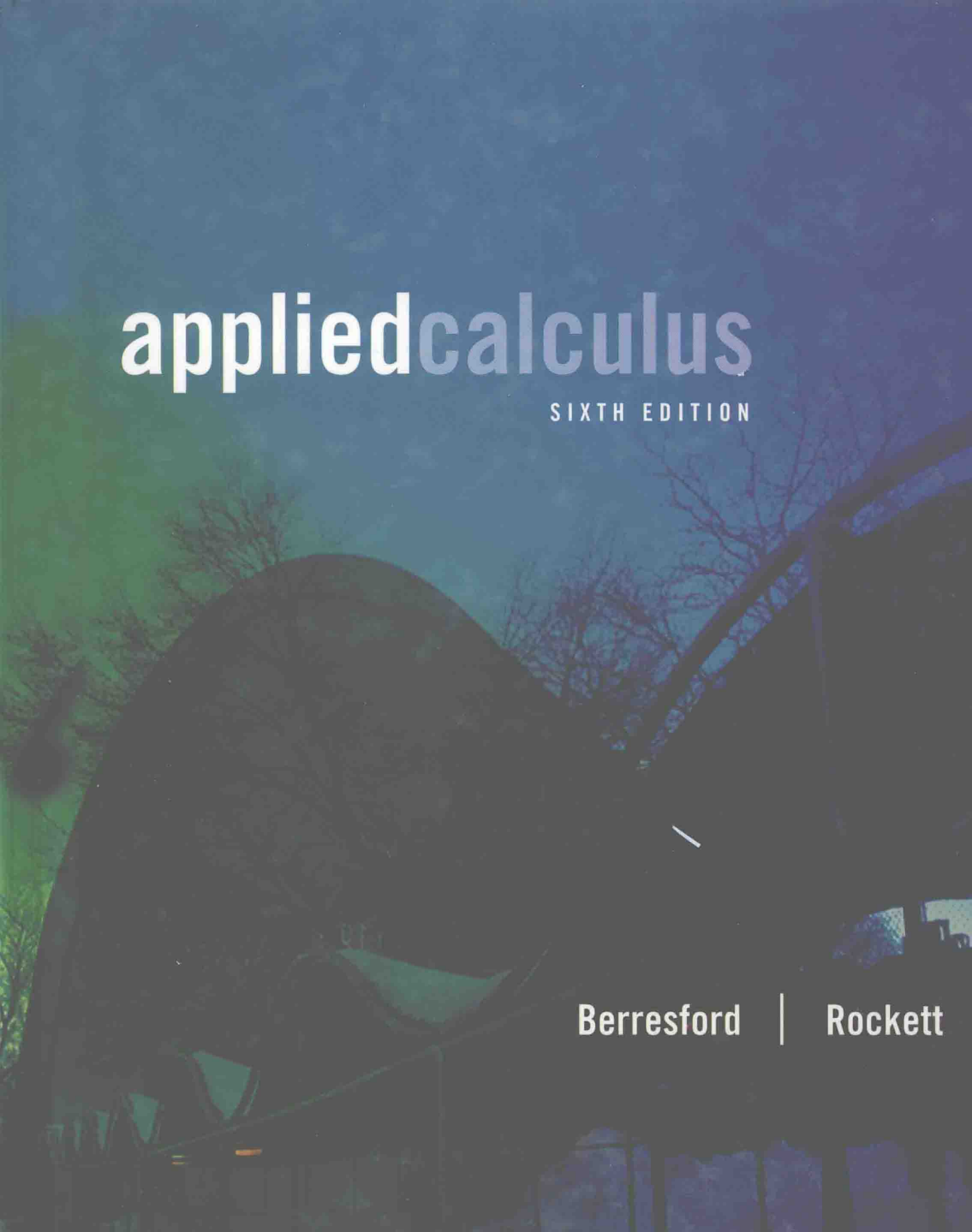
applied calculus

SIXTH EDITION



Berresford | Rockett

appliedcalculus

SIXTH EDITION

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Geoffrey C. Berresford
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PREFACE

A scientific study of yawning found that more yawns occurred in calculus class than anywhere else.* This book hopes to remedy that situation. Rather than being another dry recitation of standard results, our presentation exhibits many of the fascinating and useful applications of mathematics in business, the sciences, and everyday life. Even beyond its utility, however, there is a beauty to calculus, and we hope to convey some of its elegance and simplicity.

This book is an introduction to calculus and its applications to the management, social, behavioral, and biomedical sciences, and other fields. The seven-chapter *Brief Applied Calculus* contains more than enough material for a one-semester course, and the eleven-chapter *Applied Calculus* contains additional chapters on trignometry, differential equations, sequences and series, and probability for a two-semester course. The only prerequisites are some knowledge of algebra, functions, and graphing, which are reviewed in Chapter 1.

CHANGES IN THE SIXTH EDITION

First, what has *not* changed is the essential character of the book: simple, clear, and mathematically correct explanations of calculus, alternating with relevant and engaging examples.

New and Updated Exercises We have added many new exercises, nearly 60 in the areas of Business and Economics alone, including material on isocost and isoquant curves. We have updated well over 100 other exercises, graphs, and examples, including all of the financial problems, with current real-world data, providing sources for all.

New Design The entire book has been recast in an elegant new four-color format to clarify and unify the many features described below and to make the exposition easier and more enjoyable to read.

Other Changes We have made numerous small changes throughout the book to tighten the exposition, include further mathematical steps, and make improvements in existing graphs and add new ones.

PHILOSOPHY

We wrote this book with four principles in mind.

- ***Rapid Start*** When learning something, it is best to begin doing it as soon as possible. Therefore, we keep the preliminary material brief so that students begin calculus without delay (in Section 2.2). An early start allows more time for interesting applications throughout the course.

*Ronald Baenninger, "Some Comparative Aspects of Yawning in Betta splendens, Homo sapiens, Panthera leo, and Papoi spinx," *Journal of Comparative Psychology* **101** (4).

- ***Integration of Mathematics and Applications*** Every section has applications to motivate the mathematics being developed (see, for example, pages 102 and 131–132). There are no "pure math" sections.

