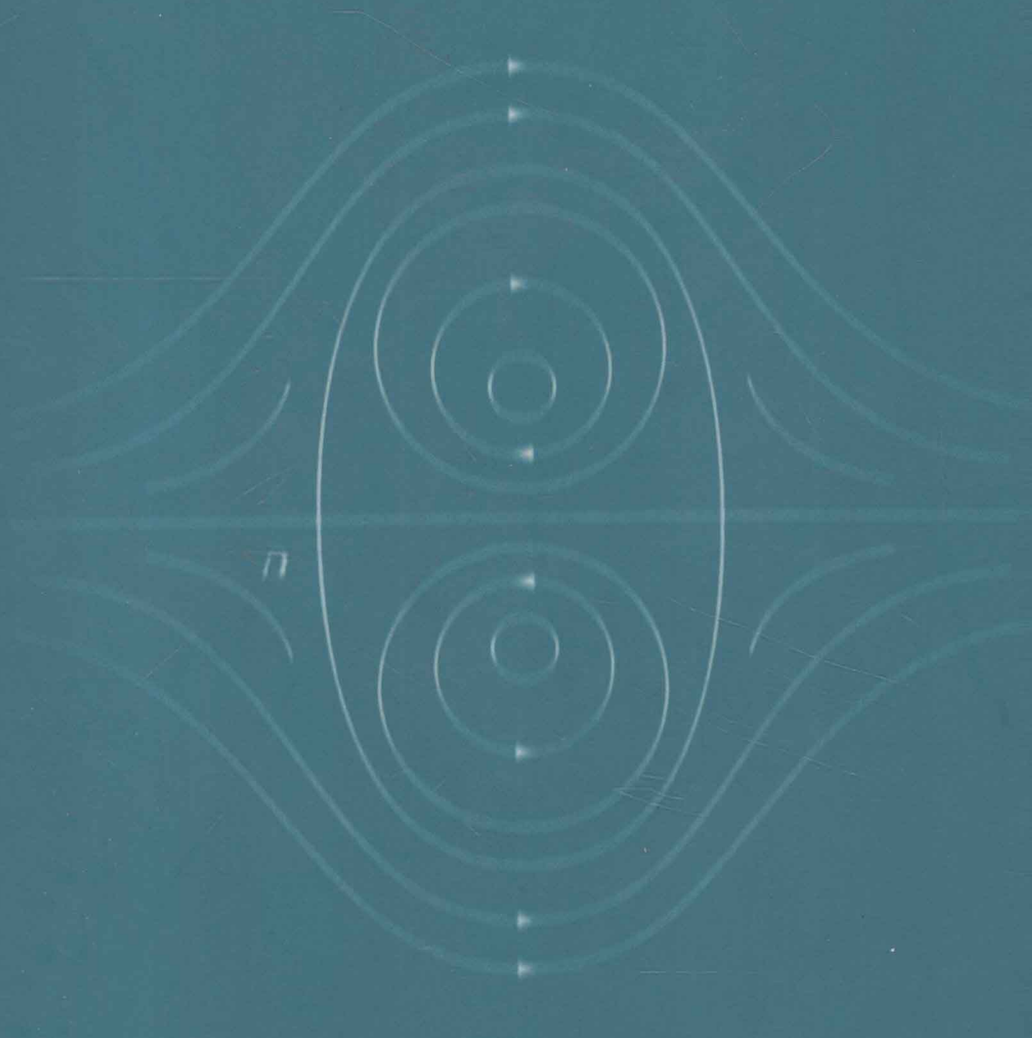
Alexandre T. Filippov

The Versatile Soliton



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Introduction

The temple of science is a multi-faceted building

Albert Einstein (1879–1955)

We all have an intuitive familiarity with the closely related notions of *oscillation, vibration* and *wave*. Any periodic motion of any object, or periodic change of its state, is called an *oscillation* or *vibration*; examples are evident in our everyday life: our pulse, the swinging of a pendulum, ticking of a clock, and so on. The scientific approach to oscillations starts by neglecting physical differences between vibrating objects, by extracting features that are common to all oscillations, and then finding mathematical laws to describe these oscillations.

Oscillations and vibrations usually generate waves. Waves on water are easily recognized as physical phenomena, but radio waves can only be imagined, making it more difficult to establish their physical reality. To imagine radio waves, one has to know some physics and mathematics. Since the theory of waves is concerned with characteristics that are common to all waves, independent of their physical nature, it is sufficient to comprehend just one example, for example, waves on water. However, we should note that the wave on water is a much more complex phenomenon than is apparent to the eye. So, in this book we will first examine much simpler waves.

