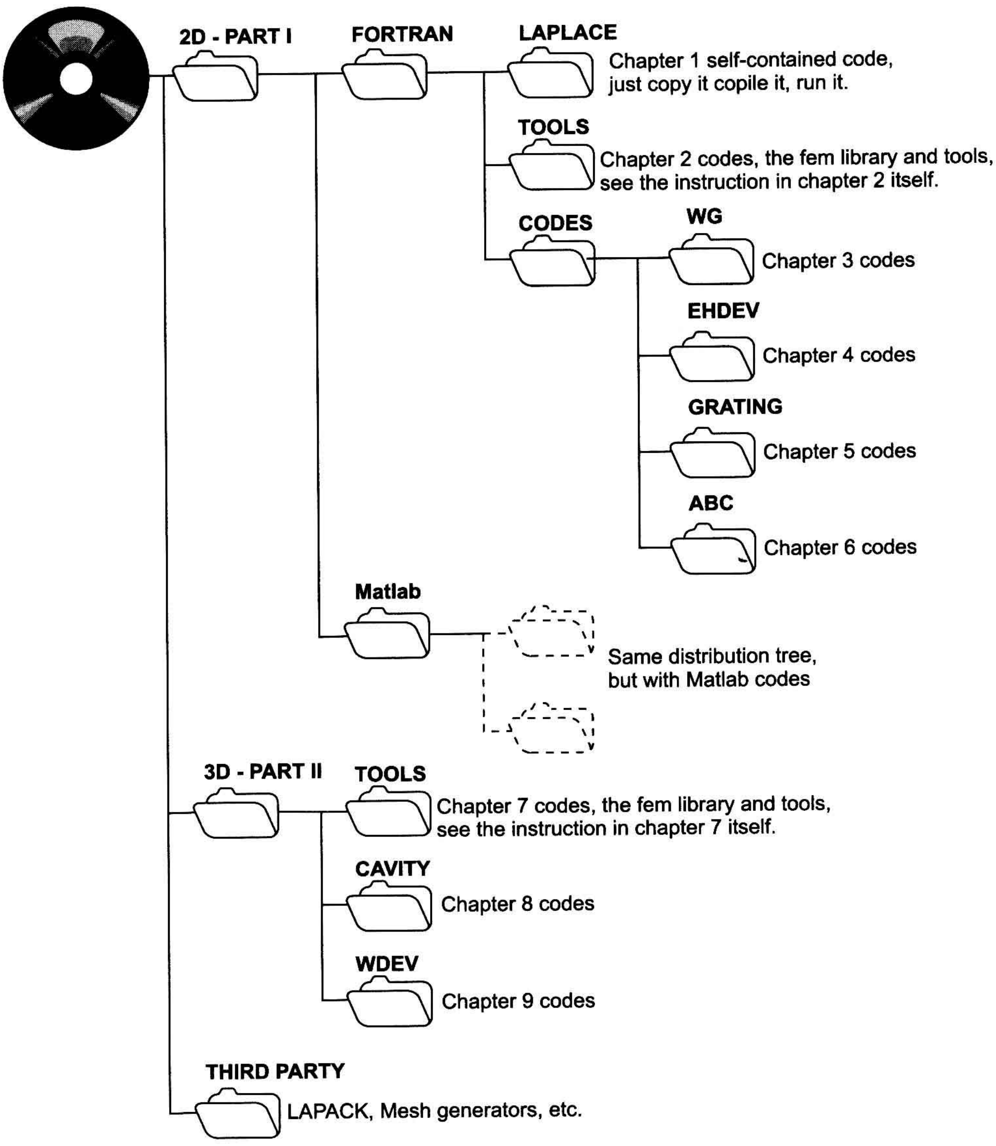

Figure 1 Accompanying CD-ROM tree.

oped in FORTRAN, converting mesh data into PostScript files. These latter can be visualized by resorting to the GhostScript/GhostView packages, whose main site is *http://pages.cs.wisc.edu/ ghost/*.

As for the first edition, a Web site located at *http://www.cem.unifi.it/* is available, publishing news and patches for the codes developed by the authors and accepting comments, suggestions, and bug reports.

Contents

PART I

Two Dimensions

CHAPTER 1

Getting Started: Shielded Microstrip Lines

The aim of this chapter is to introduce the finite element method (FEM) by means of a simple example: the evaluation of primary and secondary constants of a microstrip line shielded by a metallic box, indefinitely extending along the axial direction z (Figure 1.1).

The assumption of uniformity effectively makes the problem two-dimensional so that it suffices to analyze the structure in a transverse plane [Figure 1.2(a)].

Electromagnetic propagation in a transmission line can be fully described once its secondary constants Z (characteristic impedance) and β (propagation constant) are known. In general these can be obtained under the hypothesis of TEM or quasi-TEM propagation and lossless substrate and conductors, from the line's primary constants L (inductance per unit length) and C (capacitance per unit length) with the formulas:

$$Z = \sqrt{\frac{L}{C}} = v_p L = \frac{1}{v_p C}, \quad \beta = \omega\sqrt{LC} \tag{1.1}$$

where $v_p = \omega/\beta$ is the phase velocity of the guided wave.

Removing the dielectric substrate [Figure 1.2(b)] allows the structure to support a TEM-guided wave, which, since the medium is air, propagates with a phase velocity $v_p = c$, being c the velocity of light in air ($\simeq 3 \cdot 10^8$m/s). For this new transmission line, the following relation holds:

$$Z^{air} = \sqrt{\frac{L}{C^{air}}} = cL = \frac{1}{cC^{air}} \tag{1.2}$$

where C^{air} is the capacitance per unit length of the equivalent air-spaced shielded line. The inductance per unit length $L = L^{air}$ is unchanged since the relative permeability is $\mu_r = 1$ on the entire cross-section for both lines. Hence:

$$L = \frac{Z_0^{air}}{c} = \frac{1}{c^2 C^{air}} \tag{1.3}$$

Combining (1.1), (1.2), and (1.3) the characteristic impedance of the original shielded microstrip line can be expressed in terms of the capacitance per unit length of the two structures, with and without the dielectric slab:

$$Z = \sqrt{\frac{L}{C}} = \frac{1}{c\sqrt{C^{air}C}} \tag{1.4}$$