QUANTITATIVE TECHNIQUES

An Instructional Manual for Business and Accountancy Students

T. LUCEY

D.P. PUBLICATIONS

Quantitative Techniques

AN INSTRUCTIONAL MANUAL FOR
BUSINESS AND ACCOUNTANCY STUDENTS

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Each question used is cross referenced to the appropriate Institute or Association, the title of the paper and the date of the examination.

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Preface

AIMS OF THE MANUAL

1. This manual is designed to provide a sound understanding of Quantitative Techniques and is particularly relevant for;

a. Students preparing themselves for Professional Examinations of the following bodies; Institute of Chartered Accountants, Association of Certified Accountants, Institute of Cost and Management Accountants, and Chartered Institute of Public Finance and Accountancy.

b. Students on BEC Higher Level courses, and undergraduates reading Business Studies, Accounting and allied subjects.

c. Managers and others in industry, commerce, and local authorities who wish to obtain a working knowledge of quantitative techniques to aid them in their own work and to facilitate communication with accountants and operational research specialists.

TEACHING APPROACH

2. The manual has been written in a standardised format with numbered paragraphs, end of chapter summaries, with review questions and examination questions at the end of each chapter. This approach has been tested and found effective by thousands of students and the manual can be used for independent study or in conjunction with tuition at a college.

HOW TO USE THE MANUAL EFFECTIVELY

3. For ease of study the manual is divided into self contained chapters with numbered paragraphs. Each chapter is followed by *self review* questions, cross referenced to appropriate paragraph(s). You should attempt to answer the self review questions *unaided* then check your answer with the text.

In addition, each chapter has one or more *examination* type questions with a suggested solution, which includes notes on approach and on particular points of difficulty. Naturally examination questions are often wide ranging and include material covering more than one chapter. They are however placed at an appropriate point in the manual so that if the previous chapters have been studied all the necessary material will be available. The examination questions have mostly been drawn from past professional examinations and have been carefully selected not merely to repeat the material in the chapters but to extend knowledge and understanding. They should be considered an integral part of the manual. *Always* make some attempt at the question before reading the solution.

SEQUENCE OF STUDY

4. The manual should be studied in the sequence of the chapters. The sequence has been arranged so that there is a progressive accumulation of knowledge and any given chapter either includes all the principles necessary or draws upon a previous chapter(s).

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1 Introduction to Quantitative Techniques

1. Introduction.
This Chapter serves as an introduction to the whole book. It provides some basic definitions and clarifies some of the terminology used. The stages in an Operational Research study are explained and the contents of the book are related to the syllabuses of the major professional bodies.

2. Differences in Terminology.
The title used for this book is Quantitative Techniques. However, many other terms are also used for the concepts and techniques described in the book. These include – Management Mathematics – Operational Research or O.R. – Analytical Techniques – Elements of Financial Decisions – Business Mathematics and Management Science. Regardless of the name used it is important to realise that the same group of techniques are being described and, more importantly, the same approach is employed. This approach is brought out by the following British Standard definition of Operational Research.

3. Operational Research definition:
"The attack of modern science on complex problems arising in the direction and management of large systems of men, machines, materials and money, in industry, business, government and defence. The distinctive approach is to develop a scientific model of the system incorporating measurements of factors such as chance and risk, with which to predict and compare the outcome of alternative decisions, strategies or controls. The purpose is to help management determine its policy and action scientifically."

4. Essential Features of the O.R. Approach.
The above formal definition contains several essential features and these are:

a. Application of a model-based scientific approach,
b. Systems approach to organisations,
c. The recognition of risk and uncertainty,
d. Assistance to management decision making and control.

5. Application of a model-based scientifc Approach.
The basis of the O.R. approach is that of constructing models of problems in an objective, factual manner and experimenting with these models to show the results of various possible courses of action. A model is any representation of reality and may be in graphical, physical or mathematical terms. The type of model most frequently used in O.R. is a mathematical model, ie one which tries to show the workings of the real world by means of mathematical symbols, equations and formulae. An example of a very simple mathematical model familiar to accountants could be the following equation to estimate the total overheads for a period. Assume that total overheads comprise fixed overheads and variable overheads which are directly related to the units produced, then the equation:–

$$y = a + b x$$

is a model of the relationship of total overheads to the number of units produced where

y = Total overheads
a = Fixed overheads per period
b = Variable overheads per unit (assumed to be constant)
x = Number of units produced

The above model is obviously very simple and most practical models are of necessity much more complex. It is important to realise that however complex looking is the model and however many variables it contains, it still involves considerable simplification of reality and any results or predictions obtained from the model must therefore be used with caution and judgement.

