INTRODUCTORY ALGEBRA 2

Russell F. Jacobs

TEACHER'S EDITION

Teacher's Edition

Introductory Algebra 2

SECOND EDITION

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Introduction

RATIONALE OF THE COURSE

A basic tenet of democratic education is that each student should have the opportunity of developing his knowledge and skills to the greatest possible degree. INTRODUCTORY ALGEBRA can provide this opportunity.

The basic assumption underlying this approach to the teaching of algebra is that there are students who, by nature of their background and maturity level, can learn elementary algebra more effectively if instruction is spread over a two-year period. It is further assumed that such students can be identified and that most of them fit into the category best described as low-average in ability or achievement. The student who is ambitious, capable of a degree of abstract reasoning, and sincerely interested in learning algebra will profit most from this course.

The need for materials especially adapted to the plan of this course is evident. These materials need to meet several criteria. First, the reading level should be as low as is possible without deviating significantly from commonly accepted elementary-algebra terminology. Related to this, the amount of reading should be minimized and an open style implemented that will appeal to the student for whom the materials are written. The method of presentation should involve the student as much as possible, and there should be an abundance of graded exercises both for oral and written practice. Finally, the presentation should be paced in such a way that the materials can be used effectively over a two-year period.

ADMINISTRATIVE SUGGESTIONS

One of the first questions that arises in offering elementary algebra as a course to be studied over a period of two years relates to the matter of credit. Should a student be given credit for two years of high-school mathematics, or should he be given credit for just one year, since he has not progressed beyond elementary algebra? Another question is how to label this credit on the student's permanent record; that is, what names should be given to these courses? Some teachers and administrators fear that the second year of this course will be confused with the second college-preparatory algebra course ordinarily called Intermediate Algebra, or Algebra 2. How can this be avoided?

In Phoenix, Arizona, where the author had the experience of teaching the first such course offered, these problems had to be faced. At the outset, it was decided that students should receive credit for two years of high-school mathematics upon successful completion of the two-year course. If a student chose to drop out of the course after one year's successful work, he would still be given one year of credit in mathematics that would count toward his graduation requirements.

In order to avoid confusing the two tracks, a description of the content of the INTRODUCTORY ALGEBRA courses is included with the transcript of each student who has taken these courses. Emphasis is given to the fact that, as far as college admission is concerned, the two-year course is equivalent to one year of college-preparatory mathematics.

Some schools may wish to label the first year of the course General Mathematics. However, this has the disadvantage of the stigma that students often associate with a general mathematics course. They take pride in being enrolled in a course that is called Algebra. Some teachers and administrators may want to assign reduced credit for the two-year course through the misunderstanding that it is a "watered-down" algebra course. A careful examination of the content of the two texts, INTRODUCTORY ALGEBRA *1* and *2*, will reveal that all the usual topics of elementary algebra are treated with a reasonable degree of sophistication. Furthermore, several topics are handled in depth that cannot be included in a one-year course.

Whatever system is chosen for course designation and assigning of credit, it is most important that all people concerned understand it. This group includes students, parents, teachers, administrators, counselors, college admissions personnel, and any other persons or agencies requesting a student's transcript. Experience has shown that it does not take long for such a course to be accepted and understood locally and that the problems mentioned soon seem trivial.

SELECTION OF STUDENTS

It is fallacious to assume that all students can learn algebra successfully. For many low-ability students, some type of mathematics below the level of abstraction required for learning algebra should be provided. In selecting students for the two-year course in INTRODUCTORY ALGEBRA, the teacher should consider the following suggested guidelines.

1. average score of fourth or fifth stanine on standardized tests indicating academic promise in numerical skills, abstract reasoning, and verbalization;
2. average or below-average achievement in previous courses;
3. judgment of previous mathematics teacher and the student's counselor that the student can profit by taking this course;
4. knowledge and consent of student's parents.

The author's experience with the course has been in starting students with INTRODUCTORY ALGEBRA *1*, in the ninth grade. There is no reason to assume that the course could not be studied successfully by eighth graders if prior instruction is adequate. This approach would enable a student to complete elementary algebra in the ninth grade, thus maintaining the traditional schedule of college-preparatory courses.

TEACHING THE LOW-AVERAGE STUDENT

Teaching algebra to the student of low-average ability and achievement in mathematics requires a patient and understanding teacher. The teacher who believes that such a course has a place in the curriculum and feels an empathy for the student in the course will probably do the best job of teaching it. On many topics, progress will be slow and gains will be small. However, the course is designed to accommodate these problems by providing a variety of exercises that emphasize the discovery method and by reviewing concepts through a spiral approach.

