Ken **Dutton**
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The Art of CONTROL ENGINEERING

Prentice Hall

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Preface

The material in this book is based upon undergraduate, postgraduate and industrial courses taught at Sheffield Hallam University (England), and at The Queen's University of Belfast (Northern Ireland). It may be used to take a student from zero knowledge of control systems, by easy stages, up to an appreciation of some advanced control schemes. A feature of the text is that it considers the *implementation* of the various control schemes; although it would still be necessary to be guided by a good practitioner of the 'Art of control engineering', if applications are to be implemented safely and efficiently. Other features of the text are its layered approach to the teaching of control and its treatment of the mathematics essential to an understanding of the subject. Both of these aspects are expanded later in the preface. In brief, the pragmatic approach and the practical comments given throughout the book should prove useful to the student studying control, the educator planning a course in control studies and to practising control engineers. This preface contains the following sections, giving useful information about this book, and how it can best be used:

- A broad overview of the book
- Mathematics (notes on the book's approach to mathematics, and how it is included)
- Computer-aided control system design (CACSD) and MATLAB®
- How to use the book
- Acknowledgements (with an email address for comments)

A broad overview of the book

This book is structured to suit the way control engineering is taught in European institutions (but it is sufficiently flexible to be easily used elsewhere). It assumes little previous knowledge other than familiarity with basic physics, algebraic manipulation and elementary calculus. The major additional topics in mathematics necessary to study control engineering (such as the Laplace transform, the z-transform and matrix algebra) are covered as they are needed.

In the past, students who have been motivated to study all the control engineering options (electives) their programme of study offers, have typically needed several textbooks (including, perhaps, texts on classical control, multi-variable control, digital control and nonlinear control), some of which might be used for only a very small percentage of their course. An ambitious aim in writing this book has been to provide, in a single volume, a complete undergraduate coverage of control engineering (and also to cover some postgraduate courses).

Typically, an undergraduate student undertaking a three academic year course of study with a high control engineering content might well encounter control lectures each year. Accordingly, the chapters in this book are aimed at three different *levels*, corresponding roughly to these three years of study, as indicated in Figure P1. This means that some of the major topics might occur in each of the three levels. Of course, the higher the level, the greater the depth of treatment. A further benefit of this 'layered' approach is that some of the fundamental divisions of control engineering (such as frequency-domain methods versus time-domain methods) are largely overcome, since all these methods are studied side by side at each level, thus providing a well-integrated approach to the subject as a whole. 'Maps' of how some major topics can be followed through the text in a less integrated fashion, if desired, are given later in this preface.

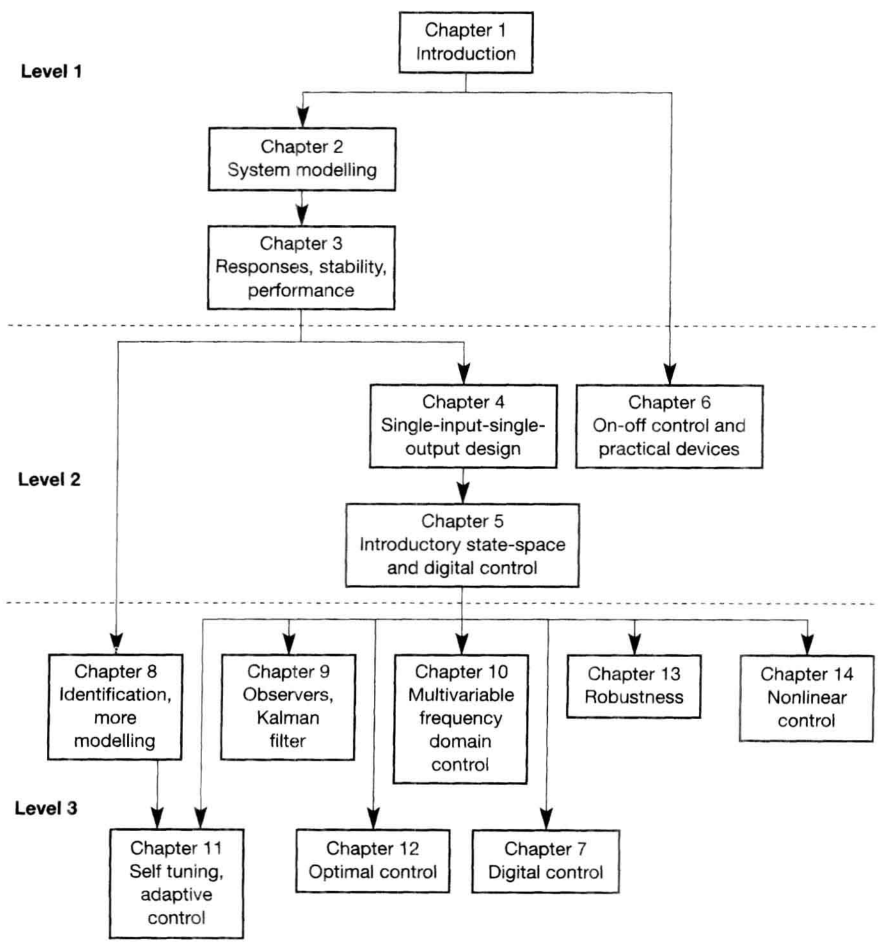

Figure P1 The layered approach of the text.