6. Systems Approach to Organisation.

The primary aim of O.R. is to attempt to identify the best way of conducting the affairs of the organisation, ie the optimum. In studying problems the O.R. man tries to optimise the operation of the organisation as a whole rather than narrow aspects of the business such as a single department or section. This is easier said than done and because of the practical necessity of dealing with manageable areas of work and thereby producing simplified and incomplete models of operations there may be a tendency to produce sub-optimal solutions, ie a solution which is optimal for a small section of the firm, but not optimal for the firm as a whole. This is another point which should be watched when considering the results of an O.R. investigation.

7. Recognition of Risk and Uncertainty.

All business planning and decision making involves forecasting future activities. This cannot be done with any certainty and so to provide the maximum possible assistance to planners and decision makers a systematic analysis of the possible extent of the risks and uncertainties involved is a vital part of any O.R. study. O.R. techniques do not of themselves remove the risks and uncertainties, but they are able to highlight their effects on the firm's operations.

8. Assistance to Management Decision-making and Control.

In general O.R. practicioners do not make the business decisions. Their role is the provision of information to assist the planners and decision makers. The skill, experience and judgement of managers cannot be replaced by formal decision making techniques. The results of an O.R. investigation are but one input of information into the decision making process. There is a strong parallel between the O.R. practicioner supplying information for management decision making and that of other information specialists such as accountants. This is why a knowledge of O.R. for the accountant (and accountancy for the O.R. man!) can be very useful and can improve the quality of the information provided. There are strong reasons why the most effective O.R. teams contain people drawn from various backgrounds – economists, accountants, mathematicians, engineers, psychologists etc. In this way there is more chance that the numerous facets of business problems can be recognised and analysed.

9. Quantification of Factors.

Not all of the factors involved in a decision making or planning situation can be quantified, but the most readily usable O.R. techniques are those based on quantifiable factors such as costs, revenues, number of units etc. These techniques are the ones most commonly included in examinations and form the basis of the syllabuses of the major professional bodies. Hence the contents and title of this book.

10. Stages in O.R. Study.

An O.R. study can be separated into six stages

a. Problem recognition and definition
b. Model building
c. Data collection
d. Problem solution
e. Interpretation of solution
f. Implementation of final solution
g. Review and maintain

11. Problem Recognition and Definition.

No technique can be applied or analysis undertaken until the problem has been recognised and carefully defined. Vague descriptions of problems are insufficient. What is required is a clear, detailed statement of what the problem really is. This is a difficult stage and thorough investigation is necessary. A superficial study may not identify the real problem. Close and friendly liaison with the people involved in the area will provide the best results.

12. Model Building.

Having defined the problem, a model must be developed which, hopefully, can be solved by an appropriate technique such as the ones described in this book. Care must be taken not to apply a standard O.R. technique to a non-standard situation. One must ensure that the model incorporates the essential features of the problem being studied.

13. Data Collection.

Data will be collected at the two previous stages, but to solve the problem much more data will be required. This data will include costs, revenues, production and stock data, etc and will be gained from past records and from estimates of the future. The data required will also include the quantification of factors not always quantified, such as risk and uncertainty.

14. Problem Solution.

For many problems, particularly those encountered in examinations, a solution can usually be obtained by standard mathematical means using a recognised O.R. technique. This does not, unfortunately, apply to all problems encountered in practice which may require the use of advanced mathematical analysis and non-standard techniques. Even where a solution can be obtained by the use of a basic model, practical problems invariably involve large amounts of data necessitating repeated calculations making the use of computers an economic proposition.

15. Interpretation of Solution.

Although many O.R. techniques produce optimal solutions, it must be realised that the model does not provide a solution to the actual real life problem, but to a simplified version of the problem represented by the model. Accordingly results must be treated with caution and there must be due regard to the problems involved in all O.R. studies which include such factors as:—

i. The appropriateness of the model to the real problem.
ii. The accuracy of the data used and assumptions made.
iii. The dangers of sub-optimisation.

