

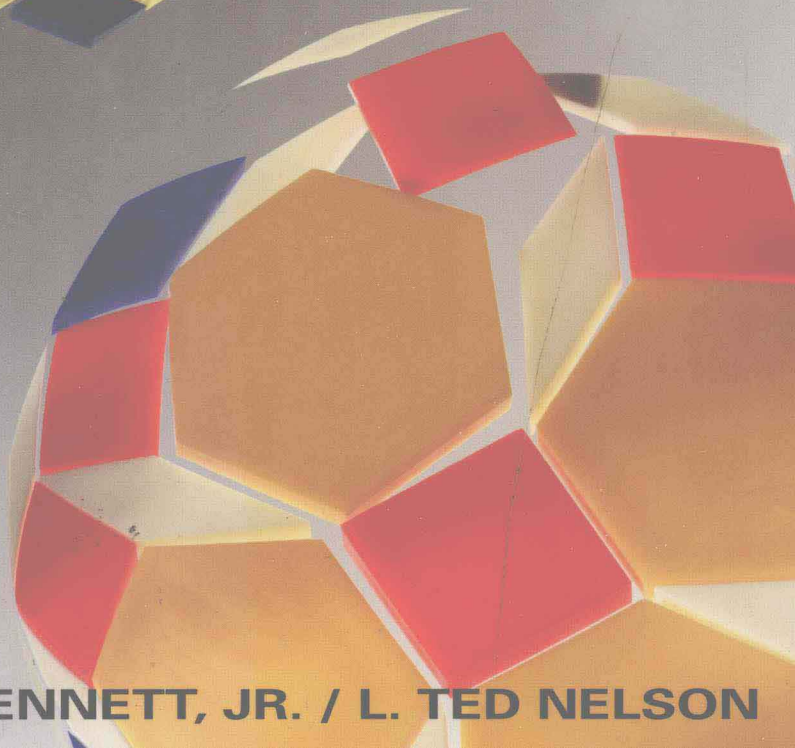
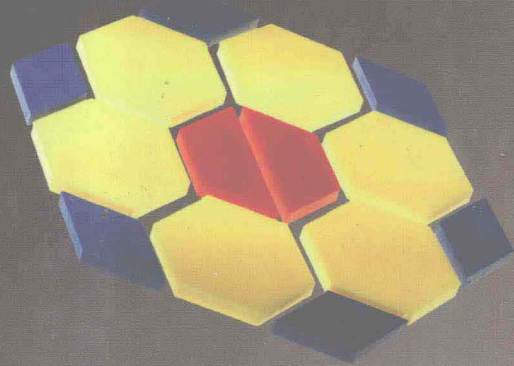


4<sup>th</sup> edition


# MATHEMATICS

FOR ELEMENTARY TEACHERS

A CONCEPTUAL APPROACH



ALBERT B. BENNETT, JR. / L. TED NELSON



4<sup>th</sup> edition

# MATHEMATICS

FOR ELEMENTARY TEACHERS

A CONCEPTUAL APPROACH

**Albert B. Bennett, Jr.**  
*University of New Hampshire*

**L. Ted Nelson**  
*Portland State University*

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## **Mathematics for Elementary Teachers: A Conceptual Approach**

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## About the Authors

It was at the University of Michigan that Albert Bennett and L. Ted Nelson and their families first met. Bennett and Nelson had been invited to participate in a National Science Foundation sponsored program of graduate studies in mathematics. Ten years later while on sabbaticals at the University of Oregon they collaborated in writing lessons to actively involve prospective teachers in learning the mathematical concepts they would be teaching. These lessons eventually led to the publication of the first editions of *Mathematics for Elementary Teachers: A Conceptual Approach* and *Mathematics for Elementary Teachers: An Activity Approach*.

Albert Bennett completed his undergraduate and masters degrees at the University of Maine in Orono. He taught mathematics at Gorham State College and became active in the summer mathematics institutes that were sponsored by the Association of Teachers of Mathematics in New England. An early bias that was reflected in his teaching of these institutes was the need to encourage intuition in the teaching and learning of mathematics. He received his doctorate in mathematics from the University of Michigan in 1966 and joined the mathematics faculty at the University of New Hampshire to teach mathematics to prospective teachers. There he organized a mathematics lab and started writing laboratory activities for teachers. In the next few years his efforts led to the publication of *Fraction Bars*, *Decimal Squares*, and articles and textbooks for elementary and middle school teachers. These publications support methods of using models and concrete materials in the teaching of mathematics.

Ted Nelson is professor of mathematics and education at Portland State University. He taught junior and senior high school mathematics after graduating from St. Cloud State University, and then continued mathematical studies at Bowdoin College and the University of Michigan, where he received his doctorate in 1968. After serving four years as the first mathematics department chair at Southwest Minnesota State University he moved to Oregon to follow his interest in teaching mathematics to teachers. Currently, his main goal is to continue development of three lab-based courses for prospective elementary teachers and eight additional lab-based courses for middle school teachers. His teaching and curriculum efforts led to a faculty achievement award for outstanding university teaching in 1988. Over the past fifteen years he has written curriculum materials and given workshops designed to bring more concrete materials, visual models, and problem solving investigations into the elementary and middle school mathematics curriculums.