Mathematics

Control engineering can easily become a highly mathematical discipline (and there are several very mathematical control texts on the market). This text is designed to be different, although it is inevitable that some parts of it will look rather mathematical on a cursory inspection. However, even in the most intensively mathematical parts, if the text is read in the order suggested, then sufficient steps of the mathematics are given to allow it to be followed.

The mathematics in this text is presented as it arises naturally, and is usually limited to that which is *necessary* for an understanding of the particular topic under consideration. For example, the Laplace transform, which is the cornerstone of the frequency-domain approaches, is introduced in a 'user-friendly' manner within the text, at the point where it becomes necessary to use it.

Sometimes, if a substantial amount of mathematical background is considered helpful, an appendix is used. For example, Appendix 1 contains all the matrix algebra necessary to follow the time-domain and multivariable control methods described in the text. In addition, it treats matrices and vectors in a helpful, 'control-orientated' manner.

Our aim, therefore, has been to integrate the mathematics with the text, since we are, after all, interested in promoting control engineering and, in our experience, seemingly abstract mathematics can 'turn off' many a student. Rigorous mathematical proofs are avoided, except where they aid an understanding of how to make the best use of the method in question. In general, the treatment is more pragmatic and practical than will be found in many books. Even so, there is always sufficient theoretical background to allow an understanding of each technique.

As one example of a full derivation which *has* been included, Appendix 6 (combined with part of Chapter 9) gives a mathematical derivation of the Kalman filter. To some, this will seem the most complicated mathematics in the book, but it has been included because the derivation is more complete than that provided by any other textbook known to the authors, and is therefore useful background material for an undergraduate text. However, 'background' is the appropriate word, and illustrates one aspect of the authors' pragmatic approach since, for an engineering course (unlike a mathematics course), it is unlikely that a full derivation of the Kalman filter (or any other very complex algorithm) would be taught line by line. Rather, it resides in the appendix where interested readers may study it, but only the outline steps would be mentioned in class, as an aid to understanding where the algorithm comes from, why it works and how it might best be used in practice.

In other places where a proof might be of interest, but the proof is given adequately elsewhere, appropriate references are quoted. However, the text never subsequently relies on knowledge of such a proof.

Each chapter in the book has an introductory section listing any new mathematical ideas which will be encountered in that chapter. In this way, the student can see where the need for some greater mathematical knowledge is imminent. However, since the new mathematics is always integrated with the text, the motivation for the study of the new topic will be clear.

Computer-aided control system design (CACSD) and MATLAB®

No new control engineering text can ignore the extent to which the availability of cheap computing power, and powerful software packages, has revolutionized the subject. This text acknowledges this by including some relatively modern analysis and design techniques which cannot be applied 'by hand' because of the volume of calculation involved (see, for example, Chapter 10). The problem is that there are many excellent software packages which can be used for control system design studies, each having its own particular strengths and weaknesses. In producing this text, it was necessary to opt for just one package, for consistency, and we chose to use MATLAB – see (The Mathworks Inc., 1993a, 1993b) in the references. For those readers not familiar with the power of modern CACSD software packages, it will be helpful to know that MATLAB was used to produce almost all of the system response plots, of every kind, shown in the book.

On the other hand, it is not assumed that the reader *must* have access to MATLAB in order to be able to make use of the book. MATLAB has been used simply as one representative CACSD package. For this reason, full details of MATLAB code are not given in the text (although full details are available in Appendix 3. What we have done is to include some sample MATLAB code in simple cases, to indicate how each design method might be specified for computer assistance. In this way, the reader who does not have access to MATLAB (or, indeed, to any other CACSD environment) can still use the vast majority of the text, while simply ignoring all references to the MATLAB files.

