


volume 57

lecture notes in pure and applied mathematics



combinatorics,  
representation theory  
and statistical methods  
in groups

Edited by  
T. V. Narayana  
R. M. Mathsen  
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# YOUNG DAY Proceedings

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*Other Volumes in Preparation*

# Combinatorics, Representation Theory and Statistical Methods in Groups

A Hélène



## FOREWORD

Alfred Young (1873-1940) was a fellow of Clare College, Cambridge, a Canon of Chelmsford, and Rector of Birdbrook, Essex (1910-1940). Although a clergyman by profession, he was intensely interested in algebra and published over 25 papers, mostly on group theory, but including one on electro-magnetism. Among them is a series of nine papers entitled "On quantitative substitutional analysis." The first of this series, published in 1900, includes a discussion (*Collected Works*, p. 78) of the diagrams and tableaux that bear Young's name today, including the concept of the positive and negative symmetric groups. In the third paper (1927) he mentions the work of Frobenius and introduces group characters. The fourth paper refers to the group matrix and the work of Issai Schur. The fifth paper treats the representations of the hyperoctahedral group. The seminormal form for the matrices of irreducible representations of  $S_n$  is presented in the sixth paper. Later papers

emphasize the applications of substitutional analysis to the theory of invariants and covariants. Young was elected to the Royal Society in 1934.

Dr. Gilbert de B. Robinson, born June 3, 1906, earned his B.A. at Toronto in 1927, and his Ph.D. under the direction of Alfred Young in Cambridge, England in 1931. He returned to Toronto and rose from Lecturer (1931) to Professor (1954-present). More recently he has served as Vice President for Research at the University of Toronto. He served with the National Research Council (1941-1945) and was President of the Canadian Mathematical Congress (1953-1957).

J. S. Frame

## PREFACE

The inspiration provided by Alfred Young's work to Invariant Theory and Group Representation Theory has been aptly described by H. W. Turnbull in the *Journal of the London Mathematical Society*, (1941). In the four decades since Young's death, his influence has spread to other branches of mathematics, particularly combinatorics and statistics, as well as to chemistry, where the Young Lattice plays an important role. When the "Collected papers of Alfred Young, 1873-1940" was published in 1977-1978 by G. de B. Robinson (University of Toronto Press, Mathematical Expositions, No. 21), it was appropriate to celebrate the occasion with a YOUNG DAY conference. The many papers presented at this conference, both invited and contributed, some of which are included here, speak for themselves. These proceedings are quite naturally dedicated to the memory of Alfred Young; but they are also dedicated, if perhaps in a minor key, to the distinguished participants of YOUNG DAY who

made this occasion a notable success. We would be remiss at this stage not to mention the wonderful hospitality of the University of Waterloo, where YOUNG DAY was held, and the generosity and hard work of Professors R. Mullin and R. A. Staal, who contributed greatly to this success.

A brief tribute to Alfred Young and Gilbert de B. Robinson was presented by J. S. Frame at the reception that followed the conference and has been given, in part, in the foreword.

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## ALFRED YOUNG AS I KNEW HIM

G. deB. Robinson

I don't know how long A. R. Turnbull had known Alfred Young before January 19, 1926 when he attended his first lecture, but his description of the class and of Young in his biography are very interesting. It was a year later in the autumn of 1927 that I arrived in Cambridge. My background in Toronto had been fairly broad but my interest in geometry had been strong, largely because of Jacques Chapelon who came every winter from Paris to lecture to us in Toronto. He had been on the staff at Liège for some time as well as an examiner at l'Ecole Polytechnique, which connection he maintained. It was A. T. DeLury who brought him and he did much to stimulate my interest in research. On my arrival at St. John's College in September, 1927 I was assigned to M. H. A. Newman as supervisor, with F. P. White as my tutor. White was a geometer and Newman a topologist and I attended lectures by both of them as well as Baker's Tea Party and a course on the theory of functions by Littlewood. Much as I enjoyed