- ***Just-in-Time Review*** Review material is placed just before it is used, where it is more likely to be remembered, rather than in lengthy early chapters that "review" material that was never mastered in the first place. Exponential and logarithmic functions are reviewed just before they are differentiated in Section 4.3, and the sine and cosine functions are reviewed just before they are differentiated in Section 8.3, as are the other trigonometric functions in Section 8.5.
- ***Continual Algebra Reinforcement*** Since many of today's students have weak algebra skills, which impede their understanding of calculus, every example has blue annotations in the right margin giving brief explanations of the steps (see, for example, pages 99–101).

FEATURES

Applications The basic nature of courses using this book is very "applied" and therefore this book contains an unusually large number of applications, many appearing in no other textbook.

- ***Application Previews*** Each chapter begins with a motivating *Application Preview* that presents an interesting application of the mathematics developed in that chapter. Each is self-contained (although some exercises may later refer to it). Topics include world records in the mile run (pages 3–4), Stevens' law of psychophysics (page 179), cigarette smoking (pages 325–326), and predicting personal wealth (page 629).
- ***Applications in Every Section*** We explore learning curves in airplane production (pages 27–28 and 32), the age of the Dead Sea Scrolls (pages 284–285), *Apple* and *Oracle* stock prices (page 318), the distance traveled by sports cars (pages 336–337), lives saved by seat belts (page 380), as well as the cost of a congressional victory (page 513). These and many other applications convincingly show that mathematics is more than just the manipulation of abstract symbols and is deeply connected to everyday life.
- ***Construction of Real-World Applications*** We demonstrate in three stages how the functions in the applied exercises are derived: In Chapter 1 we use the regression capabilities of a graphing calculator to construct functions from publicly-available data. We round to get simpler but still accurate functions. Then in Chapter 2 we begin with the (simplified) function, but often show the data so that users may check the accuracy. In later chapters we provide only the functions, having shown how they are derived. In many such cases we have had to balance accuracy with simplicity.

Explanations, Reinforcement, and Review Because calculus involves so many new ideas and techniques, we have followed the explanations with several different levels of reinforcement and review.

- ***Annotations*** As stated earlier, blue annotations to the right of most mathematical formulas and calculations provide explanations and justifications for the steps. They also state results and interpretations in words, emphasizing the important skill of *reading mathematics* (see page 201).

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- This "Be Careful" icon warns of possible misunderstandings (see page 54) or particular difficulties (see page 128).
- ***Practice Problems*** Learning mathematics requires active participation—"mathematics is not a spectator sport." Throughout the readings are short pencil-and-paper *Practice Problems* designed to consolidate understanding of a topic before moving ahead to the next, such as using negative exponents (page 23) or finding and checking an indefinite integral (page 327).
- ***Multilevel Reviews*** *Section Summaries* restate essential formulas and key concepts (see pages 204–205). *Chapter Summaries* review the major developments of the chapter keyed to particular chapter review exercises (see pages 252–253), provide *Hints and Suggestions* to highlight essential facts or "tricks" that might otherwise be overlooked, and conclude with a list of problems for a *Practice Test* on the chapter (see page 253). *Cumulative Reviews* are exercises at the end of groups of chapters to check understanding of the material developed up to that point (see page 257).

Exercises Anyone who ever learned mathematics did so by solving many many problems, and the exercises are the most essential part of the learning process.

- ***Extensive and Varied Exercises*** The exercise sets are graded from routine drills to significant applications (see, for instance, pages 287–291). *Conceptual Exercises* are short true/false, yes/no, or fill-in-the-blank quick-answer questions that many students actually enjoy at the end of a long challenging assignment (see, for example, pages 94–95). *Graphing Calculator Exercises* (optional) are marked by a . Answers to odd-numbered exercises and answers to all Chapter Review exercises are given at the end of the book, with full solutions given in the *Student Solutions Manual.*
- ***Modeling Exercises (Optional)*** We use the regression capabilities of graphing calculators to fit curves to actual data (*linear* on pages 13–15, *power* on pages 28–29, *quadratic* on pages 44–45, *exponential* on pages 64–65 and *logistic* on page 659). Many of these are financial or *Wall Street* problems, predicting, for example, Standard & Poor's industry indexes or satellite radio subscribers (pages 19 and 33).
- ***Explorations and Excursions (Optional)*** At the end of some exercise sets are advanced problems that carry the development of certain topics beyond the level of the text, covering, for example, the Beverton-Holt recruitment curve (page 21), an accessible proof of the Chain Rule based on Carathéodory's definition of the derivative (page 165), the intersection of average and marginal cost curves (page 194), competitive and complementary commodities (page 489), a derivation of the error formula for Taylor approximation (pages 715–716), and the calculus of the normal distribution, including a complete verification that the area under the standard normal curve is 1 (page 794).

Technology Using this book does not require a graphing calculator, but having one will enable you to do many problems more easily and at the same time deepen your understanding by allowing you to concentrate on concepts.

- ***Calculators*** You will need a basic calculator with keys like y^x and LN for powers and natural logarithms for many problems in this book.
- ***Graphing Calculators (Optional)*** If you have a graphing calculator, you will be able to carry out the *Graphing Calculator Explorations* and do the *Graphing Calculator Exercises* (marked by) to:
  - *explore interesting applications*, such as predicting when men and women will achieve equal pay (page 19),
  - *carry out the regression modeling exercises* throughout Chapter 1
  - *show the advantages and limitations of technology*, such as the differences between $\ln x^2$ and $2 \ln x$ (page 281).

The displays shown in the text are from the Texas Instruments *TI-84*, except for a few from the *TI-89*, but any graphing calculator or computer may be used instead. For those who do not have a graphing calculator, the *Explorations* have been designed *to be read simply for enrichment* (see the concavity and maximization explorations on pages 197 and 219).

