PLASMA PHYSICS

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BY

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PREFACE

In recent years the number of physicists working on problems in plasma physics has been growing almost exponentially. The same tendency is shown in the number of scientific publications on that subject. In contrast, the number of books on plasma physics is limited to only a few; this is, of course, to be expected in a rapidly expanding subject. However, not all the books are written by historians and it is felt that even a book which represents an attempt at an up-to-date summary may be useful, especially for postgraduate students interested in plasma physics and for older physicists who have entered this field in recent years. Writing the book has been an exciting task and I only hope that a small fraction of this feeling can be transmitted to the reader.

I would like to acknowledge the encouragement and help I have received from many of my colleagues at CERN. I am also thankful for discussions and helpful criticism to Dr. D. Finkelstein, Mr. E. R. Harrison, Dr. J. D. Lawson, Dr. L. Ornstein, Mr. C. Maissonier, Dr. A. Robson, Dr. A. Schlütter and Dr. P. Sturrock.

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ERRATA

P. 30 The title of section 1.3.2 should read: Electric vortex field and magnetic lens field.

P. 33 Line 4 from the bottom of the page: Thus, if $\dot{z}_0 = 0$, there ....

P. 84 Legend of Fig. 41: Motion of electrons for $\nu < 1$ (S) and for $\nu > 1$ (T).

P. 103 Line 1: ... when the term $4\pi \frac{e}{c} nv$ becomes equal to ...

eq. (6); r.h.s. should read: $4\pi \frac{e}{c} \frac{\partial}{\partial t} (n_e \boldsymbol{v})$

P. 104 Eq. (15): $\boldsymbol{i} = - e n_0 \boldsymbol{v}$

P. 105 Eq. (10a): r.h.s. should read: $-\frac{4\pi}{c^2} \mathrm{j}\omega \boldsymbol{i}$

Eq. (11a): l.h.s. should read: $-\operatorname{div} \boldsymbol{E}$

Eq. (12a): $\mathrm{j}\omega \boldsymbol{i} = \frac{e^2 n_0}{m} \boldsymbol{E} - \frac{e}{mc} \boldsymbol{i} \wedge \boldsymbol{B}_0$

P. 110 Fig. 56: $\alpha = 0$ pertains also to the vertical dashed line

Legend of Fig. 56 and subsequent lines of the main text: $\alpha = \omega_c/\omega_p$.

P. 130 Eq. (89a): $\frac{\mathrm{d}w}{\mathrm{d}t} = -\frac{m}{M} \frac{vv^2}{r} + ac^2 r^{-(4/\alpha+1)}$

P. 136 Eqs. (99b), (100) and (102): The term $\left(\frac{m_0}{M}\right)^{\frac{1}{4}}$ should be replaced everywhere by $3\left(\frac{m_0}{M}\right)^{\frac{1}{4}}$.

P. 143 Eq. (121): $F_m = k^2 x \frac{I^2}{c^2} \left(\ln \frac{\lambda}{2\pi r_0}\right) \sin kz$ (dynes/cm)

P. 181 Eq. (45): delete D_e

P. 186 Line 3: expression for D_p derived on p. 181.

P. 216 Footnote: ... where $b = B_1/B_0$.

P. 234 Ref. [22]) R. G. Giovanelli, Phil. Mag **40**, p. 206

P. 242 Fig. 117: C_1C_2 should be replaced by X_1X_2

CONTENTS

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Chapter 7: RESEARCH ON CONTROLLED FUSION

Chapter 8: OTHER APPLICATIONS

INTRODUCTION

Plasma physics is concerned with the behaviour of systems of many free electrons and ionised atoms where the mutual Coulomb interactions cannot be disregarded. In a restricted sense, such systems of particles consist of nearly equal numbers of positive and of negative charges. Systems of this type are examples of a medium known as plasma which in many respects behaves differently from the solid, liquid and gaseous state of matter.

All states of matter represent different degrees of organization, to which there correspond certain values of binding energy. Thus, in the solid state the important quantity is the binding energy of molecules in a crystal; in fact, a crystal could be considered as a macro- or super-molecule. If the average kinetic energy per molecule W exceeds the binding energy U (a fraction of an eV) the crystal structure breaks up, either into a liquid or directly into a gas. A similar law operates in the case of liquids, and in order to change a liquid into a gas, a certain minimum kinetic energy per molecule is required to break the bonds of the van der Waals forces. Matter can exist as a plasma, i.e. in its fourth state, when the kinetic energy W per plasma particle exceeds the ionising potential of atoms which is usually a few eV. Thus the average kinetic energy per particle determines the state in which matter exists. A precise mathematical statement of this theorem is an equation of the Saha type. However, a simple criterion can be written as

$$U_n < W_{n+1} < U_{n+1} \tag{1}$$

where U_n, U_{n+1} are the respective binding energies, expressing that matter exists in the $(n+1)$st state.

