INSTRUCTOR'S MANUAL
TO ACCOMPANY

Calculus

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PART 1

CHAPTER SUMMARIES

CHAPTER 1 - THE CARTESIAN PLANE AND FUNCTIONS

SECTION TOPICS

1.1 The Real Number Line - rational and irrational numbers, open and closed intervals, inequalities, directed distance, absolute value.

1.2 The Cartesian Plane - coordinates, quadrants, distance between two points, midpoint rule.

1.3 Graphs of Equations - solution points, point-plotting method, intercepts, symmetry, symmetry tests, points of intersection.

1.4 Lines in the Plane - slope, equations of horizontal and vertical lines, point-slope equation, slope-intercept equation and general equation of a line, perpendicular and parallel lines.

1.5 Circles and Ellipses - standard and general equations of circles and ellipses, completing the square.

1.6 Functions - independent and dependent variables, functions, domain, range, functional notation, operations with functions, composite functions, inverse functions.

Chapter Comments

This chapter is generally considered to be a review chapter and for well prepared students the first five sections can be omitted (though we recommend at least a quick review of all sections). Section 1.6 should be covered by all students since the concept of a function is introduced along with the corresponding functional notation and terminology to be used throughout the book.

Students often find inequalities difficult to cope with, so we've tried to provide clear and simple motivation for their use in Section 1.1. Throughout the book we rely heavily on a student's ability to graph a function. Therefore, care should be taken to see that students understand and can use the concepts in Section 1.3.

Although a detailed discussion of ellipses is reserved until Chapter 11, we found it useful to include the standard equation of an ellipse in Section 1.5. This allows us to use ellipses in several exercises and examples prior to Chapter 11.

Additional Topics Covered Only in Exercises

Section Exercises 1.1 Points of trisection of an interval on the real line, absolute value of the difference, product, and quotient of two numbers, the triangle inequality.

Section Exercises 1.2 Points of trisection of a line segment in the plane.

Section Exercises 1.4 The two-intercept equation of a line, the distance between a point and a line.

Miscellaneous Exercises The angle of inclination of a line in the plane, the angle between two lines.

Chapter 2 - Limits and Continuity

Section Topics

2.1 Limits - an intuitive description of limits, one-sided limits, properties of limits, quidelines for evaluating limits.

2.2 Continuity - continuity on an interval, discontinuities, removable and nonremovable discontinuities, continuity of polynomial and rational functions, the greatest integer function, properties of continuous functions, Intermediate Value Theorem, extreme values and the Extreme Value Theorem.

2.3 Limits and Asymptotes - infinite limits and vertical asymptotes, odd and even vertical asymptotes, limits at infinity and horizontal asymptotes, determinate forms for the limit of $f(x)/g(x)$, indeterminate forms ∞/∞, 0/0.

2.4 ε-δ Definition of Limits - a formal definition of the limit of a function, establishing limits by use of the definition, one-sided limits, the Squeeze Theorem, ε-δ definition of continuity, limit of a composite function, infinite limits and limits at infinity.

Chapter Comments

Sections 2.1 and 2.2 contain *intuitive* developments of two very basic concepts in calculus - limits and continuity. Care should be taken to avoid too many of the technicalities of these topics. Instead, emphasis should be placed upon examples which readily fit the intuitive feeling that students usually develop for these topics. Section 2.2 also contains an intuitive discussion of two high powered theorems: the Intermediate and Extreme Value Theorems. These can be omitted; but if discussed, do not labor over them.

Section 2.3 is crucial to the understanding of asymptotes and their subsequent use as aids to curve sketching. Cover this section carefully.

Section 2.4 is completely optional since the preceding intuitive discussion of limits is sufficient to meet the limit needs in the remainder of the text.

Additional Topics Covered Only in Exercises

Section Exercises 2.3 Slant Asymptotes.

Chapter 3 - Differentiation

Section Topics

3.1 The Derivative as the Slope of a Curve - tangent lines, four-step process for finding the slope of a curve, limit definition of derivative, differentiating by the four-step process.

3.2 The Derivative as a Rate of Change - average and instantaneous rates of change, interpretations of the derivative.

3.3 Some Rules for Differentiation - constant rule, power rule, scalar multiple rule, sum and difference rule.

3.4 Higher-Order Derivatives; Velocity and Acceleration - notation for higher-order derivatives, position equation, velocity, speed, acceleration.