- ***Programs for Graphing Calculators (Optional)*** For some examples and exercises requiring repeated calculator use we have created graphing calculator programs for use with this book, provided free to all students and faculty (see "How to Obtain Graphing Calculator Programs and Excel spreadsheet" on the next page). The programs may be used for Riemann sums (page 352), trapezoidal approximation (pages 449 and 454), Simpson's rule (pages 453 and 454), slope fields (pages 635 and 637), Euler's method (pages 678 and 680), and Newton's method (page 735).
- ***Spreadsheets (Optional)*** If you have access to a computer, you may wish to do some of the *Spreadsheet Explorations*, which explore nondifferentiable functions (pages 169–170), maximizing an enclosed area (page 217), elasticity of demand (page 315), consumption of natural resources (page 345), improper integrals (page 438), and graphing a function of two variables (page 470). Ancillary materials for Microsoft Excel are also available (see "Supplemental Resources" later in this preface).

Readability and Accuracy We have endeavored to write a book that is both accurate and understandable.

- ***Enhanced Readability*** We have put extra space around all in-line mathematics to make it stand out from the surrounding words. For reading continuity, when we refer to earlier material we try to restate the result that is being used. When references are necessary, we give explicit page numbers.
- ***Accuracy and Proofs*** All examples, problems, and answers have been carefully checked by several mathematicians. The statements of definitions and theorems are mathematically accurate. Because this book is *applied* rather than *theoretical*, we have preferred intuitive and geometric justifications to formal proofs. We provide a justification or proof for every important mathematical idea. When proofs are given, they are correct and honest.

CENGAGEbrain ***Resources on the Web*** Additional materials available on the Internet at `www.cengagebrain.com` include:

Suggestions for **Projects and Essays,** open-ended topics that ask students (individually or in groups) to research a relevant person or idea, to compare several different mathematical ideas, or to relate a concept to their lives (such as marginal and average cost, why two successive 10% increases don't add up to a 20% increase, elasticity of supply of drugs and alcohol, and arithmetic versus geometric means).

An expanded collection of **Application Previews,** short essays that were used in an earlier edition to introduce each section. Topics include *Exponential Functions and the World's Worst Currency; Size, Shape, and Exponents;* and *The Confused Creation of Calculus.*

HOW TO OBTAIN GRAPHING CALCULATOR PROGRAMS AND EXCEL SPREADSHEETS

The optional graphing calculator programs used in the text have been written for a variety of Texas Instruments Graphing Calculators (*TI-83, TI-84,* and *TI-89* among others), and may be obtained for free in either of the following ways:

- If you know someone who already has the programs on a Texas Instruments graphing calculator like yours, you can easily transfer the programs from their calculator to yours using the cable that came with the calculator and the LINK button.
- You may download the programs and instructions from the Cengage website. After turning off all pop-up blockers in your browser, go to `www.cengage.com/math/berresford`. Select this textbook by clicking on the cover or title. A new window will open. Find the CENGAGEbrain logo on the right-hand side of the page and click on **Students: Access Free Companion Content** just below it. A new window will open. In this new window, click the **Access Now** button and you will arrive at the window for the student resources for this book. The link to the Graphing Calculator Programs is listed under **Book Resources**. Click on it and then download the ZIP archive of programs and instructions. After opening this archive, select the program you want by its name and the number for the model of your TI calculator (for the *TI-84,* use the program names ending in 83). The instructions are PDF files and the programs are individually archived. Follow the directions for linking your calculator to your computer using TI-GraphLink or TI-Connect as appropriate (further information may be found at the Texas Instruments website `education.ti.com`).

The Microsoft Excel spreadsheets used in the Spreadsheet Explorations may be downloaded from the Cengage website. After turning off all pop-up blockers in your browser, go to `www.cengage.com/math/berresford`. Select this textbook by clicking on the cover or title. A new window will open. Find the CENGAGEbrain logo on the right-hand side of the page and click on **Students: Access Free Companion Content** just below it. A new window will open. In this new window, click the **Access Now** button and you will arrive at the window for the student resources for this book. Select the appropriate chapter in

the pull-down menu and the link to the Excel Spreadsheet Exploration file will then be listed under the **Book Overview** for that chapter. Click on it and then download the `.xls` file to your computer.

Instructors may also find the graphing calculator programs and Excel spreadsheets at the **Instructor Companion Site** for this book without having to "sign-on" as these are *not* "password-protected" resources.

SUPPLEMENTAL RESOURCES

This text has a wide array of valuable supplemental material.

Print The **Student Solutions Manual** (ISBN 10: 1133104010, ISBN 13: 9781133104018), prepared by Christi Verity, provides worked-out solutions to most of the odd-numbered problems in the text.

CENGAGEbrain ***On-Line*** The **Excel Guide** by Revathi Narasimhan provides a list of exercises from the text that can be completed after each step-by-step Excel example. No prior knowledge of Excel is necessary.

To access this and other course materials and companion resources, please visit `www.cengagebrain.com`. At the CengageBrain.com home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where free companion resources can be found.

This book includes **CourseMate**, an online complement to the textbook. CourseMate includes:

- An interactive eBook with highlighting, note taking, and search capabilities
- Interactive teaching and learning tools including:
  - Online quizzes
  - Conceptual applets
  - Flashcards
  - Videos
  - and more
- Engagement Tracker, a first-of-its-kind tool that monitors student engagement in the course

Go to `login.cengage.com` to access these resources, and look for the icon, which denotes a resource available within CourseMate.

The **Solution Builder** online instructor database offers complete worked solutions to all exercises in the text, allowing you to create customized, secure solutions printouts (in PDF format) matched exactly to the problems you assign in class. Access available via `www.cengage.com/solutionbuilder` or the PowerLecture CD (see PowerLecture description).

Diploma® testing software allows instructors to quickly create, deliver, and customize tests for class in print and online formats and features automatic

grading. Included is a test bank with hundreds of questions customized directly to the text, with all questions also provided in PDF and Microsoft Word formats for instructors who opt not to use the software component. The test questions are grouped by section and are a combination of multiple-choice and free-response questions. Diploma is available within the PowerLecture CD (see PowerLecture description).