Within a class, there may be quite a wide range and diversity of student ability, interest, academic achievement, and maturity. One student may work at a level of abstraction very close to the critical point necessary for succeeding in the course. Another student may have low reading comprehension. Another student may be low in computational ability, and still another may have poor work habits and a lack of interest in mathematics. Many of the students will be under-achievers, and some will be over-achievers. Many of the students will have a short attention span — particularly on abstract materials. Most of the students in a given class will have one or more of the traits mentioned. As a group, the students enrolled in this course will have poorer attendance patterns, worse disciplinary records, and less parental interest in their work.

In order to overcome some of these problems of students, the teacher must appreciate each student as an individual. Praise must be used liberally when earned even for small tasks completed successfully. Each student should be made aware of his progress at regular and frequent intervals. The teacher should try to involve each student each day in some way in the class discussion.

The teacher should make the work as concrete as possible for the student. In developing generalizations, the teacher should use many examples, and the students should be encouraged to discover results. Whenever possible, models, graphs, overhead-projector materials, measuring devices, and other multi-sensory aids should be utilized.

The teacher should vary the pace of the day's lesson — a few minutes of lecture, a few minutes of oral reading, a few minutes of class discussion, a few minutes with overhead-projector materials, a few minutes on student demonstrations, a few minutes of directed reading, and some time each day for supervised work on the next assignment. The key to a successful lesson each day is an enthusiastic teacher who keeps a fast and varied pace and involves his students in the work at hand.

THE FORMAT

Each section is written to provide material for a single day's lesson. However, in some classes, the teacher may want to spend two days on certain sections if he feels the class needs more drill or opportunity for increased understanding. Sections are designated by chapter and section number within the chapter. Thus, 3.2 refers to the second section of the third chapter.

Material is presented in a semi-programmed style with many leading, or pivotal, questions provided as appropriate. The pivotal questions are designated as **P-1, P-2, P-3,** etc. Answers to the pivotal questions are provided at the back of the students' book.

Many definitions are presented informally; the definitions are restated in the chapter summaries and at the back of the students' book under the heading Summary of Important Terms and Ideas. Each important definition and generalization is set off in a color block.

Oral exercises are provided for each section. These exercises include problems that will help to reinforce the student's understanding of the day's lesson and also prepare him for the written exercises. The oral exercises are less difficult than the written exercises.

All exercise items are numbered consecutively. There are three groups of exercises labeled A, B, and C:

A The items in the A exercises are paired by their degree of difficulty or type. The teacher may want to assign all the A exercises in a section, or he may find it desirable to assign only the odd-numbered or only the even-numbered exercises.

B The B exercises are of two types. Some will allow students to discover mathematical relationships that will be presented in later sections, and others are supplementary to the basic content of the course. The teacher will probably wish to assign or discuss B exercises.

C The C exercises are considered more difficult than the A exercises but essentially are related to the topics covered in the accompanying section. The C exercises should be assigned to the more capable individuals in a class.

Note that there are not B and C exercises for each section.

A chapter summary, which includes important terms and important ideas, is provided at the end of each chapter. Chapter review exercises are provided for each chapter. Cumulative review exercises are given at the end of every third chapter. At the end of Chapter 1, there is a review of arithmetic; and at the end of Chapter 14, there are review exercises for Chapters 1–13.

PRESENTING THE LESSON

The material is presented in this book much as a teacher would write a complete lesson plan. The teacher can lead the class through the development of each lesson using the pivotal questions to help students discover relationships. Since low-average students cannot be expected to read and understand much mathematics on their own initiative, the teacher will have to try different techniques to find what works best for his particular students. Students can be asked to read aloud. The teacher may want to read some of the material to the class. Students may be asked to read silently, recording their answers to the pivotal questions as they read. In some cases, the teacher may want to present the material in an entirely different manner from the way it is presented in the text.

In some sections, the teacher may find that it is advisable to use the oral exercises for a written assignment, since they are less difficult than the written exercises.

It is important that the teacher provide some supervised study time each day to enable students to begin work on their written assignment. One important aspect of this phase of the lesson is in helping students understand the instructions for each group of problems.