In general, a *wave* may be defined as a progression through matter of a state of motion. Waves have a close relationship with oscillations. In fact, the simplest *periodic wave* in any medium is a progression of oscillatory motion to neighboring points of the medium. Such waves are usually emitted by vibrating objects, e.g., sound waves generated by a tuning fork. The vibrating fork produces oscillations of the surrounding particles of the air; these, in turn, excite oscillations of particles farther away, and thus the sound wave is formed.

When a simple periodic wave is travelling in a medium, the particles of the medium are oscillating. If the *amplitude* of the wave (the height of the crest for waves on water) is small, the amplitude of the oscillations is proportional to the amplitude of the wave, and the shape of a larger wave is similar to that of a smaller one. For waves of large amplitude, the picture may be quite different. For example, high waves on water are much steeper than low ones. "White caps" may emerge on such waves, and eventually they may overturn. The movements of particles in such a wave are irregular and chaotic. This is an extreme demonstration of the nonlinear nature of a high wave.

To understand *nonlinearity*, one should first understand *linearity*. Characteristic properties of any linear oscillation are: (1) that its period is independent of the amplitude; (2) that the sum of two linear oscillations is also a linear oscillation; and (3) that small amplitude oscillations are linear. Linear waves have similar properties: (1) the shape and velocity of a linear wave are independent of its amplitude; (2) the sum of two linear waves is also a linear wave; and (3) small amplitude waves are linear. Large amplitude waves may become nonlinear, which means that these simple rules do not apply.

Nonlinearity results in the distortion of the shape of large amplitude waves, for example, in the formation of "white caps" on waves, in turbulence, and in many other complex phenomena, some of which will be discussed in this book. However, there is another source of distortion. It is well known that waves having different *wavelengths* (the length from crest to crest) move with different velocities — a phenomenon known as *dispersion* of a wave. A familiar effect of dispersion occurs when observing what happens to a wave when a stone is thrown into the water. Clearly, long waves move faster than short ones. You may further observe that a "hill" on water — a solitary pulse, or, a "piece of wave," so to speak — usually degrades very fast.

Remarkably, some such pulses do not disappear and can travel in water for a long time, preserving their shape and velocity. It is not easy to observe this unusual *solitary wave*, and even more difficult to explain. The first observation of this remarkable phenomenon was made more than 160 years ago, and puzzled scientists for decades. About 100 years ago the mathematical equations describing solitary waves were solved, at which point it was recognized that the solitary wave may exist due to a precise balance between the effects of nonlinearity and dispersion. Nonlinearity tends to make the hill steeper, while dispersion flattens it. The solitary wave lives "between" these two dangerous, destructive "forces."

The most striking properties of the solitary wave, in particular the fact that the wave has properties common to particles, were uncovered and understood after 1967. Indeed, the solitary wave on water is a member of a large family of particle-like waves now generically called *solitons*. In the 20th century, many solitons existing in very different physical media were discovered and studied, but their common features have become clear only in the last three decades.

Solitons are now studied in such diverse sciences as biology, oceanography, meteorology, solid state physics, electronics, elementary particle physics, and cosmology. Solitons certainly exist in crystals (dislocations), in magnetic materials (domain walls), in superconductors and superfluids (vortices), in the atmospheres of the earth (tornados) and of other planets, in the ocean (tsunami), in galaxies (vortices), and in living organisms (nerve pulses). In all likelihood, solitons also played a role in the early history of our universe. Modern theories of the smallest building blocks of matter (superstrings or more complex objects — supermembranes) predict new types of solitons that are, unfortunately, inaccessible to experiment. On the other hand, some solitons are currently used for storing and transmitting information (solitons in optical fibers) or may be used in the near future (solitons in superconductors). Sooner or later you will encounter solitons.

This book is the first attempt to explain the idea of the soliton within the context of its historical development, from first observations to recent applications. Although the reader is assumed to have some knowledge of high school physics and mathematics, the book can be read at three different levels.

No knowledge of mathematics and only an intuitive knowledge of physics is required at the first level. The reader may skip over formulas and detailed physical explanations and enjoy the descriptive history of the development of 19th century science.

To better understand the soliton, however, an effort must be made to intuitively grasp the elementary theory of oscillations and waves. Thus, at the second level, one may still skip over the formulas and concentrate only on the graphical representations of the concepts.

