

RELIABILITY OF STRUCTURES

SECOND EDITION

Andrzej S. Nowak

Kevin R. Collins



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Preface

The objective of this book is to provide the reader with a practical tool for reliability analysis of structures. The presented material is intended to serve as a textbook for a one-semester course for undergraduate seniors or graduate students. The material is presented assuming that the reader has some background in structural engineering and structural mechanics. Previous exposure to probability and statistics is helpful but not required; the most important aspects of probability and statistics are reviewed early in the text.

Many of the available books on reliability are written for researchers, and these texts often approach the subject from a very mathematical and theoretical perspective. The focus of this book is on practical applications of structural reliability theory. The book does not provide detailed mathematical proofs of the underlying theory; instead, the book presents the basic concepts, interpretations, and equations and then explains to the reader how to use them. The book should be useful for both students and practicing structural engineers and hopefully will broaden their perspective by considering reliability as another important dimension of structural design. In particular, the presented methodology is applicable in the development of design codes, development of more reliable designs, optimization, and rational evaluation of existing structures.

The text is divided into 10 chapters with regard to topics.

Chapter 1 provides an introduction to structural reliability analysis. The discussion deals with the objectives of the study of reliability of structures and the sources of uncertainty inherent in structural design.

Chapter 2 provides a brief review of the theory of probability and statistics. The emphasis is placed on the definitions and formulas that are needed for derivation of the reliability analysis procedures. The material covers the definition of a random variable and its parameters such as the mean, median, standard deviation, coefficient of variation, cumulative distribution function, probability density function, and probability mass function. The probability distributions commonly used in structural reliability applications are reviewed; these include the normal, lognormal, extreme Type I,

II, and III; uniform; Poisson; and gamma distributions. A brief discussion of Bayesian methods is also included.

In Chapter 3, functions of random variables are considered. Concepts and parameters such as covariance, coefficient of correlation, and covariance matrix are described. Formulas are derived for parameters of a function of random variables. Special cases considered in this chapter are the sum of uncorrelated normal random variables and the product of uncorrelated lognormal random variables.

Chapter 4 presents some simulation techniques that can be used to solve structural reliability problems. The Monte Carlo simulation technique is the focus of this chapter. Two other methods are also discussed: the Latin Hypercube sampling method and Rosenbluth's point estimate method.

The concepts of limit states and limit state functions are defined in Chapter 5. Reliability and probability of failure are considered as functions of load and resistance. The fundamental structural reliability problem is formulated. The reliability analysis methods are also presented in Chapter 5. The simple second-moment mean value formulas are derived. Then, the Hasofer-Lind reliability index is defined. An iterative procedure is shown for variables with full distributions available.

The development of a reliability-based design code is discussed in Chapter 6. The presented material includes the basic steps for finding load and resistance factors and a calibration procedure used in several recent research projects.

Load models are presented in Chapter 7. The considered load components include dead load, live load for buildings and bridges, and environmental loads (such as wind, snow, and earthquake). Some techniques for combining loads together in reliability analyses are also presented.

Resistance models are discussed in Chapter 8. Statistical parameters are presented for steel beams, columns, tension members, and connections. Noncomposite and composite sections are considered. For reinforced concrete members and prestressed concrete members, the parameters are given for flexural capacity and shear. The results are based on the available test data and simulations.

Chapter 9 deals with the important topic of system reliability. Useful formulas are presented for a series system, a parallel system, and mixed systems. The effect of correlation between structural components on the reliability of a system is evaluated. The approach to system reliability analysis is demonstrated using simple practical examples.

Models of human error in structural design and construction are reviewed in Chapter 10. The classification of errors is presented with regard to mechanism of occurrence, cause, and consequences. Error survey results are discussed. A strategy to deal with errors is considered. Special focus is placed on the sensitivity analysis. Sensitivity functions are presented for typical structural components.

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Introduction

I.1 OVERVIEW

Many sources of uncertainty are inherent in structural design. Despite what we often think, the parameters of the loading and the load-carrying capacities of structural members are not *deterministic* quantities (i.e., quantities that are perfectly known). They are *random variables*, and thus absolute safety (or zero probability of failure) cannot be achieved. Consequently, structures must be designed to serve their function with a finite probability of failure.

To illustrate the distinction between deterministic versus random quantities, consider the loads imposed on a bridge by car and truck traffic. The load on the bridge at any time depends on many factors such as the number of vehicles on the bridge and the weights of the vehicles. As we all know from daily experience, cars and trucks come in many shapes and sizes. Furthermore, the number of vehicles that pass over a bridge fluctuates, depending on the time of day. Since we do not know the specific details about each vehicle that passes over the bridge or the number of vehicles on the bridge at any time, there is some uncertainty about the total load on the bridge. Hence, the load is a random variable.