PERFORMANCE OBJECTIVES

The Teacher's Manual contains behaviorally-stated instructional objectives for each section of INTRODUCTORY ALGEBRA *2, Second Edition*. These objectives are called *Performance Objectives* and are stated in the *Teacher's Manual* just after the head of each section. Perhaps a discussion of the author's philosophy with regard to performance objectives is appropriate. The author believes that performance objectives can lead to improved instruction and learning if they are used properly. The purpose of performance objectives is essentially two-fold.

The use of performance objectives permits both students and teacher to understand what the learner is expected to be able to do when he successfully completes a learning activity. In other words, the terminal behavior of the learner is identified. Inherent in this purpose is the facet of learning psychology known as reinforcement and, specifically, the learner's reward in knowing he has been successful. Obviously, he cannot evaluate his own learning success unless he clearly understands what is expected of him. In fact, negative learning may occur if it becomes evident to him that his own expectations and the expectations of the teacher are not in agreement.

The second important purpose, not unrelated to the first, is the use of performance objectives in teacher evaluation of student achievement. Formal evaluation of student progress should be on the basis of criterion referenced test items. That is, the test items of the evaluation instrument should be based on and referenced to one or more performance objectives. Actually, no other evaluation procedure is defensible. The performance objectives describe the instructional program, and the evaluation of student achievement is based directly on the performance objectives.

There are several crucial decisions that a teacher must make as he or she carries out the planning for a course. First, the general goals or global objectives of the course must be identified. Next, the subject matter content must be determined on the basis of the course goals. Third, in planning the day-to-day instructional process, instructional objectives for each lesson must be stated. At this stage the teacher must decide what concepts and skills to include in the stated objectives. In the process of evaluating student performance, the teacher must decide which of the objectives to include in the evaluation instrument. For some lessons or units, it may be desirable to prepare test items for all the stated objectives. For other lessons or units, it may be appropriate to select the most important objectives in order to keep the test length within reasonable limits. Finally, the teacher must decide what is an acceptable standard of performance.

In preparing the performance objectives for INTRODUCTORY ALGEBRA, *Second Edition*, the author has attempted to focus on those concepts and skills that have been emphasized in a particular section. You, as the teacher, may have a slightly different set of objectives for your students. It is the author's hope that you will use the list of performance objectives stated in the *Teacher's Manual* as a model for preparing objectives that fit the particular needs of your own classes. In fact, all the objectives that are stated in the *Teacher's Manual* are in the so-called cognitive or psychomotor domains. No attempt has been made by the author to formulate objectives for the affective domain; although, as the teacher, you probably have many implied objectives for your students that are concerned with student interest, attitude, self-motivation, etc. Furthermore, the author has not attempted to specify what the standard of performance should be in the statement of each objective. The author believes that determination of standards of acceptable performance is the prerogative of the teacher.

The test items that are included in the booklet *Introductory Algebra 2 Tests, Second Edition* are criterion referenced. That is, each test item is based on and referenced to one or more of the stated performance objectives. Each of the tests is an attempt to sample the student's performance on the material covered. Thus, not every one of the author's stated performance objectives is tested. In order to help you identify the objective that corresponds to each test item, each item is coded for ready reference using a three digit decimal code. The following example shows the coding clearly.

Code Number	Meaning	Performance Objective
5.9.1	First Performance Objective listed in the Teacher's Edition discussion of Section 5.9.	*Given a quadratic trinomial, each student will be able to write* Yes *or* No *correctly whether it is a perfect trinomial square.*

Again, it is the author's intent that the test items will serve as a model for you to prepare your own evaluation instrument based on the performance objectives you have identified for your students. Depending upon your own requirements, you may wish to use the test booklet in its published form, or you may wish to add or delete items. By way of explanation, the author's general approach to the preparation of the tests has been to include a generous sampling of objectives with relatively few test questions per objective. The assumption in this approach is that you can generally test a student's achievement of a particular objective with one or two questions just as well as with nine or ten.

It is the author's philosophy that a criterion referenced test should be a "power" test and not a "speed" test. Thus, do not necessarily plan a class period for each test. Ideally, a student should be allowed whatever length of time he needs to demonstrate whether or not he has achieved the objectives being evaluated.

An important point about the use of performance objectives has been implied but perhaps should be explicitly stated. **The objectives should be shared with students.** An understanding of these objectives by students will help them to organize their work for successful learning and to evaluate, on their own, the extent of their learning. The performance objectives, as stated in the *Teacher's Manual,* are for the teacher's use. They are stated rather precisely and technically in order for the teacher to be able to prepare test items based upon them. It is strongly recommended that these objectives be rephrased before they are distributed for student use. Also, it would be very helpful to the student, in his interpretation of the objective, to include a sample problem or test item that is referenced to the objective. Here is an example of an objective that is restated in a form that may be more appropriate for student use with an accompanying example.

