THE FINITE ELEMENT METHOD IN *Thermomechanics*

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To my wife Su-Yong
and Jean, Euginette, Leigh

PREFACE

The rapid advances in the nuclear and aerospace technologies in the past two decades compounded with the increasing demands for high performance, energy-efficient power plant components and engines have made reliable thermal stress analysis a critical factor in the design and operation of such equipment. Recently, and as experienced by the author, the need for sophisticated analyses has been extended to the energy resource industry such as *in-situ* coal gasification and *in-situ* oil recovery from oil sands and shales.

The analyses in the above applications are of a multidisciplinary nature, and some involve the additional complexity of multiphase and phase change phenomena. These extremely complicated factors preclude the use of classical methods, and numerical techniques such as the finite element method appear to be the most viable alternative solution.

The development of this technique so far appears to have concentrated in two extremes; one being overly concerned with the accuracy of results and tending to place all effort in the implementation of special purpose element concepts and computational algorithms, the other being for commercial purposes with the ability of solving a wide range of engineering problems. However, to be versatile, users require substantial training and experience in order to use these codes effectively. Above all, no provision for any modification of these codes by users is possible, as all these codes are proprietary and access to the code is limited only to the owners.

The present approach attempts to strike a compromise between the above two extremes. The theoretical derivations presented in the proposed book, in particular Chapters 2–4, will serve as the basis for a computer code TEPSAC (*T*hermo*e*lastic-*p*lastic *S*tress *A*nalysis with *C*reep). The code, although limited to two-dimensional plane, or three-dimensional axisymmetric structures, can handle large classes of thermomechanical problems using a simplex element algorithm. The thermal and mechanical analyses are quasi-coupled by automatic transferring of data back and forth between these two major components. The full listing of the basic TEPSAC code will be included in Appendix 5 so that the readers may relate the corresponding theory in the text to the program in the Appendix, in addition to being able to use the code for their own research and development work.

In addition to the transient (or steady state) thermal–elastic–plastic-creep capability that has been included in the basic TEPSAC code, additional topics such as Fourier series approximation for nonaxisymmetric loading, elastodynamic stress analysis, thermofracture mechanics, finite strain theory, coupled thermoelastic-plasticity and program organization will also be included. These topics were carefully selected on the basis of industrial needs.

As the intention of this book is to embrace as wide a range of topics in the nonlinear thermomechanical analysis as possible, it is necessary to cut back on some of the fundamental principles of the finite element method. Topics such as element variety, in-depth study of the variational principle, integration schemes, convergence criteria, etc. will be omitted in the text. However, these are adequately discussed in many published books on the finite element method.

Chapter 1 presents only the bare fundamental principle of the finite element method. The essence of the discretization concept described in this chapter will enable the readers to appreciate the potential of the finite element method and to acquire a correct perception of its nature. The steps of the general finite element method will be presented in a way that includes the formation of some key equations. At the end of Chapter 1, readers are expected to have a firm grasp of this technique and its application in various branches of the engineering science.

Chapter 2 is devoted to the derivation of the finite element formulation of heat conduction analysis. A review of basic mathematical expressions for heat conduction in solids and convection in fluids as well as appropriate initial and boundary conditions will be presented first. It will then be followed by the discretization of axisymmetrical solids, the derivations of the element's thermal equilibrium equations and the conductivity and heat capacity matrices. An alternative method of implementing boundary conditions to the discretized models will also be included. Axisymmetric solid geometry was chosen, as planar structures can often be treated as degenerated axisymmetric geometry with a large artificial radius of curvature.

Chapter 3 deals with the finite element formulation of thermoelastic-plastic analysis, and it plays a very important part in the TEPSAC code. A brief review of relevant theories of elasticity and plasticity will be presented first. A comprehensive derivation of the element equations in both elastic and elastic–plastic cases is provided, including both the effects of thermal and the associated material properties variation. Chapter 3 also presents a detailed formulation for both isotropic and kinematic hardening behavior of the structural material. A clear distinction is made between these hardening rules and their respective applications. The solution procedure offers the reader a clear idea of how the respective mathematical expressions are implemented in the TEPSAC code.

Chapter 4 deals with the derivation of the finite element algorithm for structures subject to creep. It will be demonstrated in this chapter that the only major required modification for the creep effect on the main thermo-elastic–plastic formulation is the force matrix. The reader will follow the logical sequence of a solution presented in the book and, it is hoped, will appreciate the relative simplicity of this important aspect of nonlinear thermomechanical analysis contained in the basic TEPSAC code.

