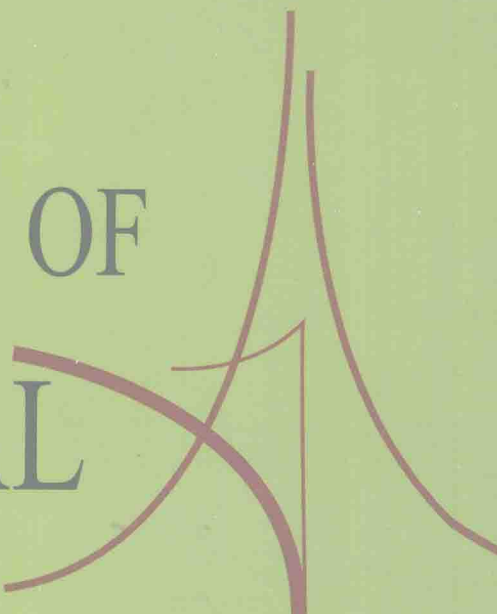


INTRODUCTION TO THE

THEORY OF CRITICAL PHENOMENA



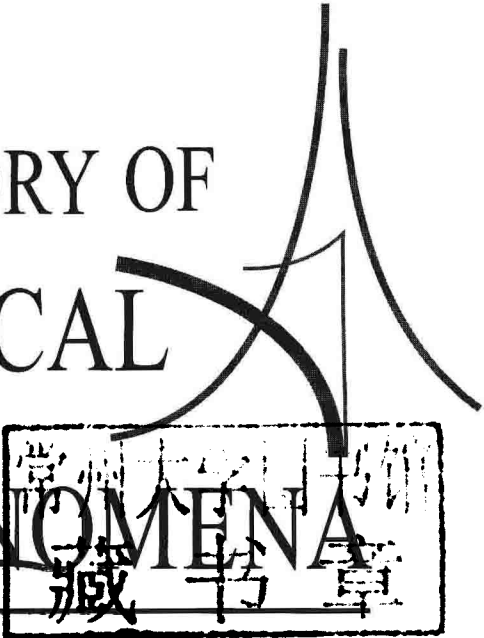
Mean Field, Fluctuations and Renormalization

2nd Edition

Dimo I. Uzunov

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THEORY OF CRITICAL PHENOMENA



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Dimo T. Uzunov

Bulgarian Academy of Sciences, Bulgaria

 **World Scientific**

NEW JERSEY • LONDON • SINGAPORE • BEIJING • SHANGHAI • HONG KONG • TAIPEI • CHENNAI

Published by

World Scientific Publishing Co. Pte. Ltd.

5 Toh Tuck Link, Singapore 596224

USA office: 27 Warren Street, Suite 401-402, Hackensack, NJ 07601

UK office: 57 Shelton Street, Covent Garden, London WC2H 9HE

British Library Cataloguing-in-Publication Data

A catalogue record for this book is available from the British Library.

**INTRODUCTION TO THE THEORY OF CRITICAL PHENOMENA (2nd Edition)
Mean Field, Fluctuations and Renormalization**

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ISBN-13 978-981-4299-48-0

ISBN-10 981-4299-48-0

ISBN-13 978-981-4299-49-7 (pbk)

ISBN-10 981-4299-49-9 (pbk)

Printed in Singapore by World Scientific Printers.

To my wife Diana and my children Ivan, Yulia, and Victor.

Foreword

It is a pleasure for me to present to the public this monograph on phase transitions and critical phenomena. In the past few years, along with our improving understanding of these phenomena, their theoretical description has made great progress evolving into more advanced forms. These also include, apart from new approximations elaborated in the framework of the mean-field theory, various novel approaches such as phenomenological and microscopic treatments of fluctuations, the scaling theory and the renormalization group theory. All of them are thoroughly explained in the present book. The author has managed to cover all these topics with a good balance, proceeding from the thermodynamics of phase transitions to the modern theory of the renormalization group. The way of presentation adopted by the author is very instructive. Detailed derivations of almost all the basic equations are included in the monograph. Thus, even readers who are too lazy or too busy to reproduce such calculations themselves can follow easily the discourse to its very end.

