Elementary Kinematics of Mechanisms

By JOHN R. ZIMMERMAN

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JOHN WILEY & SONS, INC.
NEW YORK · LONDON

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Library of Congress Catalog Card Number: 62-8767

Printed in the United States of America

Preface

This textbook is designed for the student of limited mathematical background who is making his first study of the analysis and design of mechanisms. It is presupposed that the student brings with him competence in the elements of algebra and trigonometry and a knowledge of basic drafting technique. Although it would be well if he has had a course in rigid body mechanics, the book contains sufficient introductory material on vectors and the more elementary kinematic relationships to make its use quite possible for those lacking a background in mechanics. No knowledge of calculus or advanced mathematics is required.

Emphasis is on the fundamentals of motion analysis and mechanism design rather than on descriptive detail. This does not imply that I regard the descriptive detail on such important elements as gears, cams, and flexible connectors as unimportant; on the contrary, the student should have access to and be urged to consult the well-written and lavishly illustrated publications of our industrial firms. He must develop an acquaintance both with basic principles of analysis and design and with the "hardware."

Graphical techniques have been used very often, for they are usually the more economical of effort and time. However, no deliberate attempt has been made to avoid the use of simple algebraic methods when they seem to be justified. The modern designer can ill afford strong prejudices on this matter of the graphical "versus" the algebraic approach. Whichever method achieves a sufficiently accurate solution with the least labor and mental torment is the one to use. Indeed, often it is a blend of the graphical and the algebraic approaches that is most fruitful.

Although an ample number of problems has been given in the text, one of the many excellent problem sets for kinematics now available can be used to advantage both for homework and for practicum assignments.

I shall greatly appreciate hearing from the users of this book concerning both errors and suggestions for improvements.

My debts are greatest to a number of men known to me only through their writings. To Rudolf Beyer and Kurt Hain of Germany, to N. Rosenauer and A. H. Willis of Australia, and to the Americans, Joseph S. Beggs, Ferdinand Freudenstein, A. S. Hall, Jr., A. E. Richard de Jonge, Harold A. Rothbart, and Joseph E. Shigley, grateful acknowledgment is made.

And, finally, to my wife Grace, and to my children Jan and Scott, an affectionate acknowledgment is made for putting up with it all so cheerfully.

JOHN R. ZIMMERMAN

University Park, Pennsylvania
December, 1961

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Symbols and Abbreviations

a or **A** acceleration

a or *A* usually magnitude of acceleration; for rectilinear motion it designates acceleration

$\mathbf{a}^n$ or $\mathbf{A}^n$ normal component of acceleration

$\mathbf{a}^t$ or $\mathbf{A}^t$ tangential component of acceleration

$\mathbf{a}^c$ or $\mathbf{A}^c$ Coriolis component of acceleration

a_{12} midnormal of line segment A_1A_2; b_{12} is midnormal of segment B_1B_2, etc.

a addendum

b dedendum

B backlash

C center distance

d diameter

D diameter; pitch diameter

f proper fraction

F_i input force

F_o output force

h_t whole depth of gear tooth

h_w working depth of gear tooth

k_a acceleration scale

k_s displacement scale

k_t time scale

k_v velocity scale

L connecting rod length

L belt length

m_g gear ratio

m_p contact ratio

n number of threads on worm

n number of links

N number of instant centers

N number of teeth

N angular velocity in revolutions per minute

O_{12}, O_{57}, etc. instant centers

p circular pitch

p_b base pitch

p_n normal circular pitch

P diametral pitch

P_n normal diametral pitch

P_{12}, P_{23}, etc. virtual poles

P_i input power

P_o output power

r radius

R radius; pitch radius of gear

R_b base radius of gear

R_{12}, R_{23}, etc. relative poles

Q length of arc of action

s linear displacement

s distance; magnitude of linear displacement; for rectilinear motion it designates displacement

s percent slip

t time

t circular tooth thickness

t belt thickness

T period

T_i input torque

T_o output torque

v or **V** velocity

v or *V* usually speed; for rectilinear motion it indicates velocity

Z length of path of action

α (alpha) pitch angle of bevel pinion

α (alpha) angular acceleration (positive if counterclockwise)

