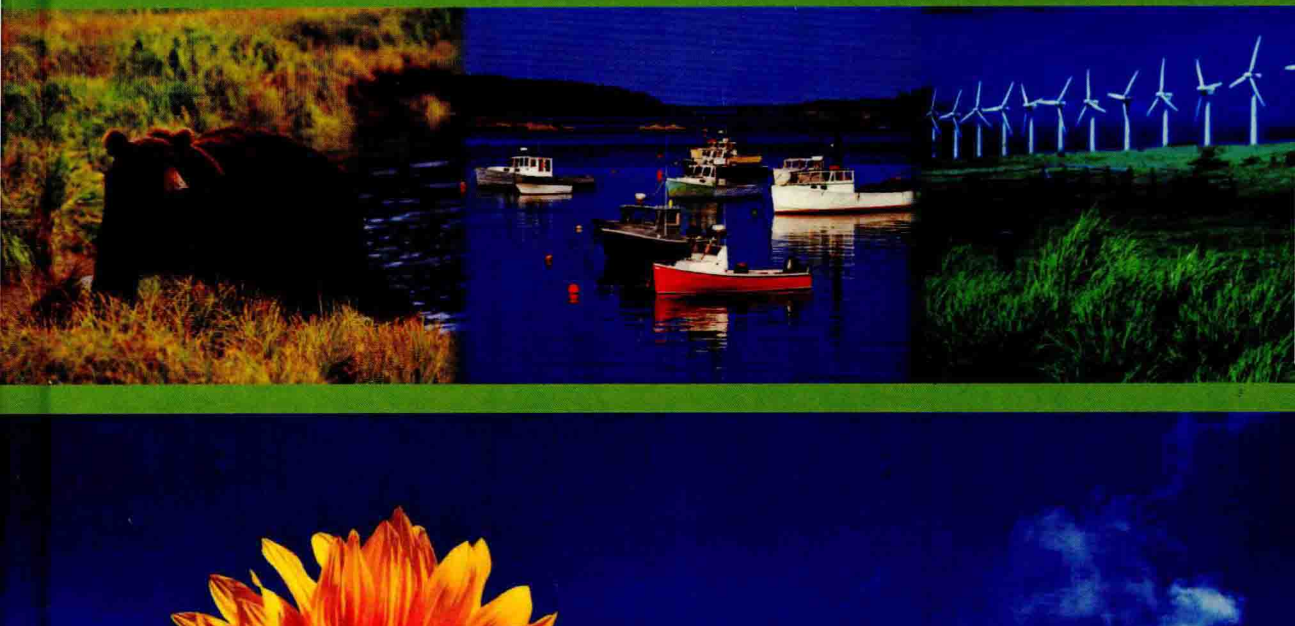
Mathematical Programming for Agricultural, Environmental, and Resource Economics



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MATHEMATICAL PROGRAMMING FOR AGRICULTURAL, ENVIRONMENTAL, AND RESOURCE ECONOMICS

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Preface

This book provides a comprehensive overview of mathematical programming models and their applications to important problems confronting agricultural, environmental, and resource economists. Mathematical programming, which includes linear and nonlinear programming models, is one of the most powerful and widely used problem-solving approaches in quantitative methods. It is used by researchers in businesses, governments, nongovernmental organizations, and academics to address problems involving the efficient allocation of scarce resources.

Unlike most mathematical programming books, the principal focus of this book is on applications of these techniques and models to the fields of agricultural, environmental, and resource economics. While applied to these important sectors of the economy, the models described here are also useful to other areas of applied economics. The three fundamental goals of the book are to provide the reader with (1) a level of background sufficient to apply mathematical programming techniques to real-world policy and business to conduct solid research and analysis; (2) a variety of applications of mathematical programming to important problems in the areas of agricultural, environmental, and resource economics; and (3) a firm foundation for preparation to more advanced, Ph.D.-level books on linear and nonlinear programming.

This book is designed to be an introductory book in applied mathematical programming. The reader is not required to have any formal background or training in this area. All techniques covered in this book are based on this assumption. Unlike more theoretical mathematical programming books, this book is written at a more basic mathematical level, which consists primarily of algebraic and geometric concepts, but a few of the later chapters include some basic calculus. The book is geared towards upper-level undergraduate and M.S-level graduate students majoring in economics, agricultural economics, environmental and resource economics, applied economics, business, and operations research. The book will also be useful to undergraduate and graduate students majoring in agricultural and food disciplines, such as food science, animal science, agronomy, and veterinarian medicine, as well as students majoring in environmental and resource studies.