# PREFACE

The fourth edition of *Mathematics for Elementary Teachers: A Conceptual Approach* continues to place emphasis on the use of models and diagrams for providing insights into mathematical concepts before stating generalizations. This edition also increases the use of technology for teaching mathematics. Recommendations by the Committee on the Undergraduate Program in Mathematics (CUPM) and the National Council of Teachers of Mathematics (NCTM) support these approaches to teaching and learning mathematics.

The primary objective of *Mathematics for Elementary Teachers: A Conceptual Approach* is to present mathematics in a format that prepares teachers to teach elementary school mathematics. Teachers need a firm foundation in the theory of mathematics as it pertains to the elementary school curriculum. They also need ideas and methods for teaching mathematics to elementary school children in a way that will provide an understanding of concepts and will generate interest and enthusiasm.

The greatest challenge in writing a text for prospective teachers is to create a proper balance between pedagogical concerns and those of a theoretical and abstract nature. Although this text focuses primarily on teaching content, the extensive use of models and diagrams accomplishes three objectives. It assists college students in understanding mathematics concepts, helps students connect and remember ideas and concepts, and provides ideas and models for teaching mathematics to elementary school children.

## Suggestions for Active Student Participation

NCTM's *Curriculum and Evaluation Standards for Mathematics* recommends that students develop mathematics by looking for patterns, making conjectures, and verifying hypotheses. Many instructors have been influenced by such recommendations and are using instructional methods which involve more active student participation and reduced time in the lecture format. Here are a few suggestions involving special features of the text for encouraging active student participation.

**MATH ACTIVITIES** This edition of *Mathematics for Elementary Teachers: A Conceptual Approach* contains a one-page math activity preceding each of 37 sections. Envelopes and a set of 12 material cards with manipulatives for the activities are packaged separately and are available from the publisher. The Math Activities can be used for the following purposes: small group instruction for introducing sections of the text; homework assignments to provide background for a section of the text; or ideas for projects or papers involving the design of an elementary school activity. The Math Activities have been included in this edition of the text for the following reasons: to introduce students to activities that develop concepts and mathematical reasoning; to provide opportunities for students to look for patterns, form conjectures, and express their thinking; and to familiarize students with activities that can be adapted to the elementary school curriculum.




**PROBLEM OPENERS** Each section of the text begins with a Problem Opener related to the content of that section. The Problem Openers may be used for small-group problem solving and class discussions. They can be used to open a lesson before the content of the section is introduced and to motivate interest in the topic. The solution to each Problem Opener and the problem-solving strategies required is contained in the *Manual*. The *Manual* also includes one or more ideas for looking back and extending each Problem Opener for additional problem solving in class or on assignments and tests.

**INVESTIGATIONS** The National Council of Supervisors of Mathematics has stated, “Students need to explore mathematics using manipulatives, measuring devices, models, calculators, and computers. . . .”<sup>\*</sup> At the end of each section of the text there is a Computer Investigation, a Calculator Investigation, or a Laboratory Investigation involving the mathematical content of the section. These investigations pose open-ended questions that require collecting data, looking for patterns, and forming and verifying conjectures. The investigations can be used for student papers or class reports. Some of the Calculator Investigations require a graphics calculator. There are also two types of Computer Investigations: those designed for discovering relationships in geometry with one of several new interactive geometry software packages; and those designed for numerical discoveries with the computer disk *Mathematics Investigator*, which is available from the publisher. This disk is described on the following pages under Supplements.

**PROBLEM-SOLVING APPLICATIONS** Each section of the text contains one or more Problem-Solving Applications, which apply the subject matter of the section and are analyzed with Polya’s four-step plan. These problems can be posed to the class for small-group problem solving. A follow-up discussion can involve comparing students’ plans for solving a problem and their solutions with those suggested in the text.

## Special Features

**WORD PROBLEMS AND PROBLEM SOLVING** Each set of Exercises and Problems contains a special subsection, Word Problems and Problem Solving, with new problems added to ensure that problem solving is reinforced throughout the text. Each problem-solving exercise is marked by the icon . One of these problems features one or more problem-solving strategies and is outlined by Polya’s four-step plan.

**PARITY OF EXERCISES** Questions in the Exercises and Problems have been augmented so that for all skills and concepts there is at least one odd-numbered and one even-numbered question.

<sup>\*</sup>National Council of Supervisors of Mathematics, “Essential Mathematics for the 21st Century” (Minneapolis, MN: NCSM Essential Mathematics Task Force, 1988), 3–4.

**PROBLEM SOLVING STRATEGIES** Chapter 1 contains eight problem-solving strategies, and several more are introduced in the remaining chapters. The problem-solving strategies used in each chapter are listed in the Contents with page references.