Extrapolating this principle to higher states of matter, so far unexplored, one may define the fifth state of matter as one in which

$$2 < W_5 < 200 \qquad \text{(MeV)}.$$

This will be a gas of free nucleons and electrons — a "nugas". The sixth state would be, consequently, defined as

$$0.2 < W_6 < 4 \qquad \text{(GeV)}$$

and would contain free mesons, mutilated nucleons and electrons. The fifth and sixth states of matter can be expected to exhibit an even greater variety of behaviour than a plasma owing to the action of short range internucleon forces in addition to long range Coulomb forces.

On the other hand, according to eq. (1) for W, plasma spans a broader energy band than any other state of matter; it encompasses about 20 octaves on the kinetic energy-scale. This width of the kinetic energy spectrum of the plasma state is the reason for much common ground between plasma physics and many other fields of physics, such as the dynamics of single charged particles (in which many-particle interactions are not considered), or the physics of electrical discharges in gases (in which interaction between charged particles and neutral atoms and molecules is of great importance), whereas some methods of description and analysis used in plasma physics belong to the subject of hydrodynamics, particularly magneto-hydrodynamics. Another physical discipline, indispensable for the theory of a plasma, is statistical mechanics and there are yet other fields from which plasma physics draws its mathematical formulation and its terminology.

Although probably more than 99.9% of matter in our Universe is ionised and therefore in the plasma state, on our planet plasma has to be generated by special physical processes and under special conditions. These processes are the subject of the physics of electrical discharges in gases and this is the reason for the parental relationship between the latter and plasma physics.

Using an anthropomorphic analogy one may say that whereas the physics of electrical discharges is more specifically concerned with the birth and metabolism of plasma, plasma physics concentrates mostly on the anatomy and motion of plasma.

On our planet the medium which often resembles an ideal plasma is a partially ionized gas. This medium enters into the experience of prehistoric humanity in three forms; as fire, as lightning and as Aurora Borealis. In this connection it is curious to note that a number of greek philosophers, starting with Empedocles of Agrigentum (about

490—430 B.C.), held that the material Univers is built of four "roots": earth, water, air and fire. This, in modern terminology, may be compared with four states of matter, solid, liquid, gaseous and the plasma state. The privilege of identifying the medium created in electrical discharges in gases as the fourth state of matter belongs to W. Crookes who writes (1879): "The phenomena in these exhausted tubes reveal to physical science a new world, a world where matter may exist in a fourth state....". At about this time it became obvious that this newly discovered state of matter is not very much at home on our dense and cold planet and that special conditions must be realized in order to generate a plasma-like medium in the laboratory. Investigation of these conditions were the subject of the physics of electrical discharges in gases. It was only when electrical and vacuum techniques developed to the point when long-lived and relatively stable electrical discharges were available that plasma physics emerged as a separate field of study.

Around 1923 I. Langmuir developed the appropriate basic theory of an ionised gas and gave the medium the name "plasma" †. During the period 1923—1938 the subject developed further due to the efforts of L. Tonks, R. Seeliger, B. Klarfeld, M. Steenbeck, A. v. Engel, L. B. Loeb, W. Bennett, F. M. Penning, J. Townsend, W. Rogowski and many others.

At the beginning of this century astrophysicists became aware of the importance played by ionised matter in the processes in outer space and subsequently some of the finest contributions to plasma physics came from their ranks. Here one may mention the work of M. N. Saha, S. Chapman, T. G. Cowling, V. C. Ferraro, S. Chandrasekhar, L. Spitzer, H. Alfvén and the german astrophysical school at the Max-Planck Institute.

In 1929 F. Houtermanns and R. Atkinson suggested that the main source of energy in stars is the fusion reactions among the nuclei of the light elements. After 1945 a similar mechanism was exploited in the construction of hydrogen bombs and at the same time some physicists became interested in a controlled release of fusion energy.

† The word plasma occurs first in the term protoplasma which was originally introduced into scientific terminology in 1839 by the czech biologist J. Purkynie for the jelly-like medium interspersed by numerous particles which constitutes the body of cells.

However, it was appreciated that the energy output from fusion reactions depends critically on the kinetic energy of the colliding nuclei and that fusion outputs of practical interest depend on one's ability to produce temperatures of at least several million degrees Kelvin. If explosions are to be avoided, then the pressure of matter at this temperature must be balanced by external forces. This is within the power of our engineers only if the density of the nuclear fuel is less than the density of our atmosphere.