Despite its introductory nature, the book places significant emphasis on real-world applications of mathematical programming to decision problems. A wide array of examples and case studies are used to convey the various programming techniques available to decision analysts. Readers will learn (1) how to set up programming models of real-world problems; (2) how to solve them graphically, algebraically, computationally, and with computer software; (3) how to interpret the results; (4) how to validate the model; (5) how to conduct sensitivity analysis; and (6) how to judge and verify the model's performance relative to the real-world decision process it depicts. Upon completing this book, students should be able to use mathematical programming in independent applied research, including applications in academic, business, nonprofit, and governmental research.

While the major focus is on applications, this book also integrates neoclassical economic theory with applied examples. The problems will almost entirely consist of areas within microeconomic theory, primarily theory of the firm, as well as applications of consumer theory, welfare economics, and environmental and resource economics. Hence, the book is a nice supplement to many courses in applied economics.

Because the overall goal of this book is to demonstrate how to use mathematical programming in real-world problem solving, the book provides many case studies from published research. Each chapter includes up to three case studies involving the use of mathematical programming in agricultural, environmental, and resource economics. The reader will be exposed to a thorough range of interesting applications, and teachers in agricultural and applied economics will find the inclusions of these case studies quite helpful in illustrating the power behind this quantitative method.

MATHEMATICAL PROGRAMMING

Mathematical programming is a branch of quantitative methods concerned with finding optimal ways to achieve a certain objective when faced with constraints on the ways the objective is achieved. For example, a farm enterprise is interested in choosing a mix of crops to grow and/or livestock to raise that will maximize profits while satisfying resource restrictions it faces on land, labor, machinery, animal numbers, and capital. An example from environmental economics is a manufacturing firm desiring to maximize profits while meeting constraints on carbon dioxide emissions. Mathematical programming models feature several common elements, including (1) an objective to be maximized or minimized; (2) activities or decision variables, which are the ways to carry out the objective; (3) objective function coefficients, which translate an overall numeric value to the objective through interaction with the values of the activities; and (4) a set of constraints that model the restrictions that the decision maker must operate within. Mathematical programming problems are modeled as a set of equations, linear or nonlinear, that define the decision-making environment.

Mathematical programming has its roots in the 1940s, when solution procedures for linear programming (LP) were developed. Linear programming was used extensively by the U.S. military during World War II, primarily to minimize various costs associated with the war effort. Techniques for solving LP problems were invented during this period by Leonid Kantorovich (LP problem), George Danzig (simplex method), and John Von Neumann (duality). After the War, mathematical programming techniques and applications were rapidly adopted in the private sector, academia, and government as a quantitative technique to handle a huge variety of problems. Today, it is one of the most widely used quantitative approaches in decision analysis.

THE USE OF THE RISK SOLVER PLATFORM FOR EDUCATION

All of the examples in this textbook have been developed as Microsoft Excel spreadsheets and can be solved using the Risk Solver Platform for Education (generally referred to in this textbook as Solver). In cooperation with Frontline Systems Inc., the developers of Solver, this program has been made available for free to students who purchase this book.

We decided to use Solver in this textbook because we have found that our students appreciated having a program that is user-friendly and works within the context of Excel spreadsheets. Therefore, students can take advantage of a variety of Excel functions in the development of their models and spreadsheet features, such as a variety of graphing options in the presentation of their results. Furthermore, the skills that students learn in Excel through the development of models for Solver can be transferred to other data analysis activities within Excel. An additional advantage of Solver is that it can be incorporated into customized programs within Excel through the use of Visual Basic for Applications (VBA). Students interested in this topic may want to consult *VBA for Modelers: Developing Decision Support Systems with Microsoft Office Excel* by S. Christian Albright.

We have observed that students find using Solver relatively easy as it builds upon skills they have already developed with Excel, and they do not have to learn a program-specific programming language. Students also tend to appreciate Solver's interactive visual menus. We recommend that instructors allow for some time at the beginning of the course for their students to get the Risk Solver for Education program installed on their personal computers—as well as having the program installed on classroom and laboratory computers, if applicable—before assigning exercises that require the program. This can be especially important if some students decide not to purchase the book until after attending a couple of classes.