Even the introduction of the renormalization group theory does not diminish the importance of the mean-field theory, as we can see, for example, from the recent development of the coherent-anomaly method. Hence, the meticulous explanation of this theory in the present book seems to be justified even from the viewpoint of modern approaches to critical phenomena. Chapter 6 on fluctuations and Chapter 7 on the perturbation theory of fluctuation fields constitute a good introduction to Chapter 8 on the renormalization group theory. The last chapter contains a brief overview of some important applications of the latter method to various problems including the Bose gas, the classical-to-quantum crossover, the random-field problem, the anisotropy effect and multicritical behavior. This part of the book

displays considerable expertise as Dr. Uzunov himself has made a series of interesting contributions in this field.

Masuo Suzuki
Professor of Statistical Physics
University of Tokyo

Preface to the Second Edition

In this second edition, Chapter 5 and Chapter 9 are considerably extended in order to ensure a more comprehensive presentation of the mean-field approximation and applications of the phase transition theory. A number of misprints and non-misleading mistakes in the first edition has been removed. Many new references are added, in particular, in new developing topics, as for instance, unconventional superconductivity, dilute Bose gases, and gauge effects in superconductors.

The rationale for a book on the theory of phase transitions and critical phenomena is still valid today — almost four decades after the advent of the ideas of scaling and universality and their integration in the Wilson–Fisher renormalization group approach. This is so, because the phase transition theory can be successfully applied to solve various problems of Physics and other natural sciences, even in economics and sociology.

Several new calculational variants of renormalization group appeared meanwhile aiming to ensure a more accurate calculation in real spatial dimensions. The oldest renormalization group method, known from the quantum electrodynamics of the fifties of 20th Century, received a vast development, too. But the Wilson-Fisher treatment has still remained most distinctly connected with the main ideas of homogeneity, scaling, and universality. It presents the basis for learning of renormalization group and performing calculations at an ample level of accuracy for the most part of problems of interest. For this reason, the second edition has not been widened with other than the Wilson-Fisher basic approach. For other renormalization schemes and applications of the phase transition theory the interested reader may use excellent books and review articles, cited in the text.

Since 1990–1991, when this book was written, until now many excellent books on phase transitions have been published, for example, by Goldenfeld (1992), Binney *et al.* (1992), Yeomans (1992), Ivanchenko and Lisianskii (1995), Zinn–Justin (1996), and Cardy (1996). A number of excellent and large collections of lectures and review articles have been also released (see the bibliography to this book), and the well-known book series of review articles *Phase Transitions and Critical Phenomena*, ed. by C. Domb and M. S. Green (after 1976, ed. by C. Domb and J. L. Lebowitz) was complemented by new volumes, containing both theory and applications.

Notwithstanding the exceptional collection of excellent text books, monographs and review articles, or, perhaps, because of its availability, the present book has its place in the literature on phase transitions and critical phenomena for several reasons. It covers the main stream of development of the theory for the last 100–120 years. As becomes apparent today, about forty years after the last revolutionary period in this theory – the advent of the renormalization group in 1971–1972, the mean field theories and some classic thermodynamic treatments, widely discussed in this book, have their considerable importance in the practical research, and cannot be substituted by the modern renormalization group methods. Further, the book ensures a gradual connection between the usual University courses and the advanced scientific literature. This might be convenient to readers among the talented undergraduates, doctorate students and non-experts. The close interrelationship between experimental and theoretical problems is discussed in details, in particular in Chapters 1 and 9. Outstanding problems are also reviewed, in particular, in Chapter 9. The numerous references, in particular, original papers, some of the review articles and books, are addressed to the attention of experts and doctorate students, involved in research on phase transitions.

The author thanks Professor Dr. Alvaro Ferraz for the hospitality at the International Institute of Physics (Natal, Brazil), where the second edition was accomplished. Besides, the author thanks Associate Professor Dr. Diana V. Shopova (Bulgarian Academy of Sciences) for the critical reading of new texts, included in the book.

D. I. Uzunov

March 2010, Natal, Brazil

Preface to the First Edition

This book is intended as an introduction to the theory of phase transitions and critical phenomena. The phase transitions are described by the methods of thermodynamics and statistical physics. On the contrary modern methods for the investigation of outstanding problems in statistical physics, thermodynamics, condensed matter physics and field theory may be partly ascribed to the development of the ideas and theoretical techniques intended to solve the questions of phase transitions. The sophistication of theoretical methods is often an obstacle to the quick understanding of important results currently reported in scientific literature. The main purpose of this book is to present a part of these methods in an intelligible form for students.