The **PowerLecture™** CD-ROM (ISBN 10: 1133104460, ISBN 13: 9781133104469) provides the instructor with dynamic media tools for teaching. Create, deliver, and customize tests (both print and online) in minutes with Diploma computerized testing featuring algorithmic equations. Easily build solution sets for homework or exams using Solution Builder's online solutions manual. Microsoft PowerPoint lecture slides and figures from the book are also included on this CD-ROM.

ENHANCED WebAssign

Technology **Enhanced WebAssign** Cengage Learning's course management system allows you to create online homework assignments that draw from thousands of end-of-chapter questions that match this textbook. Problems are enhanced with rich tutorial content like Watch It Videos and Animations, Practice It and Master It Tutorials and access to the relevant textbook sections. Flexible assignment options allow you to choose how feedback and tutorial content is released to students as well as the ability to conditionally release assignments based on your students' prerequisite assignment scores. This access card also provides students access to an interactive Premium eBook, which offers search, highlighting, and note-taking functionality, as well as links to multimedia resources. Increase student engagement, improve course outcomes, and experience the superior service offered through CourseCare. Visit us at `http://webassign.net/cengage` to learn more.

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COMMENTS WELCOMED

With the knowledge that any book can always be improved, we welcome corrections, constructive criticisms, and suggestions from every reader.

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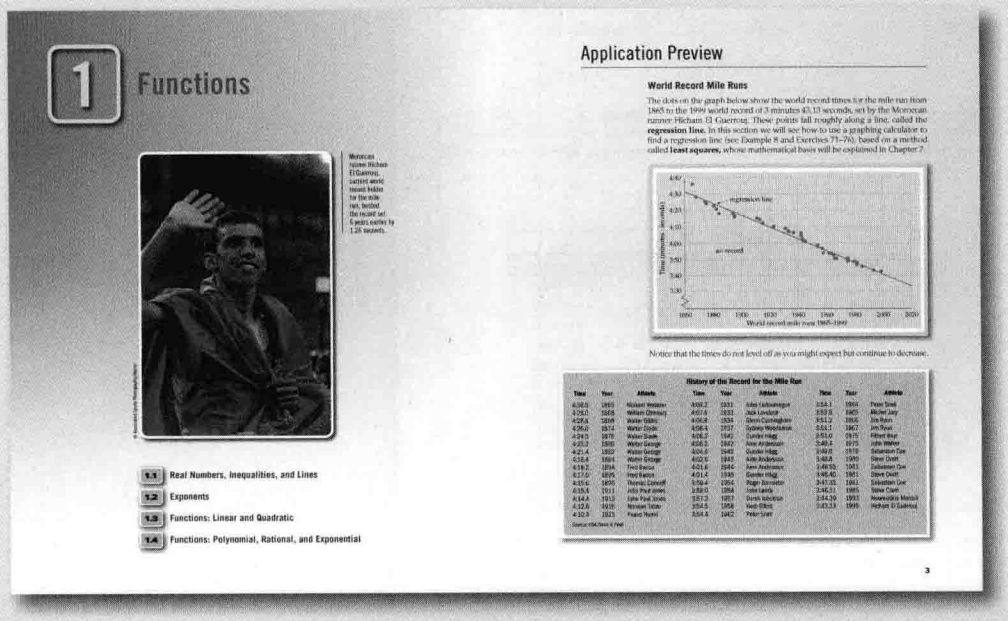
A User's Guide to Features

▼ Application Preview

Found on every chapter opener page, Application Previews motivate the chapter. They offer a unique "mathematics in your world" application or an interesting historical note. A page with further information on the topic, and often a related exercise number, is referenced.



1 Functions

Moroccan runner Hicham El Guerrouj, current world record holder for the mile run, broke the record set 6 years earlier by 1.26 seconds.

1.1 Real Numbers, Inequalities, and Lines
1.2 Exponents
1.3 Functions: Linear and Quadratic
1.4 Functions: Polynomial, Rational, and Exponential

Application Preview

World Record Mile Runs

The dots on the graph below show the world record times for the mile run from 1865 to the 1999 world record of 3 minutes 43.13 seconds, set by the Moroccan runner Hicham El Guerrouj. These points fall roughly along a line, called the **regression line.** In this section we will see how to use a graphing calculator to find a regression line (see Example 8 and Exercises 71–76), based on a method called **least squares,** whose mathematical basis will be explained in Chapter 7.

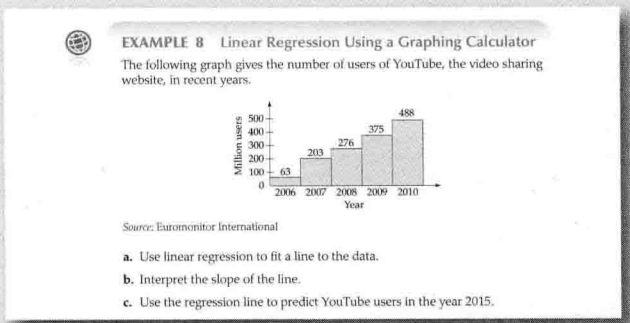
Notice that the times do not level off as you might expect but continue to decrease.

History of the Record for the Mile Run

Time	Year	Athlete	Time	Year	Athlete	Time	Year	Athlete
4:28.8	1865	Richard Webster	4:02.2	1931	Jules Ladoumègue	3:54.1	1954	Roger Bannister
4:28.0	1868	William Chinnery	4:01.6	1933	Jack Lovelock	3:53.6	1957	Michel Jazy
4:24.6	1868	Walter Gibbs	4:06.8	1934	Glenn Cunningham	3:51.3	1958	Jim Ryun
4:26.0	1874	Walter Slade	4:04.6	1937	Sydney Wooderson	3:51.1	1967	Jim Ryun
4:24.5	1875	Walter Slade	4:06.2	1942	Gunder Hägg	3:51.0	1975	Filbert Bayi
4:23.2	1895	Walter George	4:06.2	1942	Arne Andersson	3:49.4	1975	John Walker
4:21.4	1895	Walter George	4:04.6	1942	Gunder Hägg	3:49.0	1979	Sebastian Coe
4:18.4	1911	Wilfred Eveborge	4:02.6	1943	Arne Andersson	3:48.8	1980	Steve Ovett
4:19.2	1916	Fred Bacon	4:01.6	1944	Arne Andersson	3:48.53	1981	Sebastian Coe
4:17.6	1895	Fred Bacon	4:01.4	1945	Gunder Hägg	3:48.40	1981	Steve Ovett
4:15.6	1895	Thomas Conneff	3:58.4	1954	Roger Bannister	3:47.33	1981	Sebastian Coe
4:15.6	1911	John Paul Jones	3:58.0	1954	John Landy	3:46.31	1985	Steve Cram
4:14.4	1913	John Paul Jones	3:57.2	1957	Derek Ibbotson	3:44.39	1993	Noureddine Morceli
4:12.6	1915	Norman Taber	3:54.5	1958	Herb Elliott	3:43.13	1999	Hicham El Guerrouj
4:10.9	1923	Paavo Nurmi	3:54.4	1962	Peter Snell			