Objective 4.1.2

Teacher Objective: Given a perfect square rational number, each student will be able to write its two square roots.

Student Objective: When you are given a perfect square rational number, you will be able to write its positive square root and its negative square root.

Example: 16

Solution: 4, $^{-}4$

In summary, you should be sensitive to preparing performance objectives that are meaningfully stated so that you can plan appropriate teaching techniques for each lesson. Equally important, the performance objectives will then provide a basis for the selection and preparation of test items that will measure what you and your students expect them to learn.

SUGGESTED TIME SCHEDULE

Each section provides a single day's lesson. However, the teacher may find it desirable to spend two days on some sections if students have difficulty with the material.

Most teachers will set aside at least two days of instruction on each chapter for testing. (Chapter tests are provided in the test booklet for Book 2 entitled *Introductory Algebra 2 Tests, Second Edition.* The answers to these tests are given in this teacher's edition on pages K-32 to K-40 inclusive.) However, many teacher's want to evaluate students' progress at shorter intervals of time.

Some teachers may wish to devote two assignments to the chapter review exercises, while others may spend only one day on them. In most cases, two days of work could be devoted to the cumulative review exercises. There is an adequate amount of work in the text to cover 180 teaching days if the teacher plans his assignments carefully.

The following suggested time schedule is flexible because of the variables just mentioned. The sections on *Algebra and the Modern Sciences* are included, but not as a part of the minimum course.

Chapter	Days	Chapter	Days
1	14–18	8	15–18
2	11–14	9	10–14
3	10–14	10	11–14
4	13–16	11	11–13
5	13–16	12	14–17
6	12–15	13	10–13
7	12–15	14	13–16
		15	9–12

Total Days: 178–225

Teacher's Manual

Chapter 1: REAL NUMBERS

The first chapter of INTRODUCTORY ALGEBRA *2* is devoted to a review of important concepts that were introduced in INTRODUCTORY ALGEBRA *1*. The material is presented at a more sophisticated level than when first introduced. Students who mastered the basic skills in INTRODUCTORY ALGEBRA *1* will have the opportunity to practice them now. Students who may not have mastered certain skills and concepts in the first year's work may now attack them with a fresh viewpoint. A year's maturity and a new setting may help some of these students succeed.

In the first two sections of this chapter the raised dash $^-$ is used to represent negative numbers. A review is presented in Section 1.2 of the various uses of the dash symbol with a statement that the raised dash will no longer be used in the course. This seems a good point at which to dispense with it entirely.

The section on *Algebra and the Modern Sciences* at the end of Chapter 1 entitled *Measurement with the Microscope* can be presented at any time. The teacher may want to coordinate this lesson with the students' initial work on the microscope in the biology class.

1.1 Real Numbers and Absolute Value

Performance Objectives:

Given a numeral for a real number, each student will be able to identify the number as Positive or Negative.

Given a numeral for a real number, each student will be able to write a numeral for the opposite of the given number and identify the opposite as Positive or Negative.

Given the absolute value of a real number represented by the absolute value symbol, each student will be able to write the common name.

The purpose of this introductory section is to review the concepts of real number and absolute value. Students will probably relate the meaning of absolute value to the definition that was used in INTRODUCTORY ALGEBRA *1:* the absolute value of a real number is the greater of that number and its opposite. However, the more formal definition as suggested in this section may be used also. That is, $|x| = x$ when x is a positive number or zero, and $|x| = -x$ when x is a negative number. Be sure students realize that here the $-$ indicates the operation of taking the opposite, and that x may already be a negative number. Seeing the second part written as $|-x| = -(-x)$ when x is positive may also help.

1.2 Addition and Subtraction

Performance Objectives:

Given the indicated sum of two or more real numbers, each student will be able to write the common name.

Given the indicated difference of two real numbers, each student will be able to write the difference as a sum and find the common name.

Given a word statement or question that suggests a subtraction problem, each student will be able to write a phrase for the problem that is an indicated difference and then find the common name.

This section reviews addition and subtraction of positive and negative real numbers expressed as integers, common fractions, or decimal fractions. It is not necessary to require students to parrot the formal rules. However, they should have a working knowledge of them.

The meaning of subtraction in terms of addition is reviewed. This is not discussed in detail and the teacher may expand on the text.

