HOMER E. BROWN

solution of LARGE NETWORKS by MATRIX METHODS

second edition

Solution of Large Networks by Matrix Methods

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To
Mary Isabel

PREFACE

In this edition new chapters on state estimation, optimum load flow, and economic dispatch have been added. The material on transient stability has been enlarged and a sample calculation included. Where experience with the first edition indicated that it would be helpful, illustrative calculations have been added. Sets of exercises have been added at the end of each chapter.

I acknowledge the helpful suggestions for changes and additions made by former associates at North Carolina State University, William D. Stevenson, Jr., John J. Grainger, Alfred J. Goetz, and Adly A. Gergis. The chapter on economic dispatch was added at the suggestion of A. S. Al-Fuhaid, now of the University of Kuwait. He viewed the first edition in the dual role of student and, later, teacher. He made several other suggestions, most of which were included. I am especially indebted to A. M. Sasson for selecting fundamental papers on state estimation and for his guidance in this area. I am also indebted to J. O. Storry of South Dakota State University for making his dissertation on hybrid matrices available to me. I am indebted to W. Scott Meyer for a copy of an unpublished set of his notes on the calculation of the loss formula. Mary Isabel was very understanding during the writing of this edition. Her help in proofreading the original manuscript and the galley proofs was invaluable.

HOMER E. BROWN

Cary, North Carolina
March 1985

PREFACE TO THE FIRST EDITION

This book covers the class material that was given to graduate classes in network analysis at Iowa State University, Rensselaer Polytechnic Institute, Purdue University, and The Escola Federal de Engenharia de Itajubá (Brazil) when I was a visiting professor at those institutions. The material has been expanded into book form and is intended in graduate study work to indicate the methods now used in industry. It will also be helpful for practicing engineers who completed their formal education prior to the computer revolution. The methods discussed are illustrated by simple numerical examples for a better understanding of the techniques.

Although electric power systems are subjected to short circuits only a small proportion of the total time, short circuits on networks are treated first in the book because this subject is far simpler to explain and comprehend than is the solution of normal power flow problems. Because transient stability is more complex than power flow, this subject follows. Finally an introduction is given to optimization methods such as linear programming, the method of steepest ascent, and the method of eigenvalues because the next development in network analysis will surely exploit these techniques.

I should like to thank every individual who helped me in writing the book, but this would be impossible because of the great numbers. Therefore, I list only a few names for special commendation. Of the many former associates at The Commonwealth Edison Company, I express my gratitude to Conrad E. Person and Robert G. Andertich, who assisted in developing some of the techniques that are discussed in the text. For my first opportunity to be a visiting professor on a university campus while on loan and financially supported by The Commonwealth Edison Company, I am indebted to Professor Paul M. Anderson of Iowa State University and Vice-President Ludwig F. Lischer of The Commonwealth Edison Company. I am especially grateful to Dr. Eric T. B. Gross for the opportunity to be a visiting professor three times in his Power Engineering Program at Rensselaer and for his encouragement to begin writing my lectures in book form. To Dr. T. S. Lauber for reviewing the manuscript and suggesting modifications that would improve the clarity of the material, I am greatly indebted. I acknowledge the help of Dean Amadeu Casal Caminha and Professor José Abel Royo dos Santos of the Escola Federal de Engenharia de Itajubá for their corrections in the English text when translating it into Portuguese. I am also indebted to Dean Caminha for furnishing the secretarial help of Lair Elisa Fernandes and Sônia Maria Maia. I commend the women for their great care in typing in a foreign language. Finally, the writing of the book would not have been possible without the loving understanding of my wife, Mary Isabel.

HOMER E. BROWN

Itajubá, Minas Gerais, Brazil
September 1974

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GENERAL BACKGROUND

Before about 1950 matrices were used only as research tools. They systematized the arrangement of materials and generally forced the research worker to be organized. Matrices at that time in no way reduced the computational effort; however, the absence of high-speed computers limited investigations to small sets of equations involving only very small matrices.

The first generation of small-scale computers extended the use of matrices in solving network problems of limited size [1].

Networks, a broad category of studies, extend into many disciplines. The range of problems includes traffic flow in a network of city streets, stress analysis of the steel framework of large buildings, airplane wings, gas flow in pipes, the flow of electricity in a large electrical network, heat flow, and mechanical rotation.