Throughout the book, we have provided tips on how to use the tools of Excel to enhance model development and how to develop models that are easily interpreted by others. Instructors reviewing problems will appreciate the well-designed models that make the identification of problems straightforward. Readers interested in further discussions of related topics may want to consult books dedicated to this topic, such as *The Art of Modeling with Spreadsheets: Management Science, Spreadsheet Engineering, and Modeling Craft* by Stephen G. Powell and Kenneth R. Baker.

We note that some instructors may have more experience, and therefore comfort, with other mathematical programs, such as LINDO/LINGO, GAMS, AMPL, AIMMS, and MPL. Each one of these programs has its own strengths and weaknesses and we appreciate the challenges that come with learning new software. Certainly, users not previously accustomed to Solver will experience some initial challenges as they learn to navigate around its interface while setting the objective function, decision variables, constraints, Solver engine, and related parameters. To help instructors with this transition, we have developed supplemental materials that include the initial problem, a solved version of the problem, and related sensitivity analysis for every problem outlined in the book. These supplemental materials are provided as instructional aids to instructors and students and are available at www.wiley.com/college/kaiser.

One of the traditional advantages of stand-alone mathematical programs has been the ability to solve large-scale problems, especially when constraints are indexed over many different dimensions. The magnitude of this problem has decreased in recent years, and as computer power continues to grow, we encourage students and instructors to review the current editions of Frontline's Solver products (www.solver.com). Some of these Solvers can handle larger and more complex problems, not only in terms of the number of variables and constraints, but also in the incorporation of other important techniques from operations research and management science. While these advanced products come with an additional cost, Frontline traditionally has offered educational discounts.

ORGANIZATION OF THE BOOK

Since mathematical programming consists of linear and nonlinear programming, this book is divided into two major sections. Part 1 consists of six chapters involving LP and its applications. Part 2 features seven chapters involving nonlinear programming (NLP) models or linear models that relax the standard assumptions of LP.

In Part 1, the first three chapters provide a thorough overview of LP concepts, including the basic elements of the LP model, standard assumptions, tips on formulating an LP problem, sensitivity analysis, duality, and solving LP models with graphs, algebra, and Solver. While entire books have been written on these topics, Chapters 1, 2, and 3 provide enough detail and sufficient background on LP concepts to give the reader an ample foundation for applying this method to real-world problem solving. Instructors wishing to de-emphasize the theoretical concepts of LP may want to select sections from these chapters to cover in order to emphasize the remaining application chapters.

Chapters 4, 5, and 6 are concerned with applications of LP in agricultural, environmental, and resource economics. Chapter 4 examines the use of LP for farm-level decision making, and includes analyses of static and dynamic models for grain and livestock farmers, order preserving sequencing constraints, and multiperiod models. This chapter also features two research applications of farm models.

Chapter 5 examines the use of network and transportation LP models in the agricultural, food, and resources sector. The chapter also illustrates how to model product transformation problems. These models are extremely useful in developing efficient networks to minimize flows of commodities from a research application of a large transshipment model with product transformation is included in this chapter.

Chapter 6 is devoted to environmental and natural resource economic LP models. Popular models applied to problems in environmental and resource economics are presented in this chapter including application to forestry, land use planning, water conservation, and game management. In addition, a research case study looks at designing migratory corridors for grizzly bears.

Part 2 features seven chapters that cover applications of nonlinear and more advanced LP models. Chapter 7 covers integer and binary programming models. Integer programming (IP) is basically the same as LP, with the exception that some or all variables are restricted to be integers. In this chapter, the basic concepts underlying IP are presented. Specifically, the most efficient IP and general solution procedure to date, known as the branch-and-bound method, is examined. This is followed by several important applications of binary programming to the conservation of agricultural and ecologically valuable lands.

Chapter 8 provides an introduction into NLP problems that can be solved using calculus. This chapter looks at unconstrained and constrained optimization and shows how some nonlinear problems can be solved using Solver. This chapter is intended to give a conceptual foundation for NLP models. The chapter concludes by providing an application to fishery management and summarizing two research examples of NLP, one from agricultural economics and the other from environmental economics.