Many engineering applications may involve axisymmetric structures subject to nonaxisymmetric loadings. Typical are large horizontal pressure vessels or pipelines containing heavy fluids. The analysis is clearly of a three-dimensional nature. However, such analysis in the thermoelastic-plastic regime would require an enormous amount of computer time and storage. An approximation, by means of Fourier series, for the third directional variation of the physical quantities in load description and displacement field in a two-dimensional analysis code, may prove to be a great deal more economical and hence more practical. Chapter 5 presents the theory and algorithm used in conjunction with such a technique. Although this method has been proposed in the past for elastic stress analysis, the present approach extends the use of this technique into the elastic–plastic region. A numerical example on the sag of a heavy pipe illustrates the merit of this method.

Chapter 6 presents the finite element equations for stress wave propagation through structures, caused by dynamic loads such as impact or explosions. Again, the dynamic effect could be treated as a special module to the base TEPSAC code for the cases involving strong thermal effect. A numerical example is included to illustrate the physical significance of the dynamic effect.

The importance of fracture mechanics analysis on large structures has been well recognized by engineers and researchers. While the theory of linear fracture mechanics for brittle materials has been well established, the treatment of initially stable, but eventually unstable, cracks commonly occurring in ductile materials is less complete. In view of the many such recent catastrophic structural failures, emphasis on the fracture analysis of ductile materials is warranted.

Chapter 7 presents specifically some of the unique techniques that could be used to predict the growth of cracks in structures under combined thermomechanical loads. A review of the literature on linear elastic and elastic–plastic fracture mechanics is included with the presentation of brief relevant mathematical expressions, as well as the application of the finite element method. Detailed description is, however, given to the use of the "breakable" element concept which was originally developed at the author's research laboratory and has been incorporated into a special version of the TEPSAC code. This unique technique can provide users with detailed information associated with the growing crack, which is missing from many other existing methods. The subject of crack growth due to creep is also included. The existing C^*-integral method of solution is thoroughly described. Readers will find that the "breakable" element concept can be readily extended to handle creep crack growth, with some unique advantages over the C^*-integral approach.

In Chapter 8 is presented the theory and the corresponding finite element formulation on finite strain plasticity. The theoretical formulation of the problem is again made adaptable to the TEPSAC code as an optional module. It will be demonstrated that finite element analysis for finite strain plasticity can be achieved by including two additional element stiffness matrices, K_2 for the nonlinear terms in the strain–displacement relations and K_3 for the necessary correction of loading due to excessive rotational displacement components. Numerical examples on the simple elongation of a prismatic bar serve to illustrate the merit of this approach, as well as to indicate the inadequacy of the usual small strain approach for the instability of metal deformation.

Chapter 9 deals with the very important subject of coupled thermomechanics. The effect of thermomechanical coupling can be observed in many metal forming and thermal shock processes. This topic, in the author's view, has received far less attention than it deserves, presumably because of its complex nature and of the difficulty of its solution by classical methods. This chapter, however, provides the reader with a physical sense of the mathematical formulation, which starts with a review of the first and second law of thermodynamics, and the energy equations which link the thermal and mechanical effects for a deforming solid. The finite element discretization of the coupled thermoelastic–plastic equations is carried out, and shown to be related to the existing quasi-coupled approach. Again, a numerical example on the simple tension–compression of prismatic bars illustrates the physical significance of this effect, as well as providing direction for further research on this subject.

The remaining chapter of the book, Chapter 10, is devoted to a description of the basic TEPSAC code. It provides the reader with case illustrations on the use of the code. Most of the cases included are excerpts from published work.

The task of preparing a book of this breadth certainly cannot possibly be accomplished by a single person's effort. The author has the fortune and great pleasure to have had many young and dedicated research associates and students working for him during the past dozen years. The following are just a few names identifiable with specific chapters in the book: A. W. M. Bertels for his early, but very important, assistance in shaping up the TEPSA code (TEPSAC version without creep), in particular, the theories described in Chapter 3 and the concept of breakable element in Chapter 7; W. K. Tam for his work in improving the kinematic hardening theory, Y. J. Liu for his contribution in creep analysis and the formulation of the generalized creep-fracture theory in Chapter 7; R. Y. Wu for Chapter 5; D. A. Scarth for Chapter 6; Y. J. Kim for the excellent work in the improvement of the breakable element algorithm and significant contribution in Chapter 7; S. Y. Cheng for a very major contribution in formulating the finite strain theory given in Chapter 8; A. Banas and N. S. Sun for their significant work in the coupled thermoelastic–plastic theory in Chapter 9; and, more recently, the valuable assistance by G. G. Chen and Z. Wozniak for the solutions of several important numerical examples in the book. Assistance by other technical and research staff and graduate students at the author's laboratory is also gratefully acknowledged. Among these individuals are D. Fedorowich, D. Kuss, H. A. Ashour, Z. L. Gong, J. R. Yu, Z. H. Zhai, E. A. Abdel-Hadi and K. S. Bhatia.

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CONTENTS

LIST OF TABLES