3.5 Differentiability and Continuity - alternative definition of the derivative of f at c, derivatives from the left and the right, characteristics that destroy differentiability, differentiability implies continuity.

3.6 The Product and Quotient Rules - verbal and symbolic statements of the product and quotient rules, suggestions for using the product and quotient rules, proof of power rule for n a negative integer.

3.7 The Chain Rule and the General Power Rule - informal derivation of the chain rule, the general power rule as a special case of the chain rule, rule for differentiating the absolute value of a function, summary of rules for differentiating algebraic functions.

3.8 Implicit Differentiation - explicit and implicit forms of equations, finding derivatives by implicit differentiation, finding derivatives with respect to y rather than x, proof of the power rule for n a rational number.

Chapter Comments

The first two sections of this chapter are designed to give students a clear picture of both the definition of and the interpretation of the derivative of a function. A four-step limit taking process is introduced to facilitate the finding of a derivative. Special emphasis is made of the equivalence of finding; the derivative, the slope of a curve, and the instantaneous rate of change of one variable with respect to another. These first two sections should be covered carefully. We have found that this rather extensive coverage of the derivative prior to the rules for differentiating (Section 3.3) helps students retain the meaning of the derivative and prevents differentiation from becoming a meaningless mechanical process.

In Sections 3.3 and 3.4 we discuss and prove some of the basic rules for differentiation. The concepts of velocity and acceleration are used to motivate higher-order derivatives. The product, quotient, and chain rules are not included in these sections because we have found that students need time to gain confidence in using the simple rules before becoming burdened with the more complicated ones.

The power rule for $d[x^n]/dx$ is stated as true for all real numbers n. It should be pointed out that though the rule is true for any real number, the proof of such is best given in a sequential manner as addtional skills and mathematical resources are obtained. The proof for n a negative integer is given after the quotient rule (Section 3.6) and the proof for n a rational number follows the discussion of implicit differentiation (Section 3.8).

In Section 3.5, primary emphasis should be given to explaining and demonstrating some conditions which destroy differentiability. Students should learn to recognize when and where a function is not differentiable. This knowledge proves useful later in the book, especially in curve sketching.

Additional Topics Covered Only in Exercises

Section Exercises 3.6 The derivative of the triple product $y = f(x)g(x)h(x)$.

Section Exercises 3.7 The second derivative of the absolute value function, $y = |u|$.

Section Exercises 3.8 Normal lines, orthogonal curves.

Chapter 4 - Differentiation and Curve Sketching

Section Topics

4.1 Rolle's Theorem and the Mean Value Theorem - detailed examples verifying Rolle's Theorem and the Mean Value Theorem.

4.2 Increasing and Decreasing Functions - use of the first derivative to test whether a function is increasing or decreasing, critical numbers.

4.3 The First-Derivative Test for Relative Extrema - local (relative) extrema, finding critical numbers using the first derivative, testing critical numbers for relative extrema.

4.4 Concavity and the Second-Derivative Test - definitions of concave upward and concave downward, use of second derivative to determine direction of concavity, point of inflection, use of second derivative to find and test for points of inflection, Second-derivative test for relative extrema.

4.5 Newton's Method - zeros of a function, Newton's Method, iterations, situations in which Newton's Method may not converge.

4.6 Curve Sketching: A Summary - a curve sketching procedure that incorporates symmetry, asymptotes, continuity, relative extrema, and concavity.

4.7 Curvature - curvature of a circle, closest circular approximation, curvature of a graph at a point, formula for curvature.

Chapter Comments

In the first section, we discuss two basic theorems (Rolle's Theorem and the Mean Value Theorem) that are used to develop and prove many practical theorems in this and later chapters. The rest of the chapter includes the essential concepts, tests, and procedures that are needed to develop skill in sketching the graph of a function. The ability to efficiently sketch the graph of a function is extremely important and indeed, "a picture is worth a thousand words" is as true for calculus as for anything else.

Skill in curve sketching is the primary goal of this chapter and all concepts should be discussed and demonstrated in this context.

Section 4.7 is *optional* in the sense that the formula for curvature is restated whenever the curvature of a graph is required in later exercises.

Additional Topics Covered Only in Exercises

None in Chapter 4.

Chapter 5 - Applications of Differentiation

Section Topics

5.1 Applications of Extrema - a four step procedure for solving minimum-maximum problems.

5.2 Related Rates: Change/Time - a four step procedure for solving related rate problems.

5.3 The Derivative in Economics: Change/Output - marginal profit, marginal revenue, marginal cost, demand function, price elasticity of demand, elastic and inelastic demand.