Society expects buildings and bridges to be designed with a reasonable safety level. In practice, these expectations are achieved by following code requirements specifying design values for minimum strength, maximum allowable deflection, and so on. Code requirements have evolved to include design criteria that take into account some of the sources of uncertainty in design. Such criteria are often referred to as *reliability-based design criteria*. The objective of this book is to provide the background needed to understand how these criteria were developed and to provide a basic tool for structural engineers interested in applying this approach to other situations.

The reliability of a structure is its ability to fulfill its design purpose for some specified design lifetime. Reliability is often understood to equal the probability that a structure will not fail to perform its intended function.

The term “failure” does not necessarily mean catastrophic failure but is used to indicate that the structure does not perform as desired.

1.2 OBJECTIVES OF THE BOOK

This book attempts to answer the following questions:

How can we measure the safety of structures? Safety can be measured in terms of reliability or the probability of uninterrupted operation. The complement to reliability is the probability of failure. As we discuss in later chapters, it is often convenient to measure safety in terms of a reliability index instead of probability.

How safe is safe enough? As mentioned earlier, it is impossible to have an absolutely safe structure. Every structure has a certain nonzero probability of failure. Conceptually, we can design the structure to reduce the probability of failure, but increasing the safety (or reducing the probability of failure) beyond a certain optimum level is not always economical. This optimum safety level has to be determined.

How does a designer implement the optimum safety level? Once the optimum safety level is determined, appropriate design provisions must be established so that structures will be designed accordingly. Implementation of the target reliability can be accomplished through the development of probability-based design codes.

1.3 POSSIBLE APPLICATIONS

Structural reliability concepts can be applied to the design of new structures and the evaluation of existing ones. Many modern design codes are based on probabilistic models of loads and resistances. Examples include the American Institute of Steel Construction (AISC, 2011)¹ Load and Resistance Factor Design (LRFD) code for steel buildings (AISC, 2006), American Association of State Highway and Transportation Officials LRFD code (AASHTO, 2012), Canadian Highway Bridge Design Code (2006), and the European codes (EN EUROCODES, n.d.). In general, reliability-based design codes are efficient because they make it easier to achieve either of the following goals:

- For a given cost, design a more reliable structure.
- For a given reliability, design a more economical structure.

¹ Many acronyms are used in structural engineering and structural reliability. Appendix A lists the acronyms used in this book.

The reliability of a structure can be considered as a rational evaluation criterion. It provides a good basis for decisions about repair, rehabilitation, or replacement. A structure can be condemned when the nominal value of load exceeds the nominal load-carrying capacity. However, in most cases, a structure is a system of components, and failure of one component does not necessarily mean failure of the structural system. When a component reaches its ultimate capacity, it may continue to resist the load while loads are redistributed to other components. System reliability provides a methodology to establish the relationship between the reliability of an element and the reliability of the system.

1.4 HISTORICAL PERSPECTIVE

Many of the current approaches to achieving structural safety evolved over many centuries. Even ancient societies attempted to protect the interests of their citizens through regulations. The minimum safety requirements were enforced by specifying severe penalties for builders of structures that did not perform adequately. The earliest known building code was used in Mesopotamia. It was issued by Hammurabi, the king of Babylonia, who died about 1750 BC. The “code provisions” were carved in stone, and these stone carvings are preserved in the Louvre Museum in Paris, France. The responsibilities were defined depending on the consequences of failure. If a building collapsed killing a son of the owner, then the builder’s son would be put to death. If the owner’s slave was killed, then the builder’s slave was executed, and so on.

For centuries, the knowledge of design and construction was passed from one generation of builders to the next one. A master builder often tried to copy a successful structure. Heavy stone arches often had a considerable safety reserve. Attempts to increase the height or span were based on intuition. The procedure was essentially trial and error. If a failure occurred, that particular design was abandoned or modified.

As time passed, the laws of nature became better understood. Mathematical theories of material and structural behavior evolved, providing a more rational basis for structural design. In turn, these theories provided the necessary framework in which probabilistic methods could be applied to quantify structural safety and reliability. The first mathematical formulation of the structural safety problem can be attributed to Mayer (1926), Streletsii (1947), and Wierzbicki (1936). They recognized that load and resistance parameters are random variables, and therefore, for each structure, there is a finite probability of failure. Their concepts were further developed by Freudenthal in the 1950s (e.g., Freudenthal, 1956). The formulations involved convolution functions that were too difficult to evaluate by hand. The practical applications of reliability analysis were not possible until the