EARLY COMPUTATIONAL METHODS

As recently as 1955 all electrical network problems were solved either by hand or by a *network calculator*.† The network calculator was poorly named, since it did not perform any calculations. It is merely an electrical analogue device. For electrical problems the analog is direct; that is, electrical current in the problem network is represented by current in the analogue, voltage is repre-

† The name "network calculator" is a copyright of Westinghouse; GE Co. used "network analyser."

sented by voltage, and so on. The network being investigated is represented by another network on a greatly reduced scale.

The network calculator can also be used to solve problems in other fields, but problem variables must first be converted to electrical quantities. For example, the stress and strain in a steel beam could be represented by voltage and current, respectively, while mechanical inertia could be represented by inductance or capacitance.

THE COMPUTER

The second and third generation of digital computers made possible the investigation of large networks (steel structures, power system networks, etc.) by matrix methods. The superiority of the network calculator as a tool for educating electrical power system operating and system planning personnel is justified, since the network response to the various adjustments (generator angle or voltage level) can be readily observed. However, because of the larger capability of the computer programs and the almost universal availability of the digital computer, the computer is superior and more economical for detailed studies of large systems. The network analyzer on the other hand is a very specialized tool and even during peak usage was not generally available. Consequently, the network calculator has disappeared in the United States.

COMPUTATIONAL METHODS

The availability of the computer changed greatly the mathematical approach to network solutions. Longhand calculations can be carried out more readily using loop equations. The earliest computer programs analyzing the flow in networks merely automated these longhand methods [2].

Several investigators did considerable work with incident matrix and connection matrix algorithms for automatically determining independent loops in the network, since this was the most difficult part of data preparation for the loop formulation of the problem [3, 4]. Later, nodal equation methods were developed and proved to be greatly superior for the computer solution of network problems.

POWER FLOW PROBLEMS

The first computer attempts to solve network flow problems had limited success, because the programs merely automated the longhand methods using loop equations and did not exploit the capability of the computer. The greatest burden in these early programs was the preparation of the data that defined the independent loops of the network. A considerable amount of work was done to develop methods whereby the computer could automate the generation of the loop connection matrix. The method was somewhat successful but in turn added to the burden on the limited computer memory available [3, 4].

The first really successful network flow program was developed by Ward and Hale [5]. They used the nodal formulation of the problem and solved, by a modified Newton iterative procedure [6], the simultaneous quadratic equations that describe the electrical network. The programs, which followed immediately, implemented the Gauss–Seidel algorithm.

The success of the method of Ward and Hale was quickly accepted by the power industry, and a number of papers by Glimn and Stagg, Brown and Tinney, and others described modifications of the algorithm and incorporated additional features.

The increase in high-voltage interconnections between systems in the late 1960s and the availability of large computers greatly enlarged the size of systems studied. Power flow studies of larger systems by the Gauss–Seidel method require a greater number of iterations to obtain a solution or become mathematically unstable, even if the network being studied is actually a workable system. During the iterative process in the Gauss–Seidel method, the effect of adjustments in an iteration are reflected only to the neighboring nodes. The propagation of an adjustment across a large system therefore takes several iterations. Meanwhile conflicting adjustments may be taking place and are transmitted and reflected across the system.

Fortunately, as early as 1961 researchers were investigating other methods for solving network flow problems. A Newton–Raphson algorithm was developed that succeeded in solving networks that could not be solved by the Gauss–Seidel method of solution. It soon earned wide acceptance in the power industry because of its increased speed and ability to solve difficult network problems. This algorithm was the result of continued development by the Bonneville Power Administration group [7–10]. The method requires roughly the same number of iterations regardless of the size of the network.

The fast-decoupled load flow algorithm, a later development, is even faster and more stable and requires less memory than the Newton–Raphson method. This algorithm was the result of research by Despotović [11] and the team of Stott and Alsac [12].

Another load flow method that overcomes the instability of the Gauss–Seidel method is the impedance-matrix load flow algorithm [13]. The method has convergence characteristics similar to the Newton–Raphson method for the average power systems load flow problems. However, the memory requirements for the impedance matrix are very severe because the Z-matrix is full and not sparse like the Y-matrix of the Gauss–Seidel or the Jacobian matrix of the Newton–Raphson method. This severe storage problem can be overcome by tearing the system into parts and applying the diakoptics techniques of Kron [14, 15]. Because of the excessive memory requirement and the unavailability of a viable tearing algorithm, this method had limited acceptance.

The installation of large computers at power system control centers in the past several years has produced considerable interest in state-estimation load