16. Implementation of Final Solution.

After careful interpretation of the results of the O.R. study and modification where appropriate, the resulting solution would normally be implemented. Sometimes the solution may not be implemented because, although technically valid, management may consider that its implementation would not be cost effective or that it might cause too much disruption to operations and/or employees, customers, suppliers and the general public.

17. Review and maintain.

After implementation the performance of the model should be carefully monitored to ensure that it actually does work and fulfills its objectives. The review process should be at regular intervals so that appropriate adjustments can be made to meet minor changes in conditions or to recognise promptly when major changes have occurred, which render the implemented solution inappropriate. This is, of course, a form of audit procedure familiar to accountants.

18. Foundation Mathematics and Statistics.

The syllabuses at which this book is aimed are in the Professional Stage of studies and assume a basic knowledge of mathematics and statistics which would normally be gained in Foundation Level studies. Accordingly this book does not cover foundation level mathematics and statistics, but because of the step-by-step teaching approach used and the inclusion of some revision material (eg probability concepts and financial mathematics) it should be readily comprehensible to all students, including those whose basic mathematics and statistics are a little rusty.

19. Syllabus Coverage.

The syllabuses of the major professional bodies which contain Quantitative Techniques are in the Professional Stages. The syllabuses include Quantitative Techniques (Institute of Cost and Management Accountants), Elements of Financial Decisions (Institute of Chartered Accountants), Management Mathematics (Association of Certified Accountants), and Analytical Techniques (Chartered Institute of Public Finance and Accountancy). Obviously not all of these syllabuses are the same, but there is considerable overlap in the O.R./Quantitative Techniques area and this book covers all the aspects of O.R./Q.T. normally found in the examinations of these bodies. Included in the book is a major section on investment appraisal and DCF which is an important part of the Elements of Financial Decisions and Management Mathematics syllabuses of the Chartered and Certified Accountants respectively. Investment appraisal is not included in the Quantitative Techniques syllabus of the Cost and Management Accountants, but the subject does, of course, occur at other places in the ICMA examinations so that this section can usefully be studied by students of all the bodies.

20. Summary.

a. Numerous names exist for the concepts and techniques covered in this book. More important than the name is the common approach adopted.

b. This approach is characterised by a scientific, model based approach to solving problems.

c. Whatever the problem, there are common stages in all O.R. studies. These are Problem recognition, Model building, Data Collection, Solution, Interpretation and Implementation.

d. This book covers the full range of Quantitative Techniques normally encountered in professional examinations and assumes a knowledge of foundation level mathematics and statistics.

21. Points to Note.

a. O.R. should not be regarded as a compartmentalised area of study. The basic approach is applicable in virtually any situation, particularly those related to planning, control and decision making.

b. Formal mathematical notation is reduced to the absolute minimum in this book and formal proofs are not given. This is in line with the philosophy adopted by the major professional bodies for their examinations which concentrate on the application of O.R. techniques in practical situations.

SELF REVIEW QUESTIONS.

1. Define OR in your own words. Compare your definition with the B.S. definition (2).

2. What are the essential features of the O.R. approach? (4)

3. How does O.R. assist management decision making? (8)

4. What are the stages in an O.R. study? (10)

5. Give reasons why the results of an O.R. study may not be implemented. (16)

EXAMINATION QUESTIONS.

1. Most operational research applications possess certain distinguishing characteristics. These could be identified as follows:

a. A primary focus on decision making;

b. An investigation based on some measurable criteria;

c. The use of a formal mathematical model;

d. Dependence upon computing facilities.

Required: Explain each of the above characteristics. (ACCA, Management Mathematics, JUN 1976).

2. Discuss the methodology of operational research and describe the main stages in carrying out an operational research project. (ICMA, Quantitative Techniques, MAY 1978).

2 Probability and Decision Making

1. Introduction.

The laws of probability form the basis of most statistical theory and probability concepts are used in helping to solve many decision making problems. This chapter briefly reviews the basic rules of probability and shows how appropriate probability concepts can be used in helping to solve business problems. It makes no attempt to cover all aspects of this vast subject, but only those aspects which are relevant.

2. Probability Definition.

For our purposes probability can be defined as the quantification of uncertainty. Uncertainty may also be expressed as 'likelihood', 'chance' or 'risk'. Probability is represented by p, and can only take values ranging from 0, ie impossiblity, to 1, ie certainty.