Chapter 9 continues this examination of nonlinear optimization and discusses a variety of techniques available in Solver that can be used for these problems. Methods include the SOCP Barrier Solver, Evolutionary Solver, and Interval Global Solver. Sensitivity analysis of nonlinear optimization is discussed in the context of a forest example and two research applications are presented. The first example is related to agricultural economics, and the second comes from environmental economics.

Chapter 10, which deals with risk programming models, relaxes the assumption of parameter certainty. Considerable evidence exists that suggests that farmers adjust their farm plans according to their risk posture, and that profit-maximizing models, which ignore risk preferences by farmers, have failed to give accurate normative or positive economic results when applied to many farming situations. Thus, in order to properly study most farm-level decision-making problems, one must formulate the decision environment in such a way that risk and uncertainty is a critical component in the model. This chapter presents several risk programming models that have been extensively used in food and agricultural applications, including quadratic risk programming (i.e., mean-variance analysis), minimization of absolute deviations (MOTAD), target-MOTAD, chance-constrained programming, and discrete sequential programming. The chapter concludes with three research applications of risk programming in agricultural and environmental economics. This chapter is one of the more advanced in terms of mathematical complexities, and it is geared more toward graduate students than undergraduate students. Therefore, instructors of undergraduate courses may want to selectively use sections in this chapter.

Chapter 11 focuses on price endogenous programming models, which relax the assumption that price is a constant parameter. When one moves from the individual-firm level to the market level, the assumption of constant price is no longer valid. At the market level, price is determined by the interaction of market supply (the collection of all individual firms' supply curves in the market) and market demand (the collection of all individual consumers' demand curves in the market). Consequently, if one is interested in modeling a market or sector rather than an individual firm, then a "price endogenous" or "sector programming" model is necessary. Price endogenous models are also necessary at the firm level if the firm has some degree of market power because in such cases, the firm can influence price by altering its output. Several popular models are presented to illustrate price endogenous programming, along with two research applications in agricultural and environmental economics.

Chapter 12 examines goal programming (GP) models, which is a technique that relaxes the sole objective assumption. Under this approach, one can specify multiple goals or targets for the decision maker and minimize the deviations from not achieving each goal. Goal programming has been used extensively in environmental, natural resource, and agricultural economics as a planning tool. There have been numerous applications in forestry management, land use planning, pollution mitigation, and farm planning. Numerous examples of GP are presented, along with two research applications: one relating to parasite control and one to forest conservation.

Chapter 13 examines the technique of dynamic programming (DP). Dynamic programming is a method used to solve large and complicated problems by splitting them up into smaller subproblems that are both easier to solve and yield the same optimal solution as the original large problem. Three examples of the DP solution procedure are presented. In addition, two research applications from agricultural economics are summarized.

The book is intended for both upper-level undergraduate as well as introductory graduate courses in mathematical programming. Not all chapters or parts of chapters are intended for undergraduate students, and each instructor should use discretion in choosing which material to cover. The book is fairly comprehensive in addressing all the important mathematical programming topics typically covered in introductory courses. Indeed, there is probably more material in this book than can be covered in a single semester course. We intended this to be the case as it offers greater flexibility to the instructor to cover the topics the teacher prefers.

Brief Contents

Contents

Part 1

LINEAR PROGRAMMING

1

Introductory Concepts and the Graphical Approach to Linear Programming

The focus of this book is on applications of optimization models for the fields of agricultural, environmental, and resource economics. Before such techniques can be applied to real-world problems, an elementary foundation needs to be established on basic concepts, definitions, and approaches of mathematical programming in order for you to fully understand the usefulness and limitations of mathematical programming. This is not a book on theory, so the basics established here are intended to supplement the applications that follow, which are the real focus of the book. Readers interested in obtaining a broader understanding of theoretical concepts of **linear programming (LP)** may wish to consult a book in LP theory. In this and the next two chapters, we concentrate on building this foundation for LP models, and in later chapters we develop a similar set of introductory concepts for nonlinear models.