α (alpha)	pressure angle in cam mechanism
β (beta)	pitch angle in bevel gear
Δ (delta)	when preceding a quantity indicates a change in that quantity
θ (theta)	angular position; angular displacement
ϕ (phi)	angular position; angular displacement
ϕ (phi)	pressure angle of gear
ϕ_N	normal pressure angle of a gear
ψ (psi)	angular position; angular displacement
ψ (psi)	helix angle
ω (omega)	angular velocity (positive if counterclockwise)
ω_i	angular velocity of input link
ω_o	angular velocity of output link

BDC	bottom dead center
CA	constant acceleration
CR	connecting rod
ccw	counterclockwise
cw	clockwise
D-R-R-D	dwell, rise, return, dwell
D-R-D-F-D	dwell, rise, dwell, fall, dwell
log	logarithm to the base 10
LH	left hand
MCV	modified constant velocity
RH	right hand
MA	mechanical advantage
mph	miles per hour
R-R-R	rise, return, rise
rev	revolution
rpm	revolutions per minute
rps	revolutions per second
SHM	simple harmonic motion
TDC	top dead center

Contents

I Kinematic Terminology

1-1. Introductory

The art of mechanical design is directed toward the creation of new or improved machines and instruments to modify and control energy and motion. The designer accepts the task of planning and designing equipment that will not only perform the functions expected of it but will also do so with the greatest possible over-all economy. The designer's responsibility does not cease with the conception of a machine which will perform the functions expected of it; he must consider as many aspects of the machine's manufacture and use as possible, problems of ease of manufacture, appearance, safety, and so on.

Design is, therefore, a complicated enterprise; it necessitates a considerable exercise of judgment and a certain amount of trial and error. Design can never be reduced to a mechanical routine. Yet there is a pattern to it. Figure 1-1 illustrates this pattern. Although the diagram shows the logical sequence of steps in design, it must not be supposed that they always proceed in precisely this order or that they proceed independently of one another. Changes in one aspect of the design usually entail changes in others.

Even when the machine has gone into final production, the designer is not yet quite through, for there is much to be learned by keeping in touch with those responsible for its manufacture and with the machine's performance in actual service. This information can be of incalculable benefit in future work. One of the hallmarks of intelligence is the ability to learn from experience, particularly from one's mistakes.

1-2. Kinematics of Mechanisms

Kinematics of mechanisms is that discipline which deals with the motions of mechanisms, without regard to the forces which affect these motions. Two aspects of the subject can be distinguished. An investigation of the motion characteristics of a mechanism of existing design is *kinematic*

