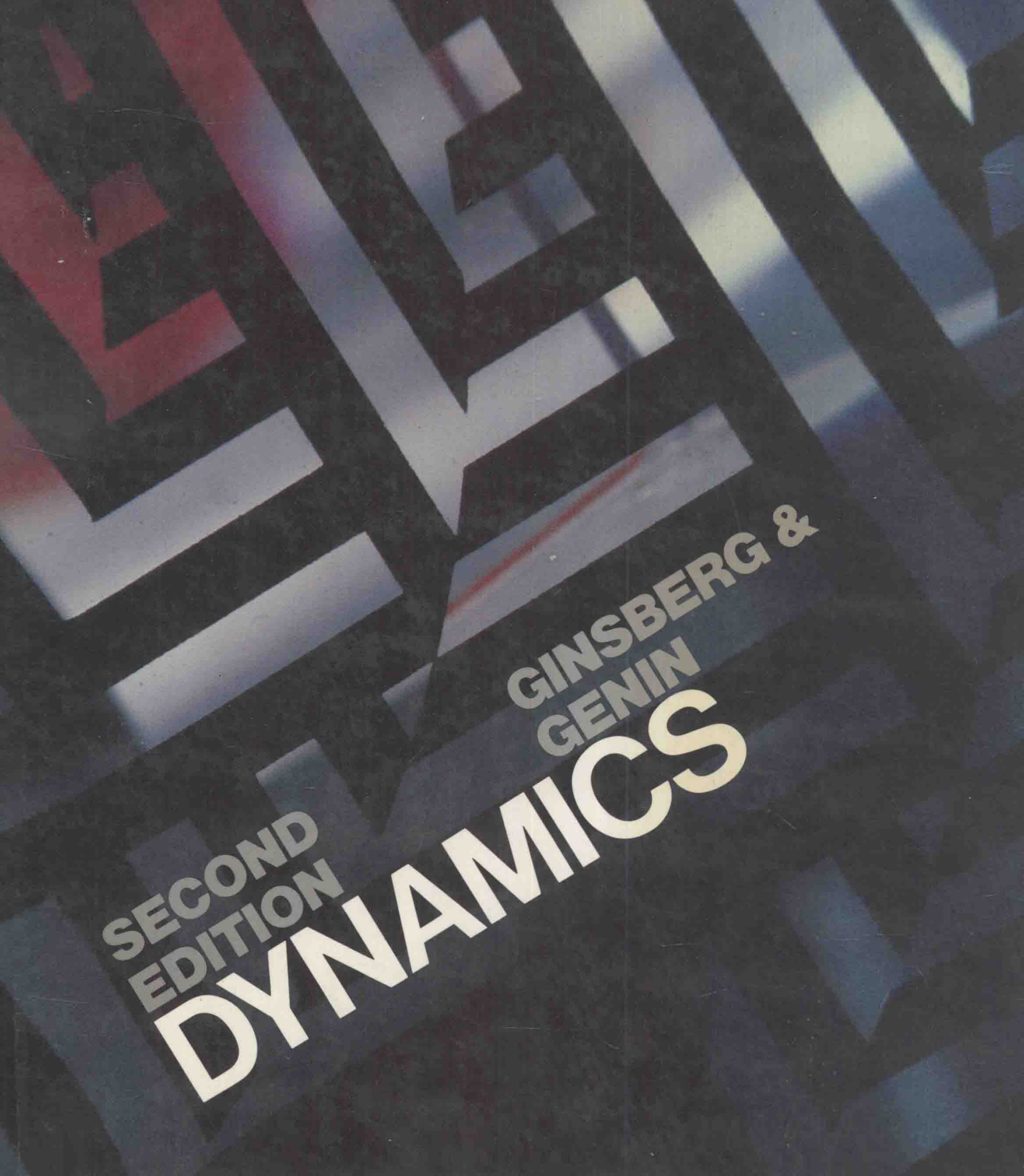
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DYNAMICS

PREFACE

The two fundamental goals of the basic engineering mechanics course in dynamics are similiar to the goals of the preceding statics course. The first is to develop an understanding of the physical laws governing the response of engineering systems to forces. The second is to enhance the reasoning powers required in engineering; that is, to develop the ability to solve problems logically, using the concept of mathematical models for physical systems. A successful experience in dynamics provides a strong foundation for a variety of subsequent courses.