Source: Track & Field

3



EXAMPLE 8 Linear Regression Using a Graphing Calculator

The following graph gives the number of users of YouTube, the video sharing website, in recent years.

Source: Euromonitor International

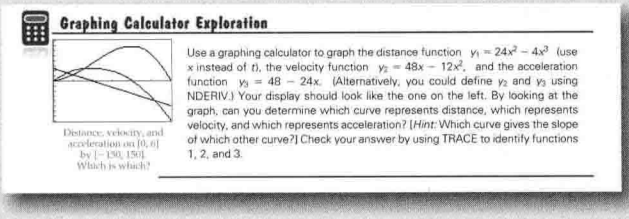
a. Use linear regression to fit a line to the data.

b. Interpret the slope of the line.

c. Use the regression line to predict YouTube users in the year 2015.

◀ Real World Icon

This globe icon marks examples in which calculus is connected to every-day life.



Graphing Calculator Exploration

Distance, velocity, and acceleration on [0, 6] by [−150, 150]. Which is which?

Use a graphing calculator to graph the distance function $y_1 = 24x^2 - 4x^3$ (use x instead of t), the velocity function $y_2 = 48x - 12x^2$, and the acceleration function $y_3 = 48 - 24x$. (Alternatively, you could define y_2 and y_3 using NDERIV.) Your display should look like the one on the left. By looking at the graph, can you determine which curve represents distance, which represents velocity, and which represents acceleration? [*Hint:* Which curve gives the slope of which other curve?] Check your answer by using TRACE to identify functions 1, 2, and 3.

◀ Graphing Calculator Explorations

To allow for optional use of the graphing calculator, the Explorations are boxed. Most can also be read simply for enrichment. Exercises and examples that are designed to be done with a graphing calculator are marked with an icon.

Spreadsheet Explorations

Boxed for optional use, these spreadsheets will enhance students' understanding of the material using Excel, an alternative for those who prefer spreadsheet technology. See "Integrating Excel" on page xx for a list of exercises that can be done with Excel.

Spreadsheet Exploration

Another function that is not differentiable is $f(x) = x^{2/3}$. The following spreadsheet* calculates values of the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ at $x = 0$ for this function. Since $f(0) = 0$, the difference quotient at $x = 0$ simplifies to:

$$\frac{f(x+h) - f(x)}{h} = \frac{f(0+h) - f(0)}{h} = \frac{f(h)}{h} = \frac{h^{2/3}}{h} = h^{-1/3}$$

For example, cell **B5** evaluates $h^{-1/3}$ at $h = \frac{1}{1000}$ obtaining $(\frac{1}{1000})^{-1/3} = 1000^{1/3} = \sqrt[3]{1000} = 10$. Column **B** evaluates this different quotient for the *positive* values of h in column **A**, while column **E** evaluates it for the corresponding negative values of h in column **D**.

B5 = =A5^(-1/3)

	A	B	C	D	E
1	h	(f(0+h)-f(0))/h		h	(f(0+h)-f(0))/h
2	1.0000000	1.0000000		-1.0000000	-1.0000000
3	0.1000000	2.1544347		-0.1000000	-2.1544347
4	0.0100000	4.6415888		-0.0100000	-4.6415888
5	0.0010000	10.0000000		-0.0010000	-10.0000000
6	0.0001000	21.5443469		-0.0001000	-21.5443469
7	0.0000100	46.4158883		-0.0000100	-46.4158883
8	0.0000010	100.0000000		-0.0000010	-100.0000000
9	0.0000001	215.4434690		-0.0000001	-215.4434690

becoming large — becoming small

Notice that the values in column **B** are becoming arbitrarily large, while the values in column **E** are becoming arbitrarily small, so the difference quotient does not approach a limit as $h \to 0$. This shows that the derivative of $f(x) = x^{2/3}$ at 0 *does not exist*, so the function $f(x) = x^{2/3}$ is *not differentiable* at $x = 0$.

Practice Problems

Students can check their understanding of a topic as they read the text or do homework by working out a Practice Problem. *Complete solutions* are found at the end of each section, just before the Section Summary.

Be Careful

The "Be Careful" icon marks places where the authors help students avoid common errors.

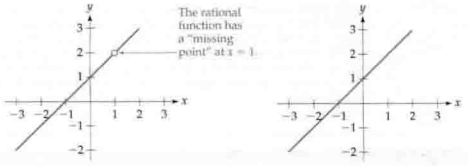
PRACTICE PROBLEM 2

What is the domain of $f(x) = \dfrac{18}{(x+2)(x-4)}$? Solution on page 65 ▶

Be Careful Simplifying a rational function by canceling a common factor from the numerator and the denominator can change the domain of the function, so that the "simplified" and "original" versions may not be equal (since they have different domains). For example, the rational function on the left below is not defined at $x = 1$, while the simplified version on the right *is* defined at $x = 1$, so that the two functions are technically not equal.

$$\underbrace{\frac{x^2 - 1}{x - 1}}_{\text{Not defined at } x = 1,\ \text{so the domain is } \{x \mid x \neq 1\}} = \underbrace{\frac{(x+1)(x-1)}{x-1}}_{\text{Is defined at } x = 1,\ \text{so the domain is } \mathbb{R}} \neq x + 1$$