On the third and deepest level, however, the reader must follow elementary mathematical computations and perhaps use the formulas to make experiments on the computer. Mathematical equations that require more advanced mathematics are collected in an appendix. Finally, for those who would like to delve deeply into the real theory of solitons, a reference list is provided to modern scientific literature on the subject.

In Part II of the book, some knowledge of elementary mathematics is required to understand the rather thorough explanation given here of nonlinear oscillations of the pendulum, dispersion of waves, phase and group velocity. Part III, which is perhaps the most interesting, may be read almost independently of Part II.

The overall presentation may appear somewhat irregular at first, but with some patience, the reader will arrive at an understanding of the deep connections between apparently different ideas and diverse phenomena. A primary goal of this book has been to uncover these connections which serve to eventually demonstrate the deep unity of science. The history of the soliton is a generic example of how a significant scientific idea develops. From a simple observation that generates controversial interpretations, an idea can become a widely-accepted mathematical theory that penetrates different areas of the natural sciences, and finally becomes embodied in physical devices that can change our everyday life. It would be much simpler to present this development in a rigid logical (or historical) order, but then the flavor of creative work, full of uncertainties, controversies and misunderstandings, would be completely lost.

Whenever possible and desirable, I try to adhere to a logical and rigorous style of exposition and give all details. Sometimes, however, I rather choose to give an overall impression and leave many details cloudy, or give only abstract sketches.

Scientific events are carefully ordered chronologically, although I move freely in time throughout this presentation. The reason for these "travels in time" is the following: I wanted to show that although the number of scientists, the instruments used, and the role of science in society have all dramatically changed in the last 150 years, the basic methods of scientific thinking have remained the same throughout different epochs.

This book has its own history. First published in 1984 in the former Soviet Union, the second Russian edition was somewhat enlarged in 1990. The present version is neither completely new nor a literal translation of the Russian original. While the "skeleton" is preserved, the "flesh" has been renewed. Several addenda are included and my aim has been to reach a western audience.

Before beginning the story of the soliton, I would like to make some general remarks concerning one of the main motifs of this book, which, although it lies in the background, essentially determines the content, the structure and even the style of this work.

The soliton is a multifaceted scientific phenomenon that is relevant to many scientific fields. A book on this subject could be written by a mathematician, by a specialist in hydrodynamics, by a solid-state physicist, by an expert in radio-engineering and electronics, or by a historian of science. I am none of these but, as a theoretical physicist, I am supposed to be able to apply fundamental physical principles to any of the natural sciences. Unfortunately, in today's world of specialization — I am a specialist in quantum field theory and high energy physics — it is impossible to have enough expertise in all the scientific areas in which the soliton concept is important and must be treated in this book.

However, theoretical physics is based on a few fundamental concepts, such as space, time, particles, waves, and on the laws of physics describing movements of particles and propagation of waves in space, as well as interactions between them (collisions of particles or emitting waves by moving particles, and so on). The laws of physics are expressed by rather abstract mathematical equations. The most difficult thing is to explain how a theoretical physicist handles real physical objects and their abstract concepts. Physics starts with simple observations of events occurring in the real world but, to find mathematical laws describing such events, the physicist has to first give a quantitative description of them. The concept of the point particle having a certain position in space was introduced in this way. By measuring distances between positions of the same particle at different moments of time, we can find the velocity of the particle. Applying the same procedure many times, we can find how the velocity depends on time and thus find the acceleration of the particle.

The first fundamental law of physics is the statement that acceleration is proportional to the force acting on the particle. This is Newton's celebrated second law of mechanics (dynamics). The most difficult thing in this law is the force. Newton knew about the force of gravity and elastic forces. Electric and magnetic forces were discovered later, but this did not change the Newtonian picture of the physical world, which was considered as a collection of moving particles with forces between them. All material bodies (gases, liquids, solids) were regarded as complexes of particles, and all physical phenomena, according to this view, could be described and explained in terms of movements of interacting particles. For example, waves in water were described as resulting from coherent oscillatory movements of small particles of water. Similarly, this applies to sound waves travelling through the air or through solids. The mathematical theory of these waves, based on the Newtonian picture, was finally formulated in the beginning of the 19th century.

At the same time, this consistent picture of the world was shaken up by experiments that proved the wave nature of light. The problem was that light waves may exist even in empty space. Physicists immediately reacted to this problem by inventing the "ether," a substance in which waves of light propagate but do not interact with matter and do not consist of any known particles. The problem of the ether became even more difficult in the Faraday and Maxwell theories of electromagnetic phenomena formed in the second half of the 19th century. The notion of ether had no place as it became clear that light also has an electromagnetic nature. By the end of that century, the concept of ether was finally abandoned by the majority of leading physicists, and Faraday's concept of the electromagnetic field was accepted.