To meet these goals, this book, like its companion volume on statics, is largely self-contained to minimize the amount of cross-referencing necessary to develop the material. We also chose this approach because we wish the book to communicate directly to the student, with a minimum of amplification and clarification required of the instructor.

To achieve this direct communication we developed a consistent approach to problem solving that could be applied to a broad class of problems. This approach recognizes two distinct aspects of the learning experience in mechanics. First, one must develop an understanding of the fundamental principles. Only then can these principles be applied selectively to the solution of a broad class of problems.

In this book the comprehension phase is addressed in a conventional manner. After each principle is derived, it is keynoted by remarks regarding common systems that illustrate its implications, as well as critical comments regarding its applicability. This is then followed by one or more solved examples. The examples directly illustrate how to employ the derived principle and, equally important, enhance the understanding of the physical meaning of the derived principles and the system responses that result from these principles. In addition to the discussion following the derivations, care is taken in the solved problems to discuss the qualitative aspects of the numerical value of the solution. Hopefully, this will also give the student better insight into how engineers think.

The application phase is addressed after a broad body of principles and techniques has been developed. This begins with the presentation of a set of sequential steps detailing the multiple operations necessary for the solution of general problems. These steps are merely a logical sequence to follow. (Certainly, they are not the only possible sequence.) With these steps, where appropriate, we indicate places where common errors are made.

By presenting a series of logical steps that systematize the approach to problem solving, we enhance the students' senses of logic and deductive reasoning. Thus, the steps are not intended for memorization. Rather, as students gain proficiency in problem solving, they will perform many of the steps intuitively.

The steps are then employed in a series of solved problems, called illustrative problems, that emphasize the synthesis of the material developed with the appropriate mathematical tools. These illustrative problems are cross-referenced directly to the steps for problem solving, to allow the student to isolate a particular aspect of the solution that may prove troublesome.

Homework problems are presented after both examples and illustrative problems; in general, the homework problems following the latter are broader in scope.

In the solution process, emphasis is placed on the logical application of the appropriate principle(s). Each solution is implemented with the aid of only that level of mathematics appropriate to solve the problem at hand.

In order to assist in the development of the methodology, and also to remove a past source of confusion for students, the number of alternative methods for treating the same type of problems was held to a minimum. Furthermore, where alternative principles in kinematics or kinetics are given, care is taken to explain their relative assets and, equally important, their relative liabilities.

Physical understanding of the subject matter is enhanced by the organization of topics. After the introductory material, the text is divided into three main groupings that discuss in turn the kinematics and kinetics of particles, rigid bodies in planar motion, and rigid bodies in three-dimensional motion. The kinetics study of each class of systems involves treatments of the equations of motion, followed by those of the principles of work and energy, and of linear and angular impulse and momentum. Because of its wide applicability, the primary tool in kinetics is the concept of equations of motion. The steps for problem solving in kinetics are generally presented at that juncture. The later material in the chapter is then developed consistently with the basic steps. In contrast, the problem-solving steps in kinematics usually appear near the end of the chapter.

It is our belief that the presentation of a systematic approach to the subject of dynamics is one of the major contributions of the book. In particular, the systematization of the methodology for treating the kinematics and kinetics of rigid bodies in three-dimensional motion is entirely new. Classroom experience with this material at Purdue University has shown that many students who otherwise would have been overwhelmed by the topic are able to comprehend and apply it. A direct result of their comprehension is that they are able to place particle and planar kinetics in proper perspective.

Indeed, we hope that the overall presentation in this book will enable the student to gain an appreciation of the orderly philosophy of dynamics.

The text concludes with a chapter that applies the principles of dynamics to the study of mechanical vibrations. Another in-depth application of basic principles is contained in the treatment of orbital mechanics at the end of Chapter 12. These topics, as well as a few others, have been marked by an asterisk to indicate that they may be omitted without loss of continuity. In addition to motivating students and providing greater technical exposure, these topics provide greater flexibility in course offerings. For example, universities offering courses on a quarterly basis could reserve treatment of three-dimensional dynamics systems, orbital motion, and vibrations for a second course on dynamics.