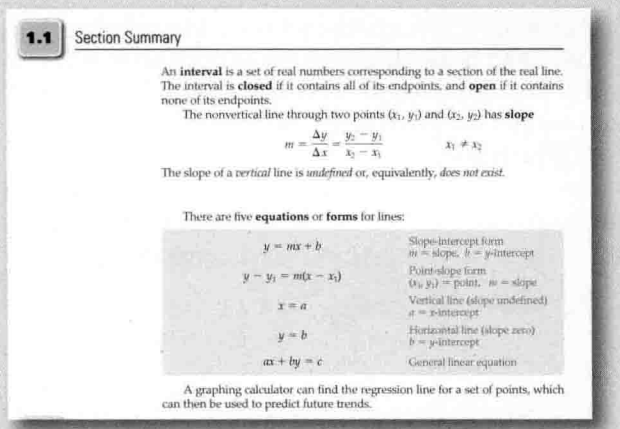
However, the functions *are* equal at every x-value *except* $x = 1$, and the graphs below are the same except that the rational function omits the point at $x = 1$. We will return to this technical issue when we discuss limits in Section 2.1.



Graph of $y = \dfrac{x^2 - 1}{x - 1}$ Graph of $y = x + 1$

▼ Section Summary

Found at the end of every section, these summaries briefly state the main ideas of the section, providing a study tool or reminder for students.



1.1 Section Summary

An **interval** is a set of real numbers corresponding to a section of the real line. The interval is **closed** if it contains all of its endpoints, and **open** if it contains none of its endpoints.

The nonvertical line through two points (x_1, y_1) and (x_2, y_2) has **slope**

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_1 \neq x_2$$

The slope of a *vertical* line is *undefined* or, equivalently, *does not exist*.

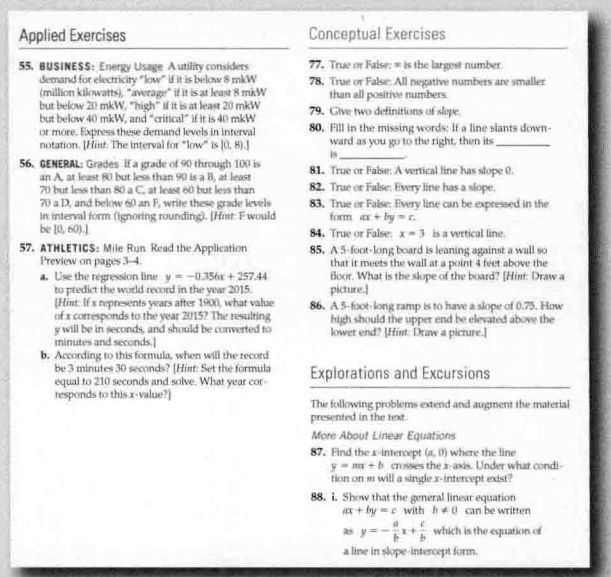
There are five **equations** or **forms** for lines:

$y = mx + b$	Slope-intercept form m = slope, b = y-intercept
$y - y_1 = m(x - x_1)$	Point-slope form (x_1, y_1) = point, m = slope
$x = a$	Vertical line (slope undefined) a = x-intercept
$y = b$	Horizontal line (slope zero) b = y-intercept
$ax + by = c$	General linear equation

A graphing calculator can find the regression line for a set of points, which can then be used to predict future trends.

Exercises ▼

The *Applied Exercises* are labeled with general and specific titles so instructors can assign problems appropriate for the class. *Conceptual Exercises* encourage students to "think outside the box," and *Explorations and Excursions* push students further.



Applied Exercises

55. BUSINESS: Energy Usage A utility considers demand for electricity "low" if it is below 8 mkW (million kilowatts), "average" if it is at least 8 mkW but below 20 mkW, "high" if it is at least 20 mkW but below 40 mkW, and "critical" if it is 40 mkW or more. Express these demand levels in interval notation. [*Hint:* The interval for "low" is $[0, 8)$.]

56. GENERAL: Grades If a grade of 90 through 100 is an A, at least 80 but less than 90 is a B, at least 70 but less than 80 a C, at least 60 but less than 70 a D, and below 60 an F, write these grade levels in interval form (ignoring rounding). [*Hint:* F would be $[0, 60)$.]

57. ATHLETICS: Mile Run Read the Application Preview on pages 3–4.

a. Use the regression line $y = -0.356x + 257.44$ to predict the world record in the year 2015. [*Hint:* If x represents years after 1900, what value of x corresponds to the year 2015? The resulting y will be in seconds, and should be converted to minutes and seconds.]

b. According to this formula, when will the record be 3 minutes 30 seconds? [*Hint:* Set the formula equal to 210 seconds and solve. What year corresponds to this x-value?]

Conceptual Exercises

77. True or False: ∞ is the largest number.

78. True or False: All negative numbers are smaller than all positive numbers.

79. Give two definitions of *slope*.

80. Fill in the missing words: If a line slants downward as you go to the right, then its _____ is _____.

81. True or False: A vertical line has slope 0.

82. True or False: Every line has a slope.

83. True or False: Every line can be expressed in the form $ax + by = c$.

84. True or False: $x = 3$ is a vertical line.

85. A 5-foot-long board is leaning against a wall so that it meets the wall at a point 4 feet above the floor. What is the slope of the board? [*Hint:* Draw a picture.]

86. A 5-foot-long ramp is to have a slope of 0.75. How high should the upper end be elevated above the lower end? [*Hint:* Draw a picture.]

Explorations and Excursions

The following problems extend and augment the material presented in the text.

More About Linear Equations

87. Find the x-intercept $(a, 0)$ where the line $y = mx + b$ crosses the x-axis. Under what condition on m will a single x-intercept exist?