This was "the beginning of the end" of the Newtonian view of the world. The 20th century began with reconsideration of the Newtonian concepts of space and time in the theory of relativity. Even deeper was the quantum revolution that made the physics of the 20th century radically different from the physics of the 19th century.

Although the general approach of physicists to physics problems has not radically changed since the end of the 19th century, modern concepts of space, time, particle, and wave are very different. In quantum theory, there is no contradiction between particle and wave pictures of light phenomena; they are complementary. Moreover, in quantum theory, movements of any particle are described by certain "waves of probability." This very abstract mathematical concept is absolutely necessary for formulating the laws of physics on the microscopic level. But, on the macroscopic level, the laws of 19th century physics are still generally applicable, except for some aspects of low temperature physics.

The soliton observed 160 years ago can be completely understood in terms of the physics and mathematics of that time. However, its most important particle-like properties could not have been discovered in that century; the idea of a "particle-like" wave was absolutely foreign to 19th century physicists. An exact and complete mathematical theory of the soliton could not have been established before the middle of the 20th century. Although the soliton is not a quantum object, its mathematical description requires tools developed in modern quantum physics, and a complete theory of solitons is a rather refined branch of modern mathematics.

I will not try to instruct the reader in the ABCs of this mathematics. Instead, I will first describe how the concept of the soliton gradually emerged from the first observations of it as a solitary wave in water and how this idea became more and more abstract and mathematical, until it eventually penetrated many branches of physics and other natural sciences.

My view of the soliton is that of the theoretical physicist. This means, in particular, that I am always looking at common features of diverse solitons, trying to uncover conceptual simplicity in apparent complexity. I will gradually introduce the most important concepts of theoretical physics and basic principles of a theoretical approach to understanding the soliton's physical reality.

Many people believe that theoretical physics is a very difficult science. A great German mathematician David Hilbert (1862–1943), who made many important contributions to mathematics, including the mathematical apparatus of quantum physics and of general relativity, once said that physics is much too hard for physicists. He obviously had in mind quantum theory, and I agree with this statement. However, theoretical physics is easier to understand than any other natural science. It deals with extremely simplified mathematical models of real phenomena and uses only a few basic concepts. The main concepts and results of theoretical physics as related to solitons can be explained without using much mathematics. Of course, some physics intuition and acquaintance with basic mathematics is desirable to follow my presentation of solitonic phenomena.

I recommend the book written by one of the creators of modern theory of superconductivity, Leon N. Cooper, *An Introduction to the Meaning and Structure of Physics*, Harper and Row, Publ. NY, 1968. The evolution of the main concepts of theoretical physics – space, time, particle, wave, field, quantum – is nicely presented in the book by Albert Einstein and Leopold Infeld, *The Evolution of Physics*, Simon and Schuster, N.Y. 1938.

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When preparing the first (Russian) version of this book I used many scientific and popular science books, as well as original and review papers. I mentioned some papers and their authors in the text, but it was impossible to mention and to thank everyone. Especially useful were my contacts with S.P. Novikov, Ya.A. Smorodinskii, and L.G. Aslamazov who made many critical and very useful remarks. The second Russian, and especially, the English version of the book were also strongly influenced by many people. I have to mention in particular Ya.B. Zeldovich, N. Zabusky, Ch. Zabusky, and E. Beschler. To all these people, I am really very grateful.

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Part I

An Early History of the Soliton

And science dawns though late upon the earth...

Percy Bysshe Shelley (1792–1822)

To understand a science it is necessary to know its history

Auguste Comte (1798–1857)
Positive Philosophy

The first officially registered scientific encounter with what is now called the soliton happened more than a century and a half ago in August of 1834 near Edinburgh, Scotland. Scottish scientist and engineer John Scott Russell (1808–1882) was well prepared to make this first observation and gave an accurate, clear, and even poetic account of his first meeting with the soliton, which he called the *translation wave* or the *great solitary wave*.

Although Russell devoted many years to further investigation of this phenomenon, his colleagues neither saw its significance nor shared his enthusiasm. Between 1844, when Russell's paper was published, and 1965, fewer than two dozen papers relating to the solitary wave were published. In 1965, American physicists Martin Kruskal and Norman Zabusky, who solved computationally an important equation related to this phenomenon, coined the term *soliton*.