Two considerations entered into the sequencing of the homework problems in each section. We tried to group problems according to the similarity of their fundamental characteristics and the resulting similarity in the techniques for solving the problem. Within each group the problems are ordered from those involving a direct application of the basic principles, to those of a more subtle nature. We have marked with an asterisk the most challenging problems in order to alert the instructor. For this reason, homework problems in a section do not necessarily appear in order of ascending difficulty.

A final technical matter for consideration here is the question of physical units. We strongly believe that the SI system of metric units is the best for engineering. Nevertheless, it is clear that a total conversion to SI is not likely to occur in the United States in the near future because a large number of engineering systems (and the tools by which they are constructed and maintained) have already been designed and built according to the U.S. Customary system of units. We have, therefore, apportioned the numerical problems approximately equally between the two systems of units. This should provide a sufficient number of homework problems for courses employing either system solely.

We gratefully acknowledge the encouragement of our colleagues, whose helpful comments proved invaluable. In particular, we wish to thank our former colleague at Purdue University, Dr. Robert W. Fox, with whom we had many delightful discussions of teaching philosophies. We also thank him as well as Hsu-Chiang Miao, Mosaad A. Foda, Karan B. Sangha, John C. Gagliardi, Joseph F. Gaziano, Michael D. Greenberg, and Hugh J. Wellington at the Georgia Institute of Technology for their assistance in solving problems. We are indebted to Rona A. Ginsberg for her participation in all phases of the manuscript preparation, as well as for her excellent editorial comments.

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Jerry H. Ginsberg
Joseph Genin

CONTENTS

CHAPTER 10

FUNDAMENTAL CONCEPTS

OBJECTIVES

1. To review the concept of creating particle, rigid body, and deformable body models.

2. To understand the definitions and physical meaning of velocity and acceleration.

3. To review Newton's laws of particle motion.

4. To review the SI and U.S. Customary systems of units.

5. To review the basic operations of vector algebra in terms of components.

6. To develop the capability to transform the description of a vector from one set of coordinate axes to a related set.

7. To understand the basic theorems describing computations in vector calculus.

An investigation of the *dynamics* of a physical system, such as a machine, reveals the relationship between the forces acting on the system and the motion of that system. Many of us develop an intuitive understanding of this subject through our daily experiences. The study of dynamics will provide quantitative predictions of the relationship between forces and motion. It will also reveal situations where our intuition is wrong.

The methods we employ to describe forces were developed in a course on statics. The primary new feature here will be that the systems are moving.

A. BASIC DEFINITIONS

The investigation of the dynamic response of a physical system can generally be broken into two parts, one treating the subject of *kinematics*, and the other the subject of *kinetics*. A kinematical study is one that deals with methods for describing the motion of a system without regard to the role of the forces causing the motion. A kinetics study considers the relationship of the forces to the kinematical variables representing the motion of the system.

A key phrase used above is *physical system*. By it we mean a collection of basic atomic elements that combine to form bodies. In the field of mechanics, for the convenience of problem solving, we tend to categorize these collections of elements into three broad groupings: particles, rigid bodies, and deformable bodies.

PARTICLE A body whose dimensions are negligible is said to be a particle. It follows, then, that a particle occupies only a single point in space.

RIGID BODY A body occupying more than one point in space is said to be rigid if all of the constituent elements of matter within the body are always at fixed distances from each other.

DEFORMABLE BODY A body is said to be deformable if its constituent elements of matter experience changes in their distances from each other that are significant to the problem being investigated.

Clearly there is a certain amount of ambiguity in these definitions. For instance, is the group of molecules that compose a gas a system of particles or a deformable body? Another basic question we could ask is whether any body can correctly be considered to be rigid, because all real materials deform when forces are exerted on them.

There are no absolute answers to these questions, for the *model* of the system we form depends on what knowledge we wish to gain about the response of the system.

B. THE MODELING PROCESS

Our general approach in studying a physical system begins by constructing a conceptual model of the system. That is, we consider its components to be either particles, rigid bodies, or deformable bodies. We then employ our knowledge of kinematics to describe the motion of the chosen model. The response of the system is determined by relating the results of the kinematical study to the force concepts of kinetics. The general approach to the subject of dynamics can be summarized in the conceptual diagram given in Figure 1.

