ADVANCES IN
APPLIED MECHANICS

Edited by

Erik Uan der Giessen

DELFT UNIVERSITY
OF TECHNOLOGY
DELFT, THE NETHERLANDS

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DIVISION OF ENGINEERING AND APPLIED SCIENCE
CALIFORNIA INSTITUTE OF TECHNOLOGY
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Preface

Advances in Applied Mechanics has a history of publishing comprehensive, state-of-the-art articles in numerous subfields of applied mechanics. But in no way does this imply that this particular area has been fully excavated. The articles in the present volume give convincing evidence that the developments often continue, requiring an update of previous advances.

Eduard Riks' article gives an up-to-date overview of the advancements made in the area of (post)buckling analysis since the often-quoted article in this series by B. Budiansky in 1974 (Vol. 14, pp. 2–65). In addition to providing a thorough outline of modern, computational techniques for buckling and postbuckling analysis of structures, this article also discusses recent methods of carrying out transient analyses after loss of stability. The latter offers new interesting insights into the notorious phenomenon of mode jumping.

The article by Paul R. Dawson and Esteban B. Marin is related to R. J. Asaro's contribution to this series (Vol. 23, 1983, pp. 1–115), which marked the beginning of a rapid expansion of numerical applications of crystal and polycrystal plasticity. The article in this volume gives an exposition of some important refinements in computational methods that make polycrystal plasticity a viable tool for actually solving engineering forming processes. Particular emphasis is placed on recent innovations in the description of crystal orientations. In addition, this paper presents numerous examples of metals with a hexagonal close packed (HCP) crystal structure.

Owing to seminal work in the 1960s, the linear-elastic properties of composite materials can now be estimated analytically from the properties of their constituents, along with the microstructure, with remarkable accuracy. Estimates of composite properties in the case of nonlinear material behavior, such as creep or plasticity, of one or more of the components are much more difficult to obtain. Analytical approaches to this problem that incorporate microstructural information have only been attempted during the last decade or so. The article by Pedro Ponte Casteñada and Pierre Suquet gives an overview of the latest developments in this field based on variational methods. This scholarly work summarizes the key theoretical tools and presents applications to numerous model materials, with an emphasis on the effect of microstructure.

The Chapter by Oleg S. Ryzhov and Elena V. Bogdanova-Ryzhova is a pioneering study of the fully developed nonlinear instability of viscous boundary layer during the final stage of transition into turbulent flow. This study is based on the important discoveries of several remarkable phenomena, first found by experiment at the Novosibirsk branch of the Russian Academy and now by theory, that the underlying mechanism actually involves generation of solitary waves under resonant forcing from flow-boundary roughness and vibration. It appears that these results will be of general interest to researchers in this important field.

In the article by Wei H. Yang, the author presents a task of integrating the existing and new bases of the mathematical theory of plasticity. In addition to the previously established conditions of convexity and normality as two pillars of support, an additional pillar, called the duality, is introduced here in terms of an equality inclusive condition which is claimed to bring the foundation to completion for the constitutive modeling of the mathematical theory of plasticity.

This series had the fortune of being cultivated during the period 1971–1982 under the editorship of Professor Chia-Shun Yih, who made outstanding contributions in realizing advances of this series with distinction and subsequently continued serving as a wise counsel until his passing on 25 April 1997. It is with our sincere appreciation of his dynamic leadership and guidance that we pay our warm tribute to his memory.

Theodore Y. Wu and Erik van der Giessen

Advances in Applied Mechanics

Volume 34

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Buckling Analysis of Elastic Structures: A Computational Approach

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I. Introduction

Twenty-two years ago, Budiansky presented in this periodical a very comprehensive overview of the theory of elastic stability as it had been established up to that date (Budiansky, 1974). This exposition of the state of the art in stability analysis was, to a large extent, based on the general theory of buckling and postbuckling behavior founded by Koiter some 30 years earlier (Koiter, 1945), a theory that had suffered from a very slow start in becoming known and accepted in the engineering community after its appearance.

In 1945, this new theory represented an important step forward in the understanding of the buckling behavior of structures as it was observed in engineering practice and experiments. It was this theory that was able to explain why the determination of the bifurcation point in the initial load deformation response could not provide enough information to predict the stability behavior of a structure with sufficient accuracy. As it was demonstrated by Koiter, the bifurcation point could not, by itself, be used to predict the failure load of the structure. To come to a better evaluation of this load, it was also necessary to establish the intrinsic properties of the bifurcation point itself. If the bifurcation point were stable, as in the case of a simply supported flat plate, the system could be loaded beyond the critical load that is associated with this point. On the other hand, if the bifurcation point turned out to be unstable, as in the case of a thin-walled cylindrical shell in compression, the structure could be expected to fail long before the critical load was reached, although it was difficult to predict with precision the load at which this would occur.

This so-called sensitivity of the failure load for initial imperfections in the geometry, boundary conditions, etc., existed if the bifurcation point itself was unstable. In contrast, imperfection sensitivity did not play a detrimental role if the bifurcation point turned out to be stable. Apart from this important observation, the theory also provided the keys to the determination of the equilibrium states that bifurcate from the initial state and a means of assessing the strength of the imperfection sensivity in the case where this sensitivity could be established.