There are several objectives of this chapter. The first objective is to define LP, explain how it is used, and outline the assumptions necessary to apply LP to agricultural, environmental, and resource economics problems. The second objective is to describe how to set up simple decision problems as LP problems, a task that is often more of an art than a science. The third objective of this chapter is to demonstrate how to solve two variable **maximization** and **minimization** LP problems using graph paper and a straight edge. The reader will also see how to use simple algebra to verify whether solutions obtained via the graphical method are indeed correct. Finally, the last objective is to discuss the notion of **sensitivity analysis**. Sensitivity analysis involves examining how sensitive the solution to a problem is with respect to the problem's parameters. The discussion in this chapter will focus on graphical techniques to accomplish various types of sensitivity analyses.

Linear programming is a category of mathematical programming models. One way to categorize mathematical programming is to divide it into two classes of models: linear and nonlinear programming. **Nonlinear programming (NLP)** is less restrictive than LP, in that, equations may have nonlinear, as well as linear forms. The majority of the chapters that follow will deal exclusively with LP models, since these models represent the majority of mathematical programming models.

Linear programming is a widely used problem-solving approach in quantitative methods. The **LP problem** is to determine the optimal value of a linear function (which defines the objectives of the problem) subject to a set of linear constraints (which defines the limits or decision environment of the problem). The term **optimal**, in this context, means minimizing or maximizing a given objective, for instance, maximizing profit, or minimizing costs. Linear programming models are used to help people and organizations make decisions. Decisions involve a process of formulating a set of alternatives to complete a goal, weighing each alternative based on some choice criterion, and selecting among these alternatives to accomplish this goal.

It should be emphasized that linear (and nonlinear) programming models are decision models or aids, not the means and ends for making the decision in question. Like all decision aids, the LP technique is there to assist people in their decision-making process. The management skills of the decision maker, which include qualitative as well as quantitative abilities, are the key attributes of the basis for one's decision. Nevertheless, quantitative techniques like LP have become powerful tools which are often used to improve managerial decision making.

1.1 APPLICATIONS OF LINEAR PROGRAMMING IN AGRICULTURAL, ENVIRONMENTAL, AND RESOURCE ECONOMICS

Linear programming has been used in a wide variety of applications of decision analysis. To provide a glimpse of such applications, which is by no means exhaustive, consider the following areas.

1. **The Diet Problem.** The problem is to determine the least-cost diet for a person, based on food prices, subject to the person receiving an adequate diet. The solution to this problem gives the combination of foods that a person should purchase to minimize food expenditures. Such applications are useful in developing countries, where food is scarce and starvation and malnutrition are major problems, as well as in food manufacturing and farming, where individuals are interested in minimizing the cost of producing food. This problem also applies to livestock producers wishing to minimize feed costs of livestock production.
2. **The Carbon Abatement Problem.** The problem is to determine the least-cost way to reduce carbon emissions by a firm in response to new legislation against global warming. The solution to this problem provides the combination of carbon-reducing activities for the firm to follow in a way that achieves the targeted reductions mandated by the law.
3. **The Product Mix Problem.** The problem is to determine the product mix (combination of outputs to be produced and sold), given limited resources, that maximizes profits, gross revenue, cash flow, net revenue, or utility for a firm. For example, a farmer needs to determine how to best allocate land among crops so that profits are maximized, given the level of control over all factors of production: for instance, the farmer owns and controls 600 acres of land, has two sons to supply family labor, owns one tractor, and so on. Small and large businesses often use LP to help determine product mixes.
4. **The Portfolio Problem.** The problem is to allocate a fixed amount of a resource (e.g., corn harvest) among alternative prospects so as to maximize the returns or minimize the risk from marketing the crop. For instance, corn could be sold at harvest, forward

marketed, stored and sold in future months, or hedged and sold on the futures market. The farmer's objective is to either maximize profit, minimize risk, or some combination of the two. Banks, investment institutions, private investors, universities, state and federal governments, and others also use LP to assist in their portfolio decision process.

5. **The Transportation Problem.** The problem is to determine how to move a product, such as oranges, produced on farms located in different geographic locations to different demand destinations in the most cost-efficient (least-expensive transportation costs) way. Linear programming applications for this class of problem are common.
6. **The Allocation Problem.** The problem is to determine how to allocate scarce resources among competing projects. For example, a conservation organization seeks to maximize the ecosystem services provided in an ecoregion, but has to select which conservation projects to fund. Given the different outcomes provided by the projects and the different objectives and priorities of the funding sources, **binary linear programming** can be used to determine which services should be used.
7. **Capital Budgeting Problem.** The problem is to invest capital, which is finite (scarce), to alternative projects. What is capital? Capital can mean money, or it can mean man-made resources, such as machinery. Business school types often define capital as some sort of money or financial measure, such as cash, stocks, bonds, savings, and so on. Economists generally define capital more broadly to include tools, equipments, factories, machinery, and all man-made items used to produce goods and services. Hence, the uses of capital budgeting may include monetary investments among alternative projects or the allocation of man-made aids to production to alternative projects.