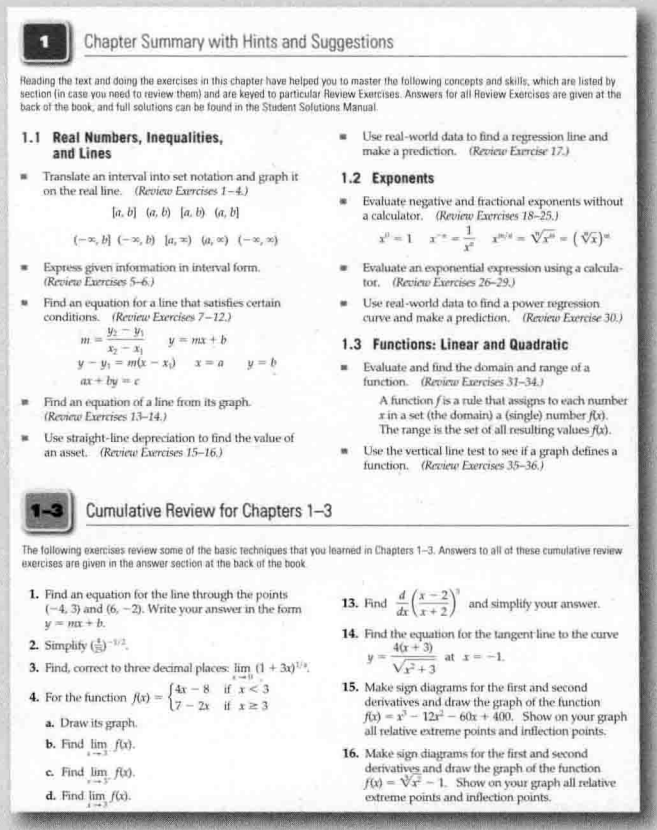
88. i. Show that the general linear equation $ax + by = c$ with $b \neq 0$ can be written as $y = -\frac{a}{b}x + \frac{c}{b}$, which is the equation of a line in slope-intercept form.

End of Chapter Material ➤

To help students study, each chapter ends with a **Chapter Summary with Hints and Suggestions** and **Review Exercises.** The last bullet of the Hints and Suggestions lists the Review Exercises that a student could use to self-test. Both even and odd answers are supplied in the back of the book.

Cumulative Review

There is a Cumulative Review after every 3–4 chapters. Even and odd answers are supplied in the back of the book.



1 Chapter Summary with Hints and Suggestions

Reading the text and doing the exercises in this chapter have helped you to master the following concepts and skills, which are listed by section (in case you need to review them) and are keyed to particular Review Exercises. Answers for all Review Exercises are given at the back of the book, and full solutions can be found in the Student Solutions Manual.

1.1 Real Numbers, Inequalities, and Lines

- Translate an interval into set notation and graph it on the real line. *(Review Exercises 1–4.)*

  $[a, b]$ (a, b) $[a, b)$ $(a, b]$

  $(-\infty, b]$ $(-\infty, b)$ $[a, \infty)$ (a, ∞) $(-\infty, \infty)$

- Express given information in interval form. *(Review Exercises 5–6.)*
- Find an equation for a line that satisfies certain conditions. *(Review Exercises 7–12.)*

  $m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad y = mx + b$

  $y - y_1 = m(x - x_1)$ $\quad x = a$ $\quad y = b$

  $ax + by = c$
- Find an equation of a line from its graph. *(Review Exercises 13–14.)*
- Use straight-line depreciation to find the value of an asset. *(Review Exercises 15–16.)*
- Use real-world data to find a regression line and make a prediction. *(Review Exercise 17.)*

1.2 Exponents

- Evaluate negative and fractional exponents without a calculator. *(Review Exercises 18–25.)*

  $x^0 = 1$ $\quad x^{-n} = \frac{1}{x^n}$ $\quad x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$
- Evaluate an exponential expression using a calculator. *(Review Exercises 26–29.)*
- Use real-world data to find a power regression curve and make a prediction. *(Review Exercise 30.)*

1.3 Functions: Linear and Quadratic

- Evaluate and find the domain and range of a function. *(Review Exercises 31–34.)*

  A function f is a rule that assigns to each number x in a set (the domain) a (single) number $f(x)$. The range is the set of all resulting values $f(x)$.
- Use the vertical line test to see if a graph defines a function. *(Review Exercises 35–36.)*

1–3 Cumulative Review for Chapters 1–3

The following exercises review some of the basic techniques that you learned in Chapters 1–3. Answers to all of these cumulative review exercises are given in the answer section at the back of the book.

1. Find an equation for the line through the points $(-4, 3)$ and $(6, -2)$. Write your answer in the form $y = mx + b$.

2. Simplify $\left(\frac{4}{25}\right)^{-3/2}$.

3. Find, correct to three decimal places: $\lim_{x \to 0} (1 + 3x)^{2/x}$.

4. For the function $f(x) = \begin{cases} 4x - 8 & \text{if } x < 3 \\ 7 - 2x & \text{if } x \geq 3 \end{cases}$

a. Draw its graph.

b. Find $\lim_{x \to 3^-} f(x)$.

c. Find $\lim_{x \to 3^+} f(x)$.

d. Find $\lim_{x \to 3} f(x)$.

13. Find $\frac{d}{dx}\left(\frac{x-2}{x+2}\right)^3$ and simplify your answer.

14. Find the equation for the tangent line to the curve $y = \frac{4(x+3)}{\sqrt{x^2+3}}$ at $x = -1$.

15. Make sign diagrams for the first and second derivatives and draw the graph of the function $f(x) = x^3 - 12x^2 - 60x + 400$. Show on your graph all relative extreme points and inflection points.

16. Make sign diagrams for the first and second derivatives and draw the graph of the function $f(x) = \sqrt[3]{x^2} - 1$. Show on your graph all relative extreme points and inflection points.

Integrating Excel

If you would like to use Excel or another spreadsheet software when working the exercises in this text, refer to the chart below. It lists exercises from many sections that you might find instructive to do with spreadsheet technology. If you would like help using Excel, please consider the *Excel Guide* (see page xiv of this preface for a description and access instructions).