The available methods of solution in 1974 were primarily analytical in nature. Computers were already in use, but their influence on the development of the solution methods was at first more in the area of the solution of complicated analytical formulations rather than systematic discretization of the governing equations from the start. At that time, there were three basic difficulties connected with the application of the theory. In the first place, the theory was an asymptotic theory, i.e., it relied on series expansions of the governing equations and therefore the solutions had a restricted range of validity. Second, if the primary solution path turned out to be nonlinear, the problem could turn out to be inaccessible for analysis, and this situation presented itself automatically if the structure was governed by a limit point rather than a bifurcation point.

There was also a third difficulty. This had to do with the structural complexity of the structures that could be analyzed. The solution of the governing equations was only possible if the geometry and material build-up of the structure under investigation remained relatively simple. If this was not the case, the obstacles for analysis could soon become insurmountable. Thus it is not surprising that the practical applications of the theory remained restricted to structures with a simple geometry such as plates, cylindrical shells, and curved stiffened panels, and this in conjunction with an elementary type of loading: uniform compression, for example. Since then, many years have passed and the situation has gradually changed. This change was brought about by the advent and evolution of the digital computer, which made it possible to develop computational tools with a range and power that were unheard of before this evolution started.

The emergence of the finite-element method is undoubtedly one of the most important advances in numerical analysis of this time, and since 1974 it has also had an impact on the modeling of the stability behavior of structures. Two schools of thought on modeling arose. The first is based on the discretization of Koiter's asymptotic theory, at times amended with extras that the increased freedom of this numerical approach allows. Early accounts of this treatment can be found in Haftka *et al.* (1971), and more recent contributions are given in Damil (1992), Arbocz and Hol (1990), Casciaro *et al.* (1992), Azrar *et al.* (1993), and Lanzo and Garcea (1996), which also contain further references. The other school is a more radical departure from the perturbation theory and is based on the continuation principle (see, for example, Riks, 1973, 1984a; Rheinboldt, 1977; Seydel, 1989; Crisfield, 1991; Kouhia, 1992), which in turn uses the principles of the numerical solution of nonlinear equations (Ortega and Rheinboldt, 1970).

The continuation approach as a general and practical tool for the solution of elastic stability problems was probably first considered in 1970 (Riks, 1970, 1972), but the finite element modeling capabilities of that time were not yet fully developed (in the nonlinear range), so that the first applications appeared years after these capabilities became available. Since then, progress in the further development and implementation of these techniques has been steady.

In what respect does the continuation method (also called the incremental method) differ from the classical perturbation method? The basic difference is that the solutions are no longer restricted to a small domain of the configuration space as with the perturbation method, but can be obtained everywhere in this space. The continuation method is thus a global method, whereas the perturbation method is a local method. With the continuation method, the initial load deformation path, as pictured in Figure 1, can be computed irrespective of degree of the nonlinearity of the problem. With this approach the stability of the solutions can be monitored during the computations, and by doing this the critical point at which loss of stability occurs (A or B) can be determined. The technique also offers a way to compute the branches of bifurcation points (under certain restrictions), so that it becomes possible to assess the postbuckling behavior of the structural model. Because of the versatile finite element modeling capabilities that are currently available, the applications of this type of analysis are no longer confined to standard problems with simple geometries, but can be extended to real world systems. Recent examples of such analyses can be found in Vandepitte and Riks (1992), Young and Rankin (1996), and Nemeth *et al.* (1996).

Thus the continuation principle in combination with the finite-element discretization method enables one to determine the static equilibrium branches (for as far as is deemed necessary) of a very general class of nonlinear structural models. Once these solutions are obtained, the stability behavior of these structures can be explained by examining the particular geometrical properties of these branches.