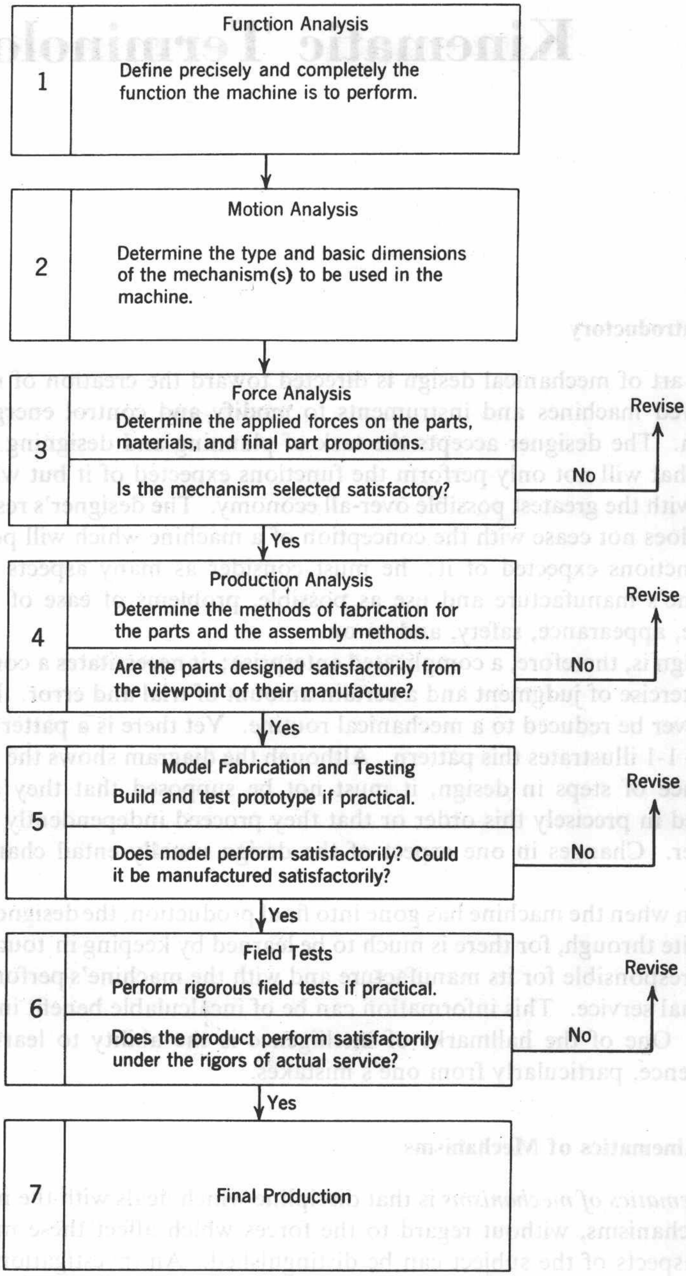

Fig. 1-1. Mechanical design procedure.

analysis. On the other hand, the design of a mechanism to provide certain desired motions is *kinematic synthesis*.

1-3. Mechanisms and Machines

A *mechanism* is an assembly of bodies, usually rigid, such that the motion of one of the bodies causes definite and predictable motions of the other bodies in the assembly. A *machine* is a mechanism or combination of mechanisms capable of transmitting and modifying mechanical energy. The primary purpose of the mechanism is to transmit and modify motion, while the machine has the additional function of transmitting and modifying mechanical energy. The internal combustion engine, for example, converts the work energy given to the pistons by the burning gases into work done by the rotating shaft driving an external load. A watch mechanism, on the other hand, is not designed to transmit energy. The proper movement of its hands is its sole function. The energy stored in the watch spring is used only to overcome the frictional resistances encountered by the members of the mechanism.

1-4. Links and Pairs

A *link* is a member of a mechanism connecting other links and having motion relative to them. Links may be supports, may guide the motion of other links, transmit motion, or serve simultaneously in more than one of these capacities. The simplest mechanisms have three links: driver, driven, and frame.

Figure 1-2 illustrates a simple three-link cam mechanism. Link 1 is the frame and supports links 2 and 3. Link 2 is the driver and imparts motion to the driven link 3.

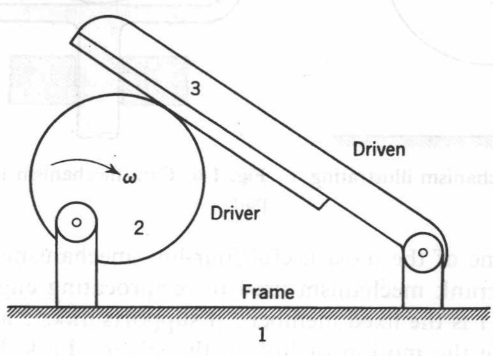

Fig. 1-2. Three-link cam mechanism.

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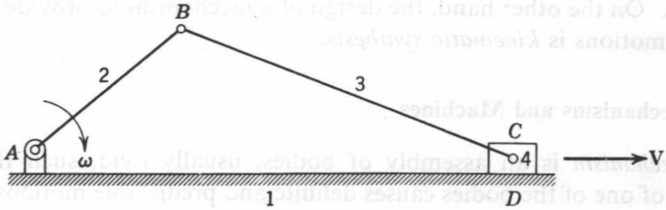

Fig. 1-3. Slider-crank mechanism.

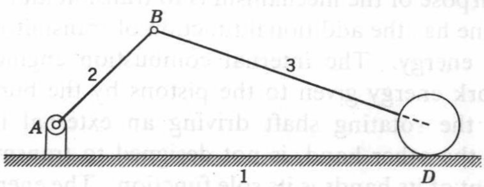

Fig. 1-4. Slider crank with ball slider.