For example, it is impossible to fly to the moon unaided and it is certain that one day we will die. This is expressed as

p (flying to the moon unaided) = 0
and p (Dying) = 1

3. Objective and Subjective Probability.

Where the probability of an event is based on past data and the circumstances are repeatable by test, the probability is known as statistical or objective probability. For example, the probability of tossing a coin and a head showing is 50% or ½ or .5. This value can be shown to be correct by repeated trials. In most circumstances objective probabilities are not available in business situations, so that subjective probabilities must be used. These are quantifications of personal judgement, experience and expertise. For example, the Sales Manager considers that there is a 40% chance (ie p = .4) of obtaining the order for which the firm has just quoted. Clearly this value cannot be tested by repeated trials. In spite of the undoubted shortcomings of subjective probabilities they are all that is normally available and so they are used to help in the decision making process. It should be emphasised that the use of probabilities does not of itself make the decision. It merely provides more information on which a more informed decision can be taken.

4. Basic Rules of Probability.

Whether the probabilities are objective or subjective, once they are established they are used in the same way according to the following rules. These rules are appropriate to statistically independent events, ie the occurence of one event is completely independent of the occurrence of any other event, eg the outcome of a die throw is completely independent of the outcome of any previous throw.

a. MULTIPLICATION RULE (AND)

This rule is used when there is a string of independent events for which each individual probability is known and it is required to know the overall probability.

Example 1. What is the probability of throwing a 3 *AND* a 6 with two throws of a die?

P (throwing a 3) = 1/6 and P (throwing a 6) = 1/6
P (throwing a 3 *AND* a 6) = 1/6 x 1/6
= 1/36

Note: The probability of 1/36 given above is the probability for throwing a 3 *followed* by a 6. If the order is unimportant ie if a 3 followed by a 6 or a 6 followed by a 3 is acceptable then the probability is doubled ie

where P (3 and 6) = P (3 followed by 6) + P (6 followed by 3)
then P (3 and 6) = 1/36 + 1/36 = 1/18

b. ADDITION RULE (OR)

This rule is used to calculate the probability of two or more mutually exclusive events. In such circumstances the probabilities of the separate events must be added.

Example 2. What is the probability of throwing a 3 *OR* a 6 with a throw of a dice?

P (throwing a 3) = 1/6 and P (throwing a 3) = 1/6
P (throwing a 3 *OR* a 6) = 1/6 + 1/6
= 1/3

c. CONDITIONAL PROBABILITY

This is the probability associated with combinations of events but given that some prior result has already been achieved with one of them.

When the events are independent of one another, then the conditional probability is the same as the probabiliy of the remaining event.

Example 3. The probability of throwing a total of 10 with 2 dice, before the events, is 1/12;

ie p (5 *and* 5) = 1/6 x 1/6 = 1/36
p (6 *and* 4) = 1/6 x 1/6 = 1/36
p (4 *and* 6) = 1/6 x 1/6 = 1/36
= 3/36
= 1/12

but if one die has been thrown and shows a 4 then the conditional probability is the probability of throwing a 6 with the other die, which is a probability of 1/6.

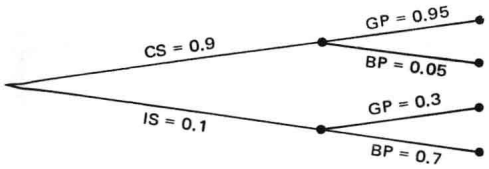
The conditional probability can be useful for the revision of probabilities as fresh information becomes available.

Example 4. From past experience it is known that a machine is set up correctly on 90% of occasions. If the machine is set up correctly then 95% of good parts are expected but if the machine is not set up correctly then the probability of a good part is only 30%.

On a particular day the machine is set up and the first component produced and found to be good. What is the probability that the machine is set up correctly?

Solution.