The physics of phase transitions is old and abundant in fascinating ideas, theories and experiments. Most of them are substantial for other fields in physics, chemical physics and even for other natural sciences. All the problems concerning phase transitions and even the main part of them cannot be comprised in only one book. I have chosen such a representation of the phase transition theory that goes along a general scheme for the description of phase transitions and critical phenomena. It includes the following topics: the thermodynamic stability theory (Chapter 2) and its connection with the classical and scaling picture of the phase transitions (Chapter 3); the Landau expansion as a further development of the thermodynamic approach and the notion of order and symmetry in condensed matter physics (Chapter 4); an introduction to statistical mechanics and the mean field approximation for homogeneous and inhomogeneous systems (Chapter 5); the fluctuation phenomena and the renormalization group method in treating the fluctuation fields (Chapters 6 – 8). I have made an attempt to establish a logical relationship between the different theories and their role

in the interpretation of experiments. Special attention is paid to the problems which are less illuminated in books and review articles published in the last twenty years. On the contrary, I have only mentioned some important problems like the phenomenological scaling and the dynamic critical phenomena, for which several excellent books and reviews exist. The text contains a number of references to the literature intended for the reader who is interested in specific problems.

In addition to the presentation of basic ideas, a big part of Chapters 2–8 is devoted to the concrete methods of calculation, characterizing the different theories and approximations. This especially concerns Chapter 7, where the perturbation theory of fluctuations is discussed in detail. I suppose that in this way the renormalization group approach will be more easily comprehended both as a concept and as a technique of calculation.

In Chapters 2–6, I often follow the deductive way of presentation, which to my opinion reveals the wide application of the quasi-phenomenological theory to the description of phase transitions.

Chapter 7 and Chapter 8 are written in an opposite manner, since when the perturbation theory and diagrammatic technique are understood for simple models, their generalization to more complex cases can be made without difficulties.

Chapter 1 and Chapter 9 have in some sense a different role. In Chapter 1 the main stages in the development of theoretical and experimental studies on the phase transitions are outlined accompanied by corresponding references to the literature. The unexperienced reader may find this difficult. In this case I recommend reading it after an acquaintance with the contents of the remaining chapters. Chapter 9 presents the application of the mean field and fluctuation theories to concrete problems. In order to keep this survey within a manageable length it was necessary to avoid, as much as possible, problems which are widely discussed in other reviews. My intention has been to throw light on issues, which are not reviewed in the existing literature. Even within this limited scope it was impossible to present within a single chapter a detailed discussion of all developments; many interesting contributions and details of the technique of calculation are given in the numerous references to original papers. The predominant part of Chapter 9 can be thought as a sum of exercises together with a brief introduction to the problems and a short discussion of the main results. I hope that this part of the book may be interesting also to specialists working in the field of phase transitions.

In the Appendices, I give some mathematical details about the methods for calculation of the functional expansions and perturbation integrals.

This book would not have been completed without the substantial support of Prof. G. Scarpetta and the colleagues from the Department of Theoretical Physics at Salerno University and G. Nadjakov Institute of Solid State Physics of the Bulgarian Academy of Sciences in Sofia. I want to thank my colleagues, the stimulating discussions with them have improved my understanding of the problems of the phase transition theory. They are: Y. Brankov, M. Bushev, K. Chao, L. De Cesare, M. E. Fisher, R. Folk, V. L. Ginzburg, B. I. Halperin, J. A. Hertz, H. Iro, I. D. Lawrie, R. Micnas, L. P. Pitaevskii, V. L. Pokrovskii, D. I. Pushkarov, Kh. Pushkarov, J. Sznajd, N. S. Tonchev, M. Suzuki, and M. Zannetti.

The critical reading of parts of the manuscript by Ms. E. J. Blagoeva, Prof. L. De Cesare, Dr. Y. T. Millev, Dr. C. Noce, Dr. I. Rabuffo, and Dr. A. Romano as well as the technical assistance by Dr. A. Saggese and Dr. V. Scarano in the final preparation of the camera-ready version of the book are gratefully acknowledged.

I am grateful for the hospitality of the Salerno University where this book has been written.

D. I. Uzunov
Salerno, January 1992

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