Now, thousands of scientific papers are published every year in which the role of solitons in physics, mathematics, hydromechanics, astrophysics, meteorology, oceanography, and biology are studied. International scientific conferences devoted to solitons and related problems are organized all over the world. More and more scientists join the "society" of soliton hunters. In fact, the concept of the soliton and its technical applications are becoming so important that the time has come for every educated person to become acquainted with the phenomenon, which is the *raison d'être* for this book.

Because the soliton's discovery, its delayed acceptance, and its relationship with technology are inextricably bound in developments that occurred through the history of science, this book will provide a framework for understanding the soliton, shining a light on those scientists whose work contributed to its "rise." So let us start from the beginning and, to better understand the reaction of Russell's colleagues to his discovery and ideas, try to imagine that we are in the first half of the 19th century, around the year 1834.

Chapter 1
A Century and a Half Ago

> The Nineteenth Century, the Iron
> And really hardhearted Age ...
>
> Aleksandr Blok (1880–1921)

> Poor our Century! How many attacks on him, what a monster is he said to be! And all this for the railways, for the steamships — his great victories not only over matter but over space and time.
>
> Vissarion Grigorievich Belinskii (1810–1842)

Only five years prior to 1834 the first railroad had been completed, and steamships were just beginning to be built. In industry, steam engines were the most popular sources of power. In fact, this period is sometimes called the "Steam Revolution." It was also the time of Napoleonic wars, social disorders and revolutions, of Goethe, Beethoven, Byron and Pushkin, and of many important scientific discoveries, including many principal discoveries in physics, the full meaning of which would become clear only gradually many years later. At the time, physics was not yet regarded as a separate science. The first physics institute was created in 1850 in Vienna. Around 1834, university professors were still reading courses in "natural philosophy," which included what we call experimental, theoretical and applied physics.

Because we are privileged now to know the role that the new concepts in physics played in the development of western civilization, it is not easy for us to imagine what educated people in the 19th century might have thought of the science of their time. In addition, the role of science in the 19th century was very different from what it would become in the second half of the 20th century; its practice was also different.

Those were the "golden" years of physics, although no one then could have imagined our supercomputers or superconducting supercolliders of elementary particles! Using extremely simple tools, however, some fundamental discoveries were made. One of the most amazing discoveries of the first part of the 19th century was made using a small piece of ordinary wire with electric current and a standard compass (magnetic arrow). Hans Christian Oersted (1777–1851) was extremely attracted to the idea of mutual interrelations among such diverse natural phenomena, as heat, sound, electricity, and magnetism. So it happens that in 1820 at Copenhagen University, while giving a lecture devoted to searching for a connection between magnetism, "galvanism" and "inorganic" electricity,[1] Oersted observed a striking phenomenon. When the electric current was allowed to travel through the wire, which was parallel to a magnetic arrow, the arrow changed direction! One of his pupils later remarked that "he was quite struck and perplexed to see the needle making great oscillations." Oersted was so astounded because the original aim of the experiment was to demonstrate the "well-known" absence of a relationship between magnetism and electricity produced by the Volta battery; apparently, nobody had ever tried to close the circuit.

Oersted's observation was immediately followed by a series of experiments clarifying quantitative details of the relationship between electricity and magnetism. The investigation of this relationship triggered an avalanche of exciting scientific developments, in which the key figure was André Marie Ampère (1775–1836). In his famous series of papers published between 1820–1825, Ampère gave the mathematical expression for the magnetic force produced by the electric current (Ampère's law) and, more generally, laid the foundation of a unified theory of electricity and magnetism which he called *electrodynamics*. In turn, Ampère's work laid the groundwork for the chain of great discoveries made by the self-taught genius, Michael Faraday.

Faraday began publishing the results of some of his experiments in 1821, but he made his main discoveries during the decade 1830–1840. In 1831 he first observed the electromagnetic induction ("electric current without batteries"), and later formulated the quantitative "Faraday's law" describing the electric current induced by a changing magnetic field. It is interesting to note that Ampère had stumbled upon this phenomenon in his experiments in 1822, but did not pay any attention to it. On the contrary, Faraday, having some hypotheses about the relationship, conducted many experiments before he finally succeeded in observing induction. He also immediately realized the possible practical value of his discov-

[1] "Galvanism" was so named after the Italian scientist Luigi Galvani (1737–1798), who discovered in 1780 electric phenomena in living creatures, typically, frogs. "Inorganic" electricity was known to ancient Greeks, but practical applications of this "type" of electricity began only after 1792 with discoveries by another Italian scientist Alessandro Volta (1745–1827). Although the ancient Greeks thought that electric and magnetic forces were of the same origin, the common belief before Oersted's discovery was that they were independent phenomena.