The dotted paths in Figure 1 illustrate a key feature of the modeling process. That is, to be sure of the validity of the model initially chosen to represent the system, it is necessary that the model display any relevant phenomena that are observed experimentally. If not, the model must be improved. In part, modeling is an educated trial-and-error procedure. That is, the modeling process is an art based on prior experience and physical intuition. On the other hand, the modeling process is also a science based on a knowledge of the purpose of the physical system and of the available analytical methods to study the phenomena exhibited by the system.

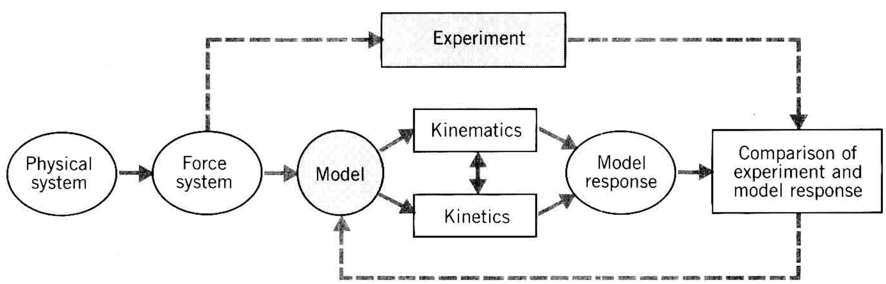

FIGURE 1

As an example, consider the case of the flight of a golf ball. We might tend to model the ball as a particle occupying a single point in space. As will be seen in our studies, there are some situations where such an approach results in a successful analysis; that is, the theoretical and experimental results are in good agreement. On the other hand, considering the golf ball to be a particle does not explain why the ball hooks, slices, and otherwise deviates from a straight course when the wind is calm. The latter phenomena stem from the aerodynamic forces produced by the spin of the golf ball. An accurate study of a golf ball's flight requires that it be modeled as a rigid body, as opposed to a simple particle. Carrying these thoughts one step further, if the problem was one of determining how a golf club imparts energy to a ball, we would quickly find that the correct model would be one where the golf ball (and club) must be modeled as deformable bodies. From the foregoing example we see that the model chosen to represent the physical system is a crucial element in the solution process.

This book does not treat the dynamics of deformable bodies. However, many of the principles and techniques that we develop here in studying particles and rigid bodies are also employed in the study of deformable bodies.

C. FOUNDATIONS OF NEWTONIAN MECHANICS

An investigation of dynamics requires a quantified description of motion. As you probably recall from your previous courses in elementary physics, the three fundamental kinematical quantities used to describe the motion of a point are position, velocity, and acceleration. These basic terms are defined with respect to a frame of reference that describes the three-dimensional space within which the motion occurs. This frame of reference can be visualized as a set of rectangular Cartesian axes xyz.

The laws of motion stated by Sir Isaac Newton (1642–1727) in his historic work *Principia* (1687) are the foundation of the study of dynamics. To employ these laws we must postulate that the frame of reference is *absolute*, which means that it is *fixed*, that is, stationary, in space. In most situations, the motion of the earth is negligible in comparison to the motion of the object of interest. The absolute reference frame in such cases may be considered to be attached to the earth.

Although considering the earth to be fixed is not exactly correct, the errors that result from this assumption are shown in Chapter 15 to be negligible for most systems. Indeed, the assumptions associated with the modeling process usually introduce errors having a greater magnitude.

The notion that anything, such as a reference frame, is fixed in space conflicts with the concepts of the theory of relativity, which tell us that nothing is stationary. Fortunately, it can be shown that the laws of the theory of relativity reduce to the simpler laws of Newtonian mechanics in the special case where bodies are moving much more slowly than the speed of light. This is certainly the case for most systems of engineering significance.