5.4 Differentials - dy as an approximation to Δy, relative and percentage errors, dx as an approximation to Δx (Newton's Method).

Chapter Comments

This chapter is designed to provide practical examples of the various interpretations and uses of derivatives. Care should be taken to emphasize the four interpretations of the derivative covered in this chapter. For instance, the minimum-maximum problems in Section 5.1 require the use of the first derivative to determine the extreme values of a function. This use corresponds to the interpretation of the derivative as the *slope* of a graph.

Sections 5.2 and 5.3 require the *rate of change* interpretation of the derivative. Specifically, the related rate problems of Section 5.2 involve time dependent variables and consequently they require the rate of change with respect to *time*. The economic problems of Section 5.3 use the derivative as a rate of change with respect to variables other than time. In general, this other variable is the number of items *produced* or *sold*.

Finally, in Section 5.4 we interpret the derivative dy/dx as a ratio of two distinct *differentials*.

It should be pointed out in Section 5.1 (see the note on page 205) that the minimum or maximum of a function on an interval may not exist or many occur at an endpoint for which the derivative is not zero. In most practical applications this phenomenon is easily recognized. It has been our experience that an extensive discussion of endpoint extrema is more confusing than helpful.

In Section 5.4, if you find a lagging student interest in using differentials for approximations, we encourage you to pursue the suggestion on page 224 where Newton's Method is viewed as an approximation by differentials. The use of calculators in this application of differentials may revive the interest.

Additional Topics Covered Only in Exercises

Section Exercises 5.3 Average cost function, average elasticity of demand.

Section Exercises 5.4 Differential forms of differentiation rules.

Chapter 6 - Integration

Section Topics

6.1 The Indefinite Integral - antidifferentiation, antiderivative, integration, integrand, constant of integration, some integration rules, general power rule for integration.

6.2 Sigma Notation - index of summation, upper and lower limits, properties of sigma notation, summation formulas.

6.3 Area and the Definite Integral - partitions, inscribed and circumscribed rectangles, lower and upper sums, norm of a partition, area as the limit of a sum, definite integral from a to b, continuity implies integrability, the distinction between indefinite and definite integrals.

6.4 The Fundamental Theorem of Calculus - properties of definite integrals, Mean Value Theorem for integrals, average value.

Chapter Comments

This chapter is crucial since it binds together the two fundamental concepts of calculus - differentiation and integration.

Make special effort to see that students understand the concepts of Section 6.1. The general power rule is given in this section so that students have ample opportunities to capture the meaning of antiderivatives and to gain skill in finding antiderivatives.

Section 6.2 merely introduces the notation that is used in developing the definition of the definite integral. Do not belabor this section.

Although we develop our definition of definite integrals from an area model, it should be pointed out that definite integrals have a variety of other applications. Don't overwork the students in this section. *However,* enough work should be given so that students retain the "limit of a sum" intepretation of a definite integral. Later uses of the definite integral rely heavily upon this interpretation.

Section 6.4 includes a feasible geometric argument leading up to the Fundamental Theorem of Calculus. The emphasis here should be that the

Fundamental Theorem provides a convenient means of evaluating a definite integral by using antiderivatives *provided* such antiderivatives can be found. Students should realize that the Fundamental Theorem does not provide a means for finding antiderivatives.

Make certain that by the end of this chapter students are aware that the indefinite integral denotes a family of functions whereas the definite integral denotes a number which is the limit of a sum.

ADDITIONAL TOPICS COVERED ONLY IN EXERCISES

None in Chapter 6.

CHAPTER 7 - APPLICATIONS OF INTEGRATION

SECTION TOPICS

7.1 Area of a Region Between Two Curves - area using vertical rectangles and dx, area using horizontal rectangles and dy.

7.2 Volumes of Solids of Revolution: Disc Method - volume of a disc, solid of revolution, volumes of solids having a horizontal axis of revolution, volumes of solids having a vertical axis of revolution.

7.3 Volumes of Solids of Revolution: Shell Method - volume of a cylindrical shell, volumes of solids having a horizontal axis of revolution, volumes of solids having a vertical axis of revolution, comparison of Disc and Shell Methods.

7.4 Volumes of Solids with Known Cross Sections - cross sections perpendicular to the x-axis, cross sections perpendicular to the y-axis.