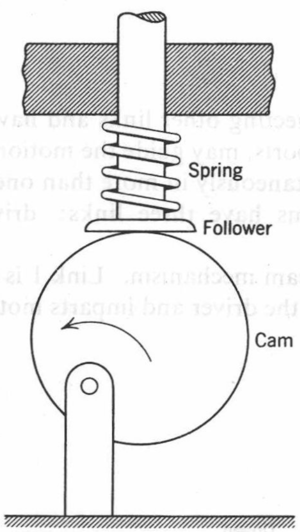

Fig. 1-5. Cam mechanism illustrating unclosed pair.

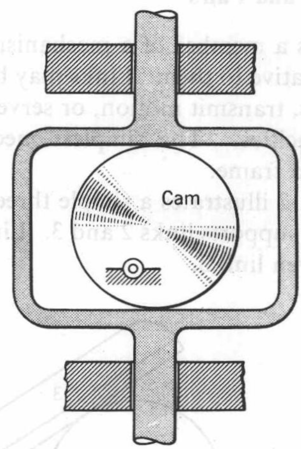

Fig. 1-6. Cam mechanism illustrating closed pair.

In Fig. 1-3 one of the most useful four-link mechanisms is shown, the familiar slider-crank mechanism used in reciprocating engines and compressors. Link 1 is the fixed member; it supports links 2 and 4 as well as serving to guide the motion of link 4, the slider. Link 2 is, if this is a compressor, the driver; it transmits motion to the connecting rod, link 3, which in turn transmits motion to the driven member, the slider, link 4.

A *pair* is a joint that permits relative motion. A pair is made up of two elements, one on each link being joined. In Fig. 1-3, for example, there is a turning pair at *A*: a joint permitting relative rotation of the crank and connecting rod. At *D*, however, there is a sliding pair: a joint permitting relative sliding between the slider and the guide. In a roller bearing there is rolling contact between the roller and each of the two races.

Pairs are classified as higher or lower. A *higher pair* is one in which there is point or line contact between the links being joined. A wheel rolling on a surface, for example, is an instance of a higher pair. A *lower pair* is one in which there is surface contact between the members being joined. A shaft rotating in a bearing, or a nut turning on a screw, is an example of lower pairing.

Frequently higher and lower pairs can be interchanged without affecting the relative motions of the links. In Fig. 1-4 the slider-crank mechanism has been redesigned by replacing the lower pairs at *D* by a higher pair. A spherical surface on the connecting rod has been substituted for the piston (thus eliminating one link), with the result that point contact between the spherical surface and the guide replaces the surface contact between the slider and guide. The design in Fig. 1-4 will be inferior from the standpoint of wear to that of Fig. 1-3, but from the viewpoint of kinematic behavior, there is no difference.

Pairs are also classified as closed and unclosed. A *closed pair* is one in which the links are held together mechanically, where only the destruction of one or more members could make possible a breaking of contact. An *unclosed pair* is one that relies upon some external means, such as a spring or gravity, to maintain contact. Figures 1-5 and 1-6 illustrate the difference. In Fig. 1-5 the follower is maintained in contact with the cam by the spring, while in Fig. 1-6 the follower is designed in such a way that contact is assured.

1-5. Kinematic Chains and Inversion

When a group of links are joined together so that, if one of the links were fixed, the others would move in definite and predictable ways, the assembly is called a *kinematic chain*. A mechanism is made from a kinematic chain by fixing one of the links; therefore there are as many possible mechanisms as there are links. The various mechanisms that can be made from a kinematic chain are its *inversions*.

This can be illustrated by noting the possible inversions of the sliding-block chain shown in Fig. 1-7. Table 1-1 lists each of the four possible inversions of this chain with common applications.