All of these problems have four general properties that are inherent in any LP model. These properties are:

1. The **objective** is to be optimized by either maximization or minimization.
2. There are **constraints** restricting the activities that are required to carry out the objective.
3. All equations are **linear**.
4. The **activities** (or **decision variables**) are generally non-negative.

To illustrate these properties, consider the following example. Suppose that a grain farmer's objective is to maximize profit by producing two types of crops: wheat and sorghum. The farmer knows that the net profit of producing wheat is \$135 per acre, while the net profit of producing sorghum is \$100 per acre. The farmer's **objective function**, then, is to maximize profit from the production of the two crops, which can be expressed mathematically as:

$$\text{Max: } Z = 135\mathit{wheat} + 100\mathit{sorghum},$$

where *wheat* is the number of acres of wheat produced and *sorghum* is the number of acres of sorghum produced. The variables *wheat* and *sorghum* are called activities in LP language. If there were no constraints placed on these activities, the optimal solution to this problem would be to produce only wheat because it has a higher unit profit (135 versus 100). Furthermore, the optimal solution would be to produce an infinite amount of wheat because there are no restrictions currently placed on the problem's activities. In reality, the farmer would likely face many restrictions such as constraints on the availability of land, labor, machinery, and raw materials needed to produce crops. For example, suppose that the farmer has a labor force of 10 people and that each acre of wheat requires two people to produce, while each acre of sorghum requires one person to produce.

The following constraint can be added to this problem to reflect the scarcity of labor for this situation:

$$2wheat + 1sorghum \leq 10.$$

This constraint has the following interpretation: each acre of wheat produced requires two people, each acre of sorghum produced requires one person, and the total amount of labor used in raising both crops cannot exceed 10 people. Similar constraints could be added to this problem to reflect the scarcity of other resources such as land, capital, raw materials, and other resource endowments. These constraints are referred to as **structural constraints**. Note that both the objective function and the resource constraint to this problem are linear. Finally, in most applications it is appropriate to add a non-negativity constraint, which requires all activities to be non-negative. In this example, this implies that the farmer cannot produce negative quantities of either wheat or sorghum. The LP model for this example is:

Max: $Z = 135wheat + 100sorghum$ Objective function,

Subject to (s.t.):

$2wheat + 1sorghum \leq 10$ Labor constraint,

$wheat, \quad sorghum \geq 0$ Non-negativity.

1.2 COMPONENTS OF THE GENERAL FORM OF THE MODEL

There are several ways to express an LP model. The first of these is called the **general form** of the model, which was used in the example above. The general form of a generic LP model for n activities and m structural constraints is:

$$\text{Max or Min: } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \quad (0)$$

s.t.:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \{\leq, =, \geq\} b_1 \quad (1)$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \{\leq, =, \geq\} b_2 \quad (2)$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \{\leq, =, \geq\} b_m \quad (m)$$

$$x_1, \quad x_2, \ldots \quad x_n \quad \geq 0 \quad (m+1)$$

The first component of the model will always be the objective function, which is expressed in equation (0). The objective function is a mathematical formulation of the decision maker's objective. The objective is expressed as a function of the activities (x_i) that are under the control of the decision maker: that is, $Z = f(x_1, x_2, \ldots, x_n)$. The objective function value (Z) measures the alternative solutions to the problem, such as profit, costs, sales, production, and so on. The objective function will either be maximized or minimized depending upon the problem. The activities (also referred to as "decision variables" or just "variables") are the unknown endogenous (model-determined) variables of the problem. The model solution provides the decision maker with the optimal activities levels. The c_is in the objective function are called the **objective function coefficients**. These are fixed parameters (or coefficients), which give the contribution of each activity to the value of the objective function. For example, if the objective is to maximize profits from the sale of two