You may want to discuss common names. Although common name means the simplest name a number or phrase can have, often there are several ways to express the common name. For example, in Problem 15 of the Written Exercises either $a - 0.6$ or $-0.6 + a$ would be acceptable.

1.3 Multiplication

Performance Objective:

Given the indicated product of two or more real numbers, each student will be able to write the common name.

In reviewing the multiplication rules, students will still need considerable convincing that multiplying two negative numbers yields a positive number. The formal proof that $(-2)(-3)$ equals 6 as shown on page 6 will have more meaning to students now that they have gained more mathematical maturity.

Be sure to assign Problems 25 and 26 in the Written Exercises. They may create in students an interest in trying to form generalizations about such products.

1.4 Division

Performance Objectives:

Given an indicated quotient of two real numbers, each student will be able to write the quotient as a product.

Given an indicated quotient of two integers, each student will be able to write the common name.

Given a phrase in one of the forms $\frac{-a}{-b}$, $\frac{-a}{b}$, *or* $\frac{a}{-b}$, *in which a and b represent real numbers,* $b \neq 0$*; each student will be able to write the common name.*

Given an open sentence of the form $ax = b$, *in which a and b are real numbers,* $a \neq 0$*; each student will be able to write the truth set.*

This is a good opportunity to emphasize again that zero cannot be a divisor. This concept is an easy one for students to forget. They may also need to be reminded that the product of a nonzero number and its multiplicative inverse is 1, not -1.

A problem such as Number 14 in the Oral Exercises will provide a good opportunity to remind students that $-p$ doesn't necessarily represent a negative number. Problems 21–28 will provide additional help on this common difficulty.

1.5 Real Number Properties

Performance Objectives:

Given a special phrase that is an example of one of the Identity Properties or one of the Inverse Properties, each student will be able to write the common name.

Given a special open sentence in which the truth set can be found by one of the real number properties, each student will be able to write the truth set.

Given an open phrase that is an indicated product of a real number and the sum of two real numbers, each student will be able to use the Distributive Property to write the product as a sum in simplest form.

Given an open phrase that is an indicated sum of two products of real numbers, each student will be able to use the Distributive Property to write the sum as a product in simplest form.

In this section the properties of real numbers are presented in a parallel arrangement for the two operations, addition and multiplication. Seeing them in this form helps the students to remember them. This section also offers the teacher the opportunity to talk about the field of real numbers and to discuss the properties of a field.

The exercises review the properties of real numbers that were studied in the first course. It will not be unusual if many students still have difficulty in identifying these properties.

1.6 Special Properties

Performance Objectives:

Given an open sentence involving a quotient of the form $\frac{a}{1}$ *or* $\frac{a}{a}$, *each student will be able to write the truth set.*

Given an open phrase of the form $-a - b$, *in which* a *and* b *represent real numbers; each student will be able to write the equivalent phrase in the form* $-(a + b)$.

Given an open phrase of the form $a - (b + c)$, *in which* a, b, *and* c *represent real numbers; each student will be able to write the equivalent phrase in the form* $a - b - c$.

Given an open phrase in one of the forms $(-a)(b)$ *or* $(-a)(-b)$, *in which* a *and* b *represent real numbers, each student will be able to find the common name.*

Given an open phrase in one of the forms $\frac{-a}{b}$, $\frac{a}{-b}$, *or* $\frac{-a}{-b}$, *in which* a *and* b *are real numbers,* $b \neq 0$; *each student will be able to write the common name.*

Some of the most important special properties that were developed in INTRODUCTORY ALGEBRA *1* are presented again. These properties will be used again and again throughout the course. Perhaps the one causing students the most difficulty is the one that shows that $a - (b + c)$ and $a - b - c$ are equivalent phrases. A careful review of this property may prevent many errors on the part of students in later problems.

In the exercises, although no divisor is assumed to be zero, you may choose a few division problems and ask students what number or numbers this assumption will necessarily exclude from the domain of the variable.

The evaluation of phrases in the Written Exercises, Problems 31–44, will be good practice for students in reviewing operations on real numbers.

1.7 Like Terms

Performance Objective:

Given an open phrase of two or more terms that is an indicated sum or difference, each student will be able to simplify the phrase by adding like terms.

Detailed steps are again shown in some examples to demonstrate the use of the Distributive Property and other number properties in collecting like terms. Point out to students that the sum of like terms can always be expressed as a product. The sum of unlike terms cannot always be expressed as a product.