7.5 Work - work done by a *constant* force, work done by a *variable* force.

7.6 Fluid Pressure - force exerted by a fluid of constant density.

7.7 Moments, Centers of Mass, and Centroids - moment of a system, equilibrium, moments about the x- and y-axes, center of mass of a system, lamina, centroid of a plane region.

7.8 Arc Length - rectifiable curves.

7.9 Surface of Revolution - lateral surface area of a frustum of a cone, area of a surface of revolution.

Chapter Comments

Each of the applications of the definite integral in this chapter can be most readily understood using the limit of a sum interpretation of the definite integral. We have found that students perform better in this chapter if they are required to set up the representative element of area, volume, work, etc. that is used in the summation process. The form of this element then clearly identifies the integrand for the definite integral needed to calculate the required values. Often students try to memorize the definite integral for each type of application. This is both time consuming and confusing for most students. We have found that practice in finding the representative elements for each application will minimize the difficulties generally encountered in applications of integration.

Additional Topics Covered Only in Exercises

None in Chapter 7.

Chapter 8 - Logarithmic and Exponential Functions

Section Topics

8.1 Exponential and Logarithmic Functions - exponential functions with base a, properties of exponents, the logarithmic function as the inverse of the exponential function, properties of logarithms, limit definition of e, continuously compounded interest, the natural logarithmic function.

8.2 Logarithmic Functions: Derivatives and Logarithmic Differentiation - algebraic and transcendental functions, derivative of the natural logarithmic function, derivative of logarithmic function to base a, logarithmic differentiation.

8.3 Logarithmic Functions: Integration, Growth, and Decay - integration formula for $\int(u'/u)dx$, differential equations, rate of decay, rate of growth.

8.4 Exponential Functions: Differentiation and Integration - derivatives of exponential functions, normal probability density function, integrals of exponential functions.

8.5 Indeterminate Forms and L'Hôpital's Rule - indeterminate forms $0/0$, ∞/∞, and $\infty-\infty$, general form of L'Hôpital's Rule, indeterminate forms 1^∞, ∞^0, and 0^0.

CHAPTER COMMENTS

Our pattern of development in this chapter is to introduce new functions or concepts by way of familiar concepts. For instance, by reversing the roles of the variable and constant in the algebraic function $f(x) = x^a$, we obtain the exponential function $g(x) = a^x$. We introduce logarithmic functions as inverses of exponential functions. The limit definition of e is motivated through a development of the formula for calculating the amount in a savings account for which interest is compounded continuously. Later, in Sections 8.2 and 8.4 the convenience of base e for exponential and logarithmic functions is repeatedly pointed out.

Section 8.1 includes a review of the properties of exponents and their relationship to the properties of logarithms.

Each calculus operation with exponential and logarithmic functions in Section 8.2 through 8.4 is followed by a practical application. Section 8.3 contains a brief introduction to differential equations.

In Section 8.5 we provide an extensive discussion of L'Hôpital's Rule and indeterminate forms. This section should be covered in sequence since L'Hôpital's Rule is used frequently in succeeding chapters.

ADDITIONAL TOPICS COVERED ONLY IN EXERCISES

Section Exercises 8.5	Comparitive rates of increase for $f(x) = x^m$, $g(x) = e^{mx}$, and $h(x) = (\ln x)^n$ where $n, m > 0$ and $x \to \infty$.
Misc. Exercises	Exponential Density Function.

CHAPTER 9 - TRIGONOMETRIC FUNCTIONS

SECTION TOPICS

9.1 Review of Trigonometric Functions - definitions of the six trigonometric functions from two viewpoints, common trigonometric identities, angle in standard position, coterminal angles, graphs of trigonometric functions, period, amplitude, shift, $\lim_{x \to 0} (\sin x)/x$.

9.2 Derivatives of Trigonometric Functions - four-step process to derive the formula for the derivative of $f(x) = \sin x$, Chain Rule for $d[\sin u]/dx$, derivative formulas for remaining trigonometric functions obtained through the use of trigonometric identities and the derivative of $\sin u$, simple harmonic motion.

9.3 Integrals of Trigonometric Functions - integration formulas derived from differentiation formulas, integrals of the six basic trigonometric functions.

9.4 Inverse Trigonometric Functions - inverse sine functions, definitions and graphs of the six inverse trigonometric functions.