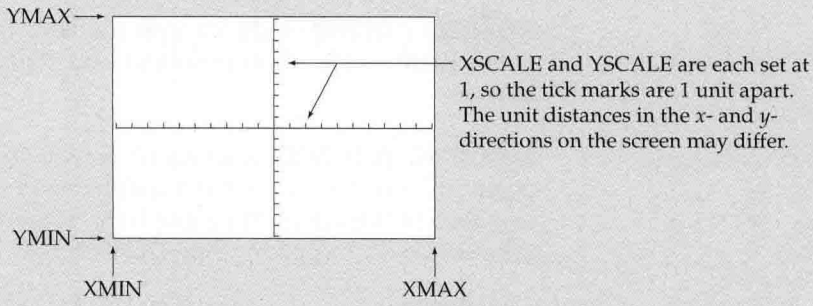
Section	Suggested Exercises
1.1	57–76
1.2	95–102
1.3	69–82, 84–92
1.4	79–94
2.1	77–78, 81–82
2.2	9–16
2.3	47–50
2.4	61–64
2.5	45–46
2.6	65, 69
2.7	11–12
3.1	68–71, 85
3.2	63–64
3.3	25–40
3.4	23–24
3.5	20
3.6	69–70
4.1	11–12, 47–49
4.2	31–50
4.3	91–93
4.4	38–39
5.1	41–42
5.2	45–46, 55–58
5.3	13–18, 83–84, 87–88
5.4	32, 35–36, 69
5.5	31–32
5.6	77–78
6.1	63–64
6.2	65, 66, 68
6.3	41–42
6.4	9–18, 27–37
7.1	29–30, 38–42
7.2	47–48, 53–56
7.3	29–32
7.4	13–18, 27–32
7.5	29–36
7.6	31–32, 35–36
7.7	41–42
8.1	9–20
8.2	36–41
8.3	73–80
8.4	49–54
8.5	13–16, 20–26
9.1	71
9.2	54
9.3	36–38, 41–44
9.4	11–24, 27–30
10.1	49–58
10.2	9–12, 21–22
10.3	35–36
10.4	11–24, 33–38
11.1	29–36
11.2	37–41
11.3	23
11.4	21–26

Graphing Calculator Basics

While the (optional) *Graphing Calculator Explorations* may be carried out on most graphing calculators, the screens shown in this book are from the Texas Instruments *TI-84* calculator.* Any specific instructions work with the entire family of *TI-83* and *TI-84* calculators. (We occasionally show a screen from a *TI-89* calculator, but for illustration purposes only.) To carry out the Graphing Calculator Explorations, you should be familiar with the terms described in *Graphing Calculator Terminology* below. To do the *regression* (or *modeling*) examples and exercises in Chapter 1 (again optional), you should be familiar with the techniques in the following section headed *Entering Data.*

GRAPHING CALCULATOR TERMINOLOGY

The viewing or graphing **WINDOW** is the part of the Cartesian plane shown in the display screen of your graphing calculator. **XMIN** and **XMAX** are the smallest and largest x-values shown, and **YMIN** and **YMAX** are the smallest and largest y-values shown. These values can be set by using the **WINDOW** or **RANGE** command and are changed automatically by using any of the **ZOOM** operations. **XSCALE** and **YSCALE** define the distance between tick marks on the x- and y-axes.



Viewing Window [−10, 10] by [−10, 10]

The viewing window is always [XMIN, XMAX] by [YMIN, YMAX]. We will set XSCALE and YSCALE so that there are a reasonable number of tick marks (generally 2 to 20) on each axis. The x- and y-axes will not be visible if the viewing window does not include the origin.

Pixel, an abbreviation for *pic*ture *el*ement, refers to a tiny rectangle on the screen that can be darkened to represent a dot on a graph. Pixels are arranged in a rectangular array on the screen. In the above window, the axes and tick marks are formed by darkened pixels. The size of the screen and number of pixels vary with different calculators.

*Screens may differ depending on the calculator's operating system.

TRACE allows you to move a flashing pixel, or *cursor,* along a curve in the viewing window with the x- and y-coordinates shown at the bottom of the screen.

> *Useful Hint:* To make the x-values in TRACE take simple values like .1, .2, and .3, choose XMIN and XMAX to be multiples of one less than the number of pixels across the screen. For example, on the *TI-84,* which has 95 pixels across the screen, using an x-window like [–9.4, 9.4] or [–4.7, 4.7] or [940, –940] will TRACE with simpler x-values than the standard windows stated in this book.

ZOOM IN allows you to magnify any part of the viewing window to see finer detail around a chosen point. **ZOOM OUT** does the opposite, like stepping back to see a larger portion of the plane but with less detail. These and other **ZOOM** commands change the viewing window.

VALUE or **EVALUATE** finds the value of a previously entered expression at a specified x-value.

SOLVE or **ROOT** finds the x-value that solves $f(x) = 0$, equivalently, the x-intercepts of a curve. When applied to a difference $f(x) - g(x)$, it finds the x-value where the two curves meet (also done by the **INTERSECT** command).

MAX and **MIN** find the maximum and minimum values of a previously entered curve between specified x-values.

NDERIV or **DERIV** or **dy/dx** approximates the *derivative* of a function at a point. **FnInt** or **∫f(x)dx** approximates the definite integral of a function on an interval.

In **CONNECTED MODE** your calculator will darken pixels to connect calculated points on a graph to show it as a continuous or "unbroken" curve. However, this may lead to "false lines" in a graph that should have breaks or "jumps." False lines can be eliminated by using **DOT MODE.**

The **TABLE** command lists in table form the values of a function, just as you have probably done when graphing a curve. The x-values may be chosen by you or by the calculator.

The **Order of Operations** used by most calculators evaluates operations in the following order: first powers and roots, then operations like **LN** and **LOG,** then multiplication and division, then addition and subtraction—left to right within each level. For example, 5 ^ 2x means (5 ^ 2)x, *not* 5 ^ (2x). Also, $1/x + 1$ means $(1/x) + 1$, *not* $1/(x + 1)$. See your calculator's instruction manual for further information.

Be Careful Some calculators evaluate $1/2x$ as $(1/2)x$ and some as $1/(2x)$. When in doubt, use parentheses to clarify the expression.

Much more information can be found in the manual for your graphing calculator. Other features will be discussed later as needed.

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