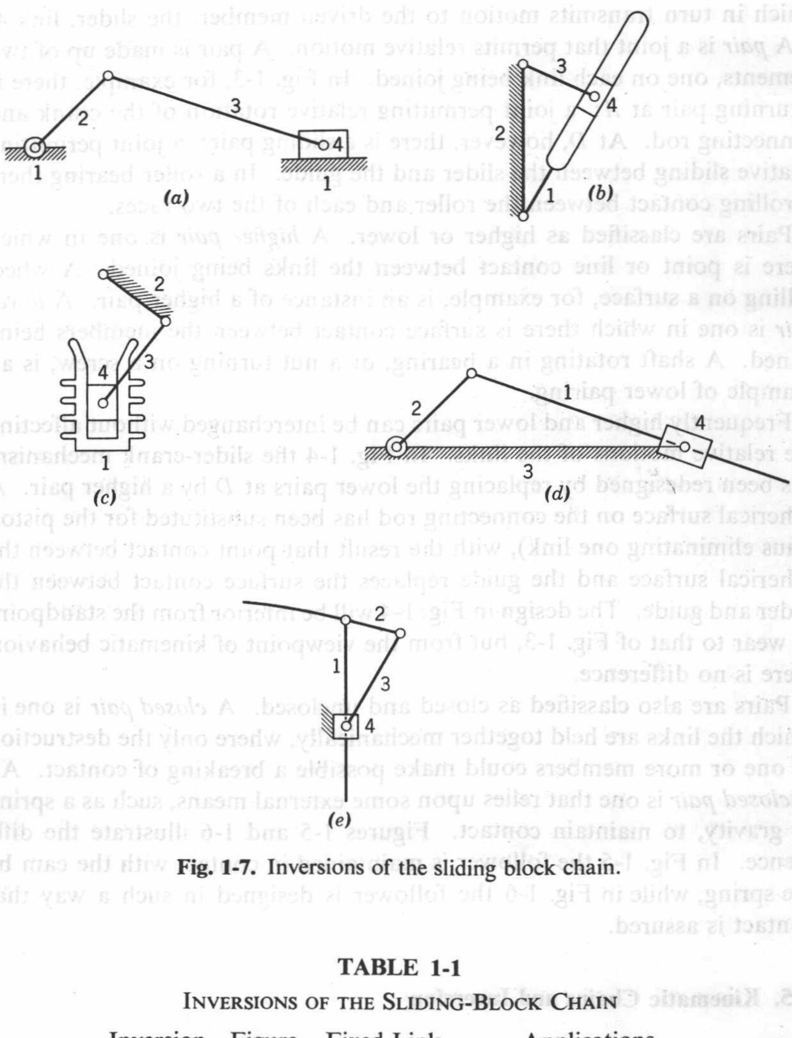

Fig. 1-7. Inversions of the sliding block chain.

TABLE 1-1

INVERSIONS OF THE SLIDING-BLOCK CHAIN

Inversion	Figure	Fixed Link	Applications
1st	1-7*a*	1	Reciprocating engines and compressors
2nd	1-7*b*	2	Quick-return
	1-7*c*	2	Rotary engine
3rd	1-7*d*	3	Small oscillating steam engine
4th	1-7*e*	4	Hand pump (not a widely used inversion)

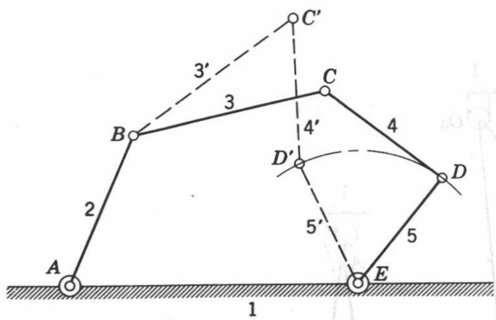

Fig. 1-8. Unconstrained chain.

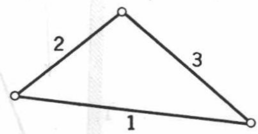

Fig. 1-9. Structure.

The definition of kinematic chain included the qualification that, with one link fixed, the others are obliged to move in definite ways. Unless this condition is adhered to, the assembly will fail to be a kinematic chain. An example of this is shown in Fig. 1-8. Here is a five-link assembly which fails to qualify. To test it, one of the links (link 1) has been fixed. For a single position of link 2, there are an indefinite number of positions in which links 3, 4, and 5 might be arranged; two of which have been shown. This chain is *unconstrained.* To give it constrained motion, a second link, such as link 5, could be used as a second input link.