9.5 Inverse Trigonometric Functions: Derivatives and Integrals - derivatives of inverse trigonometric functions, integrals involving the inverse trigonometric functions.

9.6 Hyperbolic Functions (Optional) - definitions of hyperbolic functions hyperbolic identities, differentiation and integration formulas, inverse hyperbolic functions, derivatives and integrals involving inverse hyperbolic functions.

Chapter Comments

Even though Section 9.1 is listed as a review section, it should be covered carefully. Most calculus students have been exposed to trigonometry prior to calculus, but they usually have forgotten many of the basic definitions and identities. All of the basic definitions and identites in this section should be memorized. Additional trigonometric identities are included on the inside of the cover of the text. Graphing trigonometric functions is an important skill and the use of periods, amplitudes, and shifts is very useful in this regard.

To be efficient with trigonometry, memorizations is essential. We strongly recommend that students memorize the derivative and integral formulas for each of the six basic trigonometric functions as well as those formulas involving the three inverse functions arcsine, arctangent, and arcsecant.

Section 9.6 is optional. On occasion some of these functions are encountered later in the text. *However,* in such instances the necessary definitions or formulas are included. If Section 9.6 is omitted, it should be pointed out to students that they already have all the necessary tools to operate with the hyperbolic and inverse hyperbolic functions since these functions are merely exponential and logarithmic functions, respectively.

Additional Topics Covered Only in Exercises

Section Exercises 9.6 Equation of a tractrix.

Chapter 10 - Techniques of Integration

Section Topics

10.1 Review of Fundamental Integration Formulas - rewriting integrands in forms which fit the fundamental formulas.

10.2 Completing the Square - writing quadratic expressions in the form $u^2 \pm a^2$ or $a^2 - u^2$.

10.3 Substitution - making a change of variable to eliminate radicals of the form $\sqrt[n]{ax+b}$.

10.4 Partial Fractions - linear factors, repeated factors, irreducible quadratic factors, rules for decomposing a rational function into its partial fractions.

10.5 Trigonometric Integrals - power rules for the six trigonometric functions, integrals of the form $\int(\sin^m x\cos^n x)u'\,dx$, integrals of the form $\int(\sec^m x\tan^n x)u'\,dx$.

10.6 Trigonometric Substitution - trigonometric substitutions for integrals involving $\sqrt{a^2-u^2}$, $\sqrt{u^2+a^2}$, or $\sqrt{u^2-a^2}$.

10.7 Integration by Parts - suggestions for assigning values to v' and u in the formula $\int uv'\,dx = uv - \int vu'\,dx$, repeated use of integration by parts, integration formulas for $\sec^3 u$ and $\sqrt{u^2 \pm a^2}$, partial fractions involving repeated quadratic factors.

10.8 Miscellaneous Substitutions - substitutions that eliminate radicals of different orders, the substitution $u = \tan(\theta/2)$ for rational integrands involving trigonometric functions, reciprocal substitutions.

10.9 Summary and Integration by Tables - a list of ninety common types of integrals together with their antiderivatives *and* the integration technique used to obtain each antiderivative, reduction formulas.

10.10 Numerical Integration - Trapezoidal Rule, Simpson's Rule, approximation of error in the Trapezoidal and Simpson's Rule.

10.11 Improper Integrals - infinite limits of integration, infinite integrands.

Chapter Comments

This chapter is the *heart* of the second term of calculus. It requires a great deal of perseverance by both students and professors. We usually notice an improvement in student performance when we stress the overall picture of Chapter 10 before beginning Section 10.2

It should be emphasized that up to this point our integration efforts have been confined to the development and use of a collection of nineteen fundamental integration formulas. Obviously, these few formulas do not cover all types of integrands. Our overriding objective for this chapter is to expand the use of these formulas by a variety of techniques that allow us to rewrite integrands in forms that fit the nineteen basic rules.

It should be pointed out that the so called "integration techniques" of Sections 10.2, 10.3, 10.4 and 10.6 are not integration techniques all but are instead algebraic or trigonometric procedures for rewriting integrands in forms that fit the basic integration formulas. This view of the situation is usually enlightening to students and a certain amount of intrique can be developed in this pursuit of *new ways to use old formulas*.

The important issue in integration by parts is the assignment of factors to u and v' in the integration by parts formula.

Section 10.8 includes some useful substitutions; however, do not labor over this section. Time can be spent more profitably on integration by tables, a topic which is too often omitted in calculus texts. This is