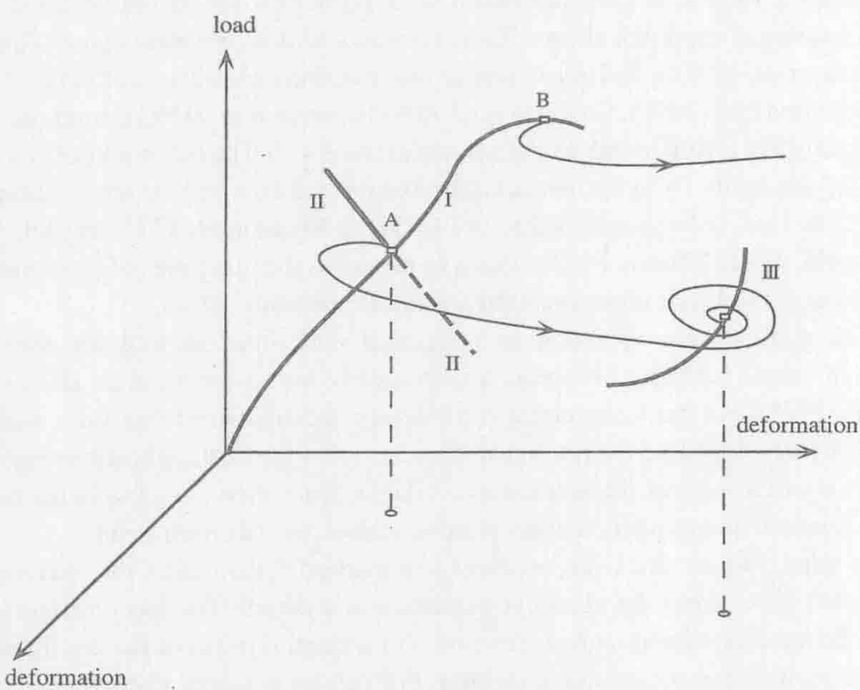

FIG. 1. Initial stable response followed by collapse.

But a stability analysis as described above is still a product of the classical quasistatic approach whereby only the solutions of the equilibrium equations are reviewed. According to the philosophy behind this approach, loss of stability is a dynamical transformation of state (see Figure 1) that starts at an unstable critical equilibrium state (a bifurcation point or limit point) but will end in a state in which the structure is no longer usable. Consequently, the critical state at which the motion starts *is of interest*, but what happens after this state is reached *is not*. However, it can be asked whether this quasistatic point of view can always be maintained.

It has been known for a long time that unstable buckling does not always lead to an unserviceable state of the structure. To the contrary, it can happen that after such an event further loading is still possible. This occurs, for example, with plates and stiffened panels, and in these particular cases the phenomenon is called mode jumping. Thus it is not possible, at least not in the general case, to predict beforehand what will happen when an unstable critical point is reached. The passage through the critical state may have the result that the structure will end up in a new state with irreparable damage, but it may also happen that the structure remains in operation with no damage at all. In general, the actual outcome is dependent on the problem at hand and can only be predicted by an extended analysis, i.e., by taking the transient motion into account.

The methods for integrating the equations of motion in the field of solid mechanics are actually quite well developed (Belytschko and Hughes, 1983; Argyris and Mlejnek, 1991), and the computer resources that are available at the present time no longer hinder the use of these methods. Consequently, in this chapter we not only consider the use of continuation methods for the solution of the equations of equilibrium, but also the use of transient methods to provide an answer to the question of how an actual buckling process works and where to it will lead. We believe that there are many problems in engineering practice where the answer to this question is badly needed.

The discussion will closely follow the ideas that were developed in two recent publications (Riks *et al.*, 1996; Riks and Rankin, 1997), but we aim here at a more complete presentation. Just as in the two references mentioned, we will introduce the numerical procedures by first giving a review of the elementary bifurcation theory for ordinary nonlinear equations that depend on a single parameter. The first part of this review is purely a geometrical introduction of this subject that is meant to serve as a preparation for the development of the static methods to be discussed later. The second part is focused on the stability or loss of stability that occurs at bifurcation or limit points, which is necessary for the understanding of what happens when such a point is reached.

The introduction to the computational procedures begins with a short description of the path-following method to be used for the computation of the static equilibrium branches, together with additional techniques that are needed for the analysis of these solutions. The computation of the buckling motion that starts at the point of loss of stability is the next subject, and the synthesis of this technique with the previous methods ultimately leads to the appropriate computational strategy, which makes it possible to numerically simulate the quasi-static as well as the dynamic elements of a complete buckling event.

A good way to demonstrate the feasibility of a new strategy is to verify its predictive power by means of test results of well-documented buckling experiments. Therefore, the chapter will end with the description of the numerical simulation of two well-known buckling tests: the mode-jumping experiment of a plate strip carried out by Stein (1959a) and the classical tests on cylindrical shells in compression that were carried out by Esslinger and collaborators in 1970 (Esslinger, 1970; Esslinger *et al.*, 1977).

II. Basics, the Geometrical Point of View

A. Notation, Assumptions, Governing Equations

The structural models that will be studied here are supposed to be purely elastic. It is further assumed that an appropriate discretization procedure is available that allows us to represent the state of the structure in terms of a vector of finite dimension. In the following two sections we will first concentrate on the solutions of the static equilibrium.

To understand what type of loadings will be considered here, it is convenient at first to make the distinction between the configuration space $\mathbb{C}_{N+K}$, in which the state $\mathbf{d}^e$ of the structure is described and the computional space $\mathbb{D}_N$ in which the freedoms, $\mathbf{d}$, that are to be determined are described. Here N is the dimension of $\mathbb{D}_N$ and $(N + K)$ is the dimension of $\mathbb{C}_{N+K}$. Thus the computational space $\mathbb{D}_N$, which is a subspace of $\mathbb{C}_{N+K}$, refers to all of the freedoms that are determined by the governing equations, while the configuration space refers to the same freedoms plus the variables that are prescribed by the kinematical boundary conditions. In this manner, the configuration of the structure under load can, at all times, be represented by an $(N + K)$-dimensional vector,

$$\mathbf{d}^e = \sum_{i=1}^{N+K} d_i \mathbf{e}_i, \tag{2.1}$$

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