The assembly of Fig. 1-9 fails also, but for another reason. There can be

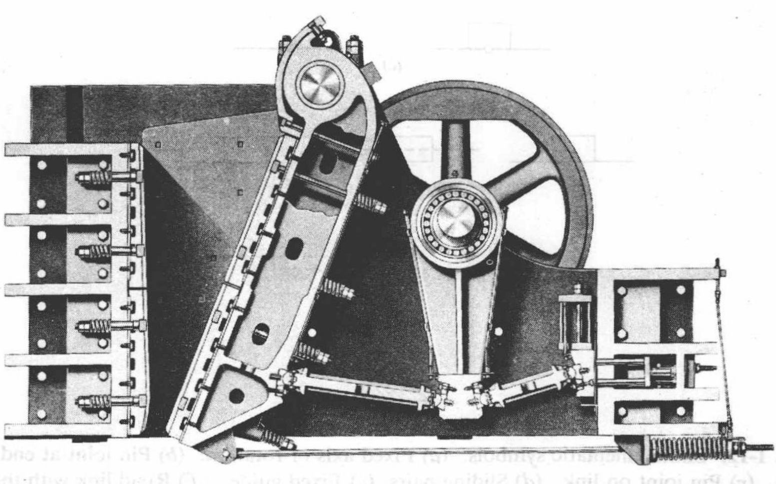

Fig. 1-10. Jaw crusher mechanism. (Courtesy Allis-Chalmers Manufacturing Co., Milwaukee, Wisc.)

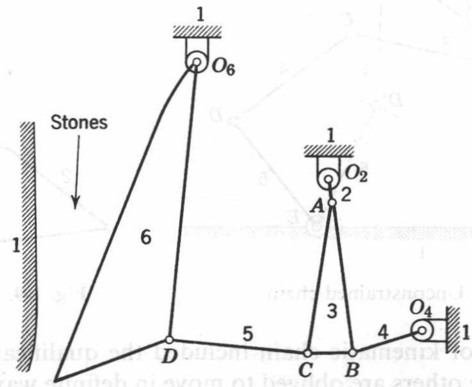

Fig. 1-11. Kinematic representation of jaw crusher mechanism.

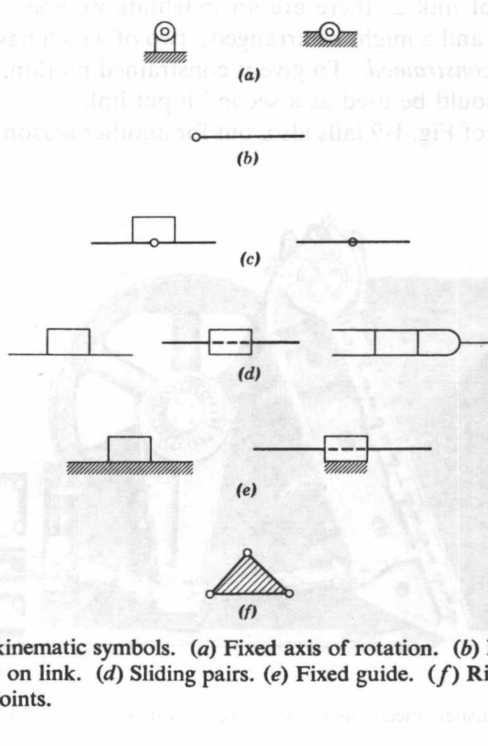

Fig. 1-12. Some kinematic symbols. (*a*) Fixed axis of rotation. (*b*) Pin joint at end of link. (*c*) Pin joint on link. (*d*) Sliding pairs. (*e*) Fixed guide. (*f*) Rigid link with three noncollinear pin joints.

no relative motion of the members. It is a structure; and, as a structure, it could serve as a single link in a kinematic chain, but not as a kinematic chain.

1-6. Schematic Diagrams

To take the trouble to show the members of a mechanism in their true shapes would be a laborious and unnecessary chore. Only a few of the dimensions of the members of a mechanism are of any relevance to a kinematic analysis. For example, in Fig. 1-10 a jaw crusher mechanism is shown in great detail. The kinematic representation of this mechanism in Fig. 1-11 is a sharp contrast in its stark simplicity. An enormous amount of detail has been omitted in making this kinematic representation, yet it shows all the dimensions essential to a motion analysis of the mechanism.

A number of the most commonly used kinematic symbols are shown in Fig. 1-12.

REFERENCE

1. R. S. Hartenberg and J. Denavit, "Men and Machines," *Machine Design*, May 3, June 14, and July 12, 1956.

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