The search for a mechanism of a controlled release of fusion energy became, therefore, synonymous with the study of high temperature, low density plasmas. This was a new lease of life for plasma physics which was becoming rather unfashionable and, as one of my friends put it, regarded by most other physicists as a rather charming subject, full of small, colourful experiments, where there was little left to discover and whose only real justification was the amusement of those who bothered to waste their time on it.

With the goal of a fusion reactor as an incentive, plasma physics became a subject of interest to many physicists and engineers. However, it should be appreciated that, inspite of recent developments, most of the problems in plasma physics are understood only qualitatively and that quantitative solutions apply only in idealised cases. It seems that many of these problems will be solved within a decade or two. In such a case a book on the subject cannot aim at becoming a classic but rather a narrative, trying to collect and systematise a rich and rather bewildering literature in this field. This book represents such an effort. In this introductory chapter we shall briefly describe the treatment of the subject as developed in this book.

As in any problem which deals with a large ensemble of individuals, plasma physics uses two complementary modes of description: the analysis of the movement of a single particle and the fluid model. These two treatments are the subjects of chapters 1 and 2.

These modes of description are subsequently applied to equilibrium configurations (chapter 3), to wave-motion and instabilities in plasma (chapter 4) and finally to shocks in plasma and motion of plasma bunches (chapter 5).

In order to complete our description of plasma it is important to know how an equilibrium configuration is established. However, this problem can be solved only if one can find a suitable description of the various collision, diffusion and radiation processes that are operative in arriving at an equilibrium. This is the aim of chapter 6.

The six chapters provide us with models of plasma processes which are used in chapters 7 and 8 to describe some of the applications of plasma physics to the research on the controlled fusion of light nuclei, to electronics and to other problems in applied physics and in engineering.

MOTION OF ELECTRONS AND IONS IN ELECTRIC AND MAGNETIC FIELDS

Introduction

This chapter is concerned with the study of the motion of charged particles in the various fields of force and in combinations of such fields, that are of interest in the study of plasmas.† The most important of these motions is the interaction of charged particles with various magnetic fields. The last section of this chapter will be devoted to radiation emitted by single charges.

1.1. Motion in an Electrostatic Field

If the field is irrotational it may be described by a potential V, and a particle trajectory is constructed in the general case from the

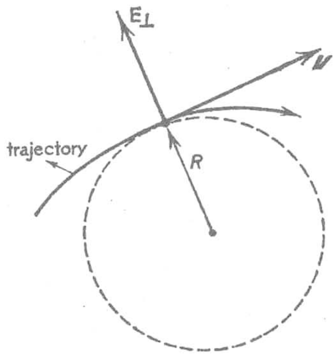

Fig. 1. The projection of a trajectory of a charged particle in an electrostatic field on its oscillating plane.

equation for the radius of curvature R. Thus (see Fig. 1)

$$m\frac{v^2}{R} = eE_{\perp} \qquad (1)$$

and provided that $v = 0$ for $V = 0$ one may write (for non-relativistic

† Lists of symbols are given at the end of each chapter.

energies)

$$\frac{2eV}{R} = e\left(\frac{\partial V}{\partial r}\right)_{\perp}$$

and therefore,

$$R = \frac{2V}{\left(\frac{\partial V}{\partial r}\right)_{\perp}} \quad \text{(cm)}. \tag{2}$$

This formula can be used very simply in a step by step graphical plotting of the trajectory. It is also used in automatic trajectory tracing in an electrolytic tank (ref. [1])).

Motion of this type is of great interest in electron optics; however, in plasma physics it enters only in connection with the microscopic description of Coulomb scattering.

1.2. Motion in a Magnetostatic Field

In a homogeneous magnetostatic field B a charged particle moves

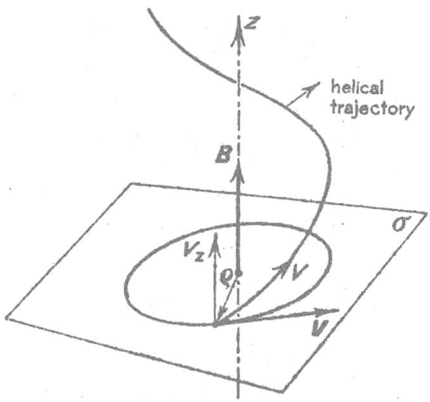

Fig. 2. Trajectory of a charged particle in a uniform magnetostatic field.

on a helical trajectory (Fig. 2) with an angular frequency

$$\omega_c = \frac{e}{mc} B. \tag{3}$$