**CALCULATORS** Beginning with Chapter 1, there are instructions for using graphics and nongraphics calculators. Calculator paragraphs and exercises throughout the text are marked with a calculator icon.

**PUZZLERS** The text contains over 40 mathematics puzzles, which are listed in the Contents. Most occur at the end of the Exercises and Problems and are marked by an icon. Answers for the puzzles are found at the back of the text.

**HISTORICAL HIGHLIGHTS** Highlights in mathematical history appear throughout the text to suggest the origins of important ideas and provide background on the lives of some of the world's greatest mathematicians.

**VISUAL MODELS** Most mathematical concepts are illustrated by diagrams and geometric figures before mathematical terminology and abstractions are introduced.

**MENTAL CALCULATING AND ESTIMATING** The chapters on whole numbers, integers, rational numbers, and real numbers contain mental calculating and estimating techniques. The exercises involving mental calculating and estimating techniques are marked by an icon.

**BOXED FEATURES** Key definitions, rules, and properties are displayed in boxes.

**KEY TERMS** Words and terms that are defined or explained appear in boldface type in the paragraphs.

**CHAPTER REVIEWS** New words are listed, and key ideas are summarized at the end of the chapter.

**CHAPTER TESTS** Each chapter concludes with a test for evaluating skills and an understanding of the concepts in that chapter.

**BIBLIOGRAPHY** A bibliography of additional references is included at the end of each chapter.

**ANSWER SECTION** Selected answers for the section-opening Math Activities (marked with ★), and answers for the Puzzlers, the odd-numbered Exercises and Problems, and the chapter tests are at the back of the book.

## Supplements

**INSTRUCTOR'S RESOURCE MANUAL** The *Instructor's Resource Manual for Mathematics for Elementary Teachers: A Conceptual Approach* contains extensions for all problem openers and answers for the problem openers and extensions; answers for all even-numbered Exercises and Problems; answers for inves-

tigations (computers, calculators, and laboratory); chapter tests with answers (two tests for each chapter); transparency masters (various grids and dot paper); and a description of the *Mathematics Investigator*.

**MATHEMATICS INVESTIGATOR** The *Mathematics Investigator* is software containing 14 programs which are designed for the computer investigations at the end of the exercises in the text. Students may use this disk for gathering data and running simulations for the investigations listed below. These investigations pose questions to generate interest in various mathematical topics and encourage students to formulate and investigate their own conjectures. Instructors may use the disk to demonstrate computer simulations and the process of forming conjectures and looking for counterexamples. There are disks for the Macintosh and the IBM PC computers. Many of the features of these computers, such as pull-down menus, editing (cutting, pasting, copying text to other files), and printing, can be used with the programs on this disk.

#### MATHEMATICS INVESTIGATOR MENU

Triangular Numbers	Differences of Squares	Integer Differences
Palindromic Sums	Repeating Decimals	Standard Deviations
Palindromic Differences	Factorizations	Dice Roll Simulations
Palindromic Decimals	Frequency of Primes	Coin Toss Simulations
Consecutive Numbers	Number Chains	

**ACTIVITY BOOK** *Mathematics for Elementary Teachers: An Activity Approach, 4th edition*, contains an activity set corresponding to each section of the text. Each activity set is a sequence of inductive activities and experiments that enable the student to build an understanding of mathematical ideas through the use of models and the discovery of patterns. The activity sets augment the ideas presented in the corresponding sections of the text. Over 50 Material Cards, some with colored manipulatives, are packaged with *Mathematics for Elementary Teachers: An Activity Approach*. A section on Ideas for the Elementary Classroom at the end of each chapter includes a suggested elementary school activity and a list of selected sources. There are puzzlers throughout the book, and the activity sets are followed by Just for Fun enrichment activities.



# TO STUDENTS

It is important for you as a future teacher to realize that mathematics is a way of thinking rather than a collection of rules. It is the conceptual grasp that will help both you and your students learn and apply mathematics. The shaping of mathematical thinking and a mindset for solving problems needs to begin early in the school curriculum, and the responsibility for this lies with the elementary school teachers.

This book has been written to help you understand the basic concepts of mathematics so that you can help others enjoy mathematics. You will be encouraged to become actively involved in visualizing mathematical concepts, solving problems, performing mental calculations, and using the appropriate technology (calculators and computers). You will discover that most mathematical concepts can be developed by using manipulatives and visual images and that mathematics is an interesting part of your life and the world around you.

The content of this book reflects the curriculum standards set by the National Council of Teachers of Mathematics for grades K through 8, which establish the direction of school mathematics for the 1990s. In the past, elementary school mathematics programs stressed computational rules and speed and accuracy in computing. The focus is now changing, and today's children need to develop conceptual understanding, the ability to reason and communicate through mathematics, and the ability to solve problems. When estimations and approximations are not sufficient, computations can be done with calculators and computers. Children who are able to see mathematics conceptually have a better chance of solving problems and acquiring confidence in their ability to reason.

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
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