This is displayed in the form of a PROBABILITY TREE or DIAGRAM as follows:



$\therefore$ CSGP = 0.9 x 0.95 = 0.855
CSBP = 0.9 x 0.05 = 0.045
ISGP = 0.1 x 0.3 = 0.03
ISBP = 0.1 x 0.7 = 0.07
1.00

CS = Correct set up
IS = Incorrect set up
GP = Good Part
BP = Bad Part

$\therefore$ Probability of getting a good part
= CSGP + ISGP = 0.855 + 0.03 = 0.885

$\therefore$ Probability that machine is correctly set up after getting 1 good part is

$$\frac{\text{CSGP}}{\text{CSGP + ISGP}} = \frac{.855}{.885} = \underline{0.966}$$

Note: Good parts may be produced when the machine is correctly set up and also when it is incorrectly set up. In 100 trials there will be 855 occasions when it is correctly set up and good parts are produced (shown as CSGP above) and 30 occasions when it is incorrectly set up yet good parts are produced (shown as ISGP above). Thus if a good part is produced we can state that there are 855 occasions out of 885 when the good part is from a correctly set up machine.

5. Expected Value.

The basic rules of probability so far outlined help to quantify the options open to management and thereby may help in coming to a decision. Where the options have values (so much profit, contribution etc) as well as probabilities, the concept of expected value is often used. The expected value of an event is its probability times the outcome or value of the event over a series of trials.

Example 5. Two projects are being considered and it is required to calculate the expected value of each project. The project data have been estimated as follows:—

	PROJECT A			PROJECT B		
	£	p	EV £	£	p	EV £
Optimistic Outcome	6000	x .2 =	1200	6500	x .1 =	650
Most likely Outcome	3500	x .5 =	1750	4000	x .6 =	2400
Pessimistic Outcome	2500	x .3 =	750	1000	x .3 =	300
PROJECT EV			£3700			£3350

EV = Expected Value

On the basis of Expected Value, Project A would be preferred as it has the higher value.

6. Expected Value – Advantages and Disadvantages.

Expected Value is a useful summarising technique, but suffers from similar advantages and disadvantages to all averaging methods.

Advantages:

a. Simple to understand and calculate.

b. Represents whole distribution by a single figure.

c. Arithmetically takes account of the expected variabilities of all outcomes.

Disadvantages:

a. By representing the whole distribution by a single figure it ignores the other characteristics of the distribution, eg the range and skewness.

b. Makes the assumption that the decision maker is risk neutral, ie he would rank equally the following two distributions:

		£	p	
	Pessimistic Outcome	18000	.25	
	Most likely Outcome	20000	.5	EV = £20 000
	Optimistic Outcome	22000	.25	
and	Pessimistic Outcome	6000	.2	
	Most likely Outcome	18000	.6	EV = £20 000
	Optimistic Outcome	40000	.2	

It is of course unlikely that any decision maker would rank them equally due to his personal attitude to risk.

7. Optimisation of Levels of Activitiy under Conditions of Uncertainty.

Expected Value concepts can be used to calculate the maximum stock or profit level when demand is subject to random variations over a period.

Example 6. A distributor buys perishable articles for £2 per items and sells them at £5. Demand per day is uncertain and items unsold at the end of the day represent a write off because of perishability. If he understocks he loses profit he could have made.

A 300 day record of past activity was as follows:

Daily Demand (Units)	No. of Days	p
10	30	.1
11	60	.2
12	120	.4
13	90	.3
	300	1.0

What level of stock should he hold from day to day to maximise profit?

Solution:

It is necessary to calculate the *Conditional Profit* (CP) and *Expected Profit* (EP).

CP = profit that could be made at any particular conjunction of stock and demand situation, eg if 13 articles were bought and demand was 10 then

CP = (10 x 5) – (13 – 2) = £24.

EP = CP x probability of the demand
eg the CP above is £24 and p (demand = 10) = .1
EP = £24 x .1 = **£2.4**

CONDITIONAL AND EXPECTED PROFIT TABLE

Stock Options

Demand	p	10		11		12		13	
		CP £	EP £	CP £	EP £	CP £	EP £	CP £	EP £
10	.1	30	3	28	2.8	26	2.6	24	2.4
11	.2	30	6	33	6.6	31	6.2	29	5.8
12	.4	30	12	33	13.2	36	14.4	34	13.6
13	.3	30	9	33	9.9	36	10.8	39	11.7
	1.0		30		32.5	*	34.0		33.5

Optimum

Table 1.

The optimum stock position, given the pattern of demand, is to stock 12 units per day.