The sum $2x + 3y$ cannot be expressed as a product. However, the sum $2x + 2y$ is equivalent to the product $2(x + y)$.

It is important for students to realize that the symbols $+$ and $-$ separating terms of a phrase such as $x^2 - 3x + 5$ are read as symbols of operation. However, such phrases can always be expressed as sums.

$$x^2 - 3x + 5$$

is equivalent to

$$x^2 + (-3x) + 5.$$

The device shown for checking each term of a phrase as it is used in collecting terms may be helpful to some students. Do not however, insist that it is a necessary part of the mathematics involved.

1.8 Open Phrases and Multiplication

Performance Objectives:

Given an indicated product involving real numbers and open phrases, each student will be able to write the common name.

Students often get confused about multiplication of terms or monomials. After being convinced that they must look for *like* terms to collect in a phrase they make the mistake of thinking that a common name cannot be found for the product of unlike terms.

A common error also is for a student to give $6x$ as the equivalent phrase of $(2x)(3x)$.

You can always use various properties of real numbers to demonstrate the correct results of multiplying two monomials. Students often must be reminded of the power of these properties and the necessary conclusions that result from them. The word, *monomial* has not been introduced yet. It will be defined in a later chapter.

1.9 Addition and Multiplication Properties of Equality

Performance Objective:

Given an equation with the variable on one side of the equality symbol, each student will be able to use the Addition and Multiplication Properties of Equality to find the truth set and then perform a check.

These are the two main properties that are used in finding truth sets of open equalities. Students should be required to show detailed steps in their work until they show a good knowledge of equation solving. Remind students that the check is for the purpose of discovering any computational errors. They should see that, as the application of the various number properties produces a sequence of equivalent open sentences, the check is not a *logical* necessity.

1.10 Truth Sets of Open Sentences

Performance Objective:

Given an equation with the variable on both sides of the equality symbol, each student will be able to show the steps in finding the truth set and perform a check.

In this lesson the Addition Property of Equality is applied to open phrases. It should be understood that this property can be used in this manner if you are sure the open phrase represents a real number.

In finding truth sets of sentences in this section requiring the use of both the Addition and Multiplication Properties of Equality, students should be warned to apply the Multiplication Property in the last step.

1.11 Solving Problems

Performance Objective:

Given a word problem involving consecutive integers; each student will be able to represent each unknown by a variable or open phrase, translate the problem into an open sentence, find the truth set of the sentence, and use the truth number to solve the problem.

Consecutive integer problems have been chosen here as a means of reviewing problem-solving techniques. These problems are important because they demonstrate many concepts that students should know at this point.

Students should be reminded that they must read each problem carefully in order to provide the information that is required. A common mistake occurs in problems in which consecutive negative integers are involved. A student may find that -5 is the lesser of two consecutive integers and go on to say that -6 is the greater.

1.12* Algebra and the Modern Sciences: Measurement with the Microscope

This section involves some rather elementary algebraic work with formulas as related to using the microscope for measuring small objects. It has been included in this book as a good example of the relationship between mathematics and a modern science. The importance of coordinating instruction in mathematics with instruction in the physical sciences cannot be over emphasized.

It would be helpful for INTRODUCTORY ALGEBRA teachers to consult with biology and other science teachers so that this section can be introduced at an appropriate time. Note that although a very brief discussion of the microscope is given here, instruction in its parts and operation should be left to the science instructors.

Two formulas related to the circle are needed for this work, $d = 2r$ and $A = \pi r^2$. The micron is defined and the basic correspondences between the micron, the meter, and the millimeter are given.

One relationship that will cause students some difficulty is: the ratio of the lengths of the diameters of two fields is the reciprocal of the ratio of the magnifications of the two objective lenses. Several examples will probably be needed to clinch this point.

Tables are used as an aid in leading the student through the steps in finding the diameter of the high-power field. The diameter of the high-power field can be used by the scientist to estimate the size of an object being viewed through a microscope.

Chapter 2: PROPERTIES OF ORDER

As the title indicates, this chapter presents the various properties of order and shows their application to finding truth sets of inequalities. Since work with inequalities is so closely related to equalities, it will give students an opportunity for further review of these skills. This may also be the first time that some students gain insight into the methods necessary for finding truth sets of sentences by a systematic process.

The main working tools for this chapter are the Addition and Multiplication Properties of Order.

2.1 Graphs of Inequalities

Performance Objectives:

Given a number line graph, each student will be able to write a set description of the numbers represented by the graph.