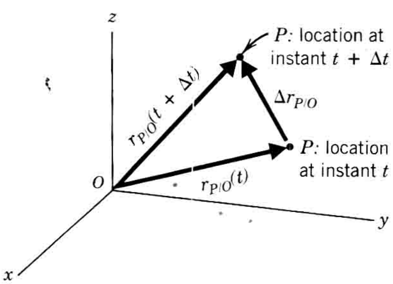

FIGURE 2

Let us now return to the basic kinematical quantities. The location of a point P within an arbitrarily chosen xyz reference frame cannot be specified until we know both the length and the orientation of the line from the origin O to point P. For example, the location of point P with respect to the origin O in Figure 2 is described by the *position vector* $\bar{r}_{P/O}$. (The notation should be read as "r of P with respect to O").

You will note that Figure 2 indicates that as point P moves in space its position vector changes, so that $\bar{r}_{P/O}$ is a function of time. The vector $\Delta\bar{r}_{P/O}$ represents the change in the vector $\bar{r}_{P/O}$ in the time interval from t to $t + \Delta t$. According to the rules for adding vectors diagrammatically, from Figure 2 we may write

$$\bar{r}_{P/O}(t) + \Delta\bar{r}_{P/O} = \bar{r}_{P/O}(t + \Delta t)$$

so that

$$\Delta\bar{r}_{P/O} = \bar{r}_{P/O}(t + \Delta t) - \bar{r}_{P/O}(t)$$

Division of $\Delta\bar{r}_{P/O}$ by Δt yields a quantity that resembles the time derivative of a scalar function

$$\frac{df}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta f}{\Delta t} = \lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$$

A similar definition is used for differentiation of a vector function, where the time rate of change of the position vector $\bar{r}_{P/O}$ is called the *velocity* $\bar{v}_P$. This is the second of the three basic kinematical quantities. Thus

$$\bar{v}_P \equiv \frac{d}{dt}\bar{r}_{P/O} \equiv \lim_{\Delta t \to 0} \frac{\Delta \bar{r}_{P/O}}{\Delta t} \equiv \lim_{\Delta t \to 0} \frac{\bar{r}_{P/O}(t + \Delta t) - \bar{r}_{P/O}(t)}{\Delta t} \tag{1}$$

The notation for velocity has been simplified slightly by not indicating that $\bar{v}_P$ is defined with respect to point O. This is permissible because the velocity with respect to all fixed points is the same, as will be proven in Chapter 11, Section E.

The velocity $\bar{v}_P$, being a vector, has magnitude and direction. The magnitude is defined as the *speed* v_P: $v_P = |\bar{v}_P|$. The speed expresses a rate of motion (e.g., 50 km/hr) and is a scalar quantity. When we are requested to determine how fast a point is moving, it is synonymous with being asked to find the speed. The direction of $\bar{v}_P$ tells us in which direction the point P is traveling. We shall have more to say about speed and velocity in the next chapter.

The third of the basic kinematical quantities, acceleration, is defined in a manner that is similar to eq. (1) for velocity. The *acceleration* $\bar{a}_P$ is the time rate of change of the velocity. This means

$$\bar{a}_P = \frac{d}{dt}\bar{v}_P = \lim_{\Delta t \to 0} \frac{\bar{v}_P(t + \Delta t) - \bar{v}_P(t)}{\Delta t} \tag{2}$$

The kinematical quantities of position and velocity are pieces of information that our eyes and brain are attuned to sense. Were it not for Newton's laws of motion, it would not be obvious that we should also be interested in acceleration. Because Newton's laws are the basis for our future studies we restate them here, in modern language.

FIRST LAW A particle will remain at rest or move with constant speed along a straight line, unless it is acted upon by a resultant force.

SECOND LAW When a resultant force is exerted on a particle, the acceleration of that particle is parallel to the direction of the force and the magnitude of the acceleration is proportional to the magnitude of the force.

THIRD LAW Each force exerted upon a body is the result of an interaction with another body, the forces of action and reaction being equal in magnitude, opposite in direction, and collinear.

In our studies we shall find that the first law is included in the second; it merely emphasizes that the state of motion of a particle can be changed only by the action of forces. The third law helps to define what forces are acting on a body.

The first and third laws are the only ones needed to solve problems in statics. The added feature in dynamics is the application of the second law. This law may be reworded to state that the force $\bar{F}$ acting on a particle is proportional to the acceleration $\bar{a}_P$ of the particle. As you probably know,