8. Value of Perfect Information.

Assume that the distributor in Example 6 could buy market research information which was perfect, ie it would enable him to forecast the exact demand on any day so that he could stock up accordingly. How much would the distributor be prepared to pay for such information? To solve this type of problem, the profit with perfect information is compared with the optimum EP from Table 1.

Profit with perfect Information

			£
When demand is 10, stock 10	Profit	= (10 x £3) x .1 =	3.0
When demand is 11, stock 11	Profit	= (11 x £3) x .2 =	6.6
When demand is 12, stock 12	Profit	= (12 x £3) x .4 =	14.4
When demand is 13, stock 13	Profit	= (13 x £3) x .3 =	11.7
			£35.7

As the EP from Table 1 was £34, the distributor could pay up to £1.70 (£35.7 – 34) for the information.

9. Solution by Marginal Probability Formula.

An alternative approach to solving problems such as Example 6 above is to work out the marginal profit, MP, ie the profit from selling one more unit, and the marginal loss, ML, ie the loss from not selling the marginal unit and to calculate

the relationship between MP and ML in terms of probability.

Example 7. Using the same data as in Example 6.
Solve by using marginal profitability and marginal loss relationships.

From Example 6
MP = £3 and ML = £2

It follows that additional items will be stocked whilst the following relationship holds:

$$p(\text{making additional sale}).\ MP > p(\text{not making additional sale}).\ ML$$

If P denotes the probability of making the additional sale then (1—P) = probability of NOT making the sale.

$$\begin{aligned} \text{at breakeven point } P(MP) &= (1-P)(ML) \\ P(MP) + P(ML) &= ML \\ P(MP + ML) &= ML \\ P &= \frac{ML}{MP + ML} \end{aligned}$$

Inserting the data from Example 6 we obtain

$$P = \frac{2}{3 + 2}$$

$$= .4 \text{ ie the probability at Break Even Point}$$

This probability is compared with the probability of demand at the various levels, the optimum position being the highest demand with a probability greater than P, ie

Demand	Probability	
Greater than 10 units	1.00	
Greater than 11 units	.9	
Greater than 12 units	.7	Break Even probability = .4
Greater than 13 units	.3	

12 items is the most profitable stock level.

10. Summary.

a. Objective probabilities are rarely available in business decision making situations so that subjective probabilities, ie the quantification of judgement, are used.

b. The two basic rules of probability are the multiplication rule (AND) and the addition rule (OR).

c. Expected Value (EV) has many uses in decision making situations and is calculated by multiplying the value of the outcome(s) by the probability. In general the option with the highest EV would be chosen.

d. Although useful, EV ignores the range and skewness of distributions, so that it needs to be used with judgement.

11. Points to Note.

a. The use of probability concepts and expected value is widespread in profes-

sional examinations so that this chapter should be studied carefully.

b. Questions involving probability may appear in any part of the syllabus (inventory control, investment appraisal, network analysis etc).

SELF REVIEW QUESTIONS.

1. *Distinguish between objective and subjective probabilities. (3)*
2. *Define the Multiplication and Addition rules of Probability. (4)*
3. *How is expected Value calculated? (5)*
4. *What are the advantages and disadvantages of using Expected Value as a decision criterion? (6)*
5. *Assuming that data on past demand and probabilities are available how might a value be imputed for receiving perfect information? (8)*

EXAMINATION QUESTIONS.

1. *A large restaurant purchases cakes daily from a local bakery. The cakes cost 10 pence each and sell at 15 pence each in the restaurant, but any which have not been sold there at the end of each day are sold through another outlet for 8 pence each. The relative frequency distribution of restaurant sales (obtained from an analysis of past sales data) is given below:*

Daily restaurant sales of cakes Dozens (1 dozen = 12 cakes)	Relative frequency
30	*0.01*
31	*0.09*
32	*0.16*
33	*0.25*
34	*0.30*
35	*0.11*
36	*0.08*

You are required to state:

a. The optimum quantity which the buyer should purchase in order to maximise expected profits;

b. how much the buyer could afford to pay for perfectly correct information about daily sales.

(ICMA, Quantitative Techniques MAY 1976).

2. *Makalu Ltd. uses a large number of machines which incorporate a special component, liable to failure. Its present policy is to replace each component individually following failure. Records have been kept of the working lives, before failure, of 500 of the components and have revealed the following results:*