For those readers who do have access to MATLAB, several MATLAB m-files are provided, which can be downloaded from Addison Wesley Longman's web site at http://www.awl-he.com ('online samples and supplements') which support the written text. Their file names generally link up with the numbers of the figures in the text, so it is easy to find the m-file that will generate a particular figure. By modifying copies of these files, the reader is able to experiment with controller parameter values, or to try other modifications to the various control system designs and simulations presented in the book. Several of the m-files thus constitute extremely useful templates for many of the techniques of control system design and simulation. Appendix 3 describes MATLAB, and the MATLAB software configuration needed to run these m-files. Appendix 4 describes the related package SIMULINK® (The Mathworks Inc., 1994a), which is used for digital computer simulation (although most of the simulations in the downloadable m-files just use MATLAB). The ASCII text file *readme.txt* on the disk contains further information.

Details of one suitable philosophy for including MATLAB- and SIMULINK-based material into the teaching of this text can be found in Dutton and Barraclough (1996).

How to use the book

Figure P1 gives an overview of how the material in the book is structured. It also shows broadly which chapters contain some material necessary for the study of later chapters. At the introductory level, it will be necessary to study the first three chapters (or parts of them, at least) for most purposes. Conversely, at the highest level, Chapters 7 to 14 are almost independent of each other, making it very easy to select topics as required. Also, parts of some of these chapters can easily be omitted if desired, thus widening the choice even more. The divisions shown between the levels can be moved to some extent, to suit any particular programme of study.

As a guide to the selection of material for various purposes, we offer the following suggestions (which can be extended or contracted as required, to suit the course of study). Whatever aspects of control engineering are to be studied, our basic belief is that an underlying knowledge of the frequency-domain techniques (often called 'classical control') is the best foundation from which to begin. Even if a course concentrates on the time-domain (state-space) methods, the importance of the 'feel' which comes from the frequency-domain study of the input–output behaviour of systems, and of the effects of adding simple controllers into feedback loops, cannot be overestimated. This means that, in our opinion, most of Chapter 1, plus a selection of topics from Chapters 2, 3 and 4, will almost always be desirable.

If a course heavily biased towards the frequency-domain methods is required, then the selection shown in Figure P2 is suggested. The various optional sections indicated can be chosen to suit the time available. The course could terminate at any stage after the 'Level 2' material.

For a course biased towards the time-domain methods, taking into account the previous comments, Figure P3 suggests a suitable route through the text. Again, the course could terminate at any stage after the 'Level 2' material.

For a purely digital control course, it will still be necessary to study the background of frequency-domain and/or time-domain methods, so as to be able to design that which is to be digitally implemented (apart from the purely digital

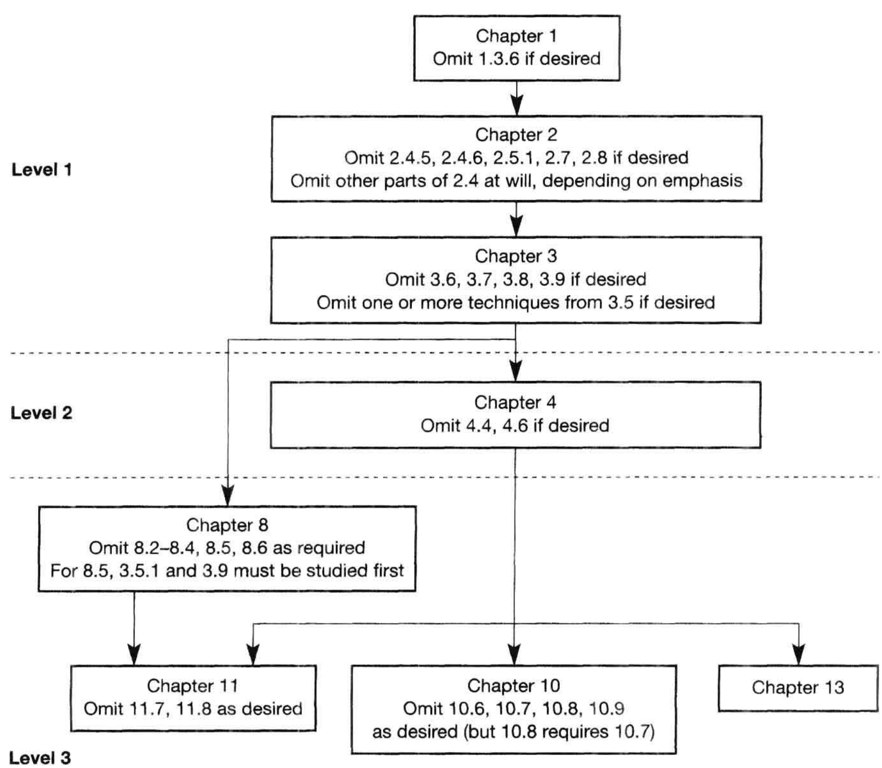

Figure P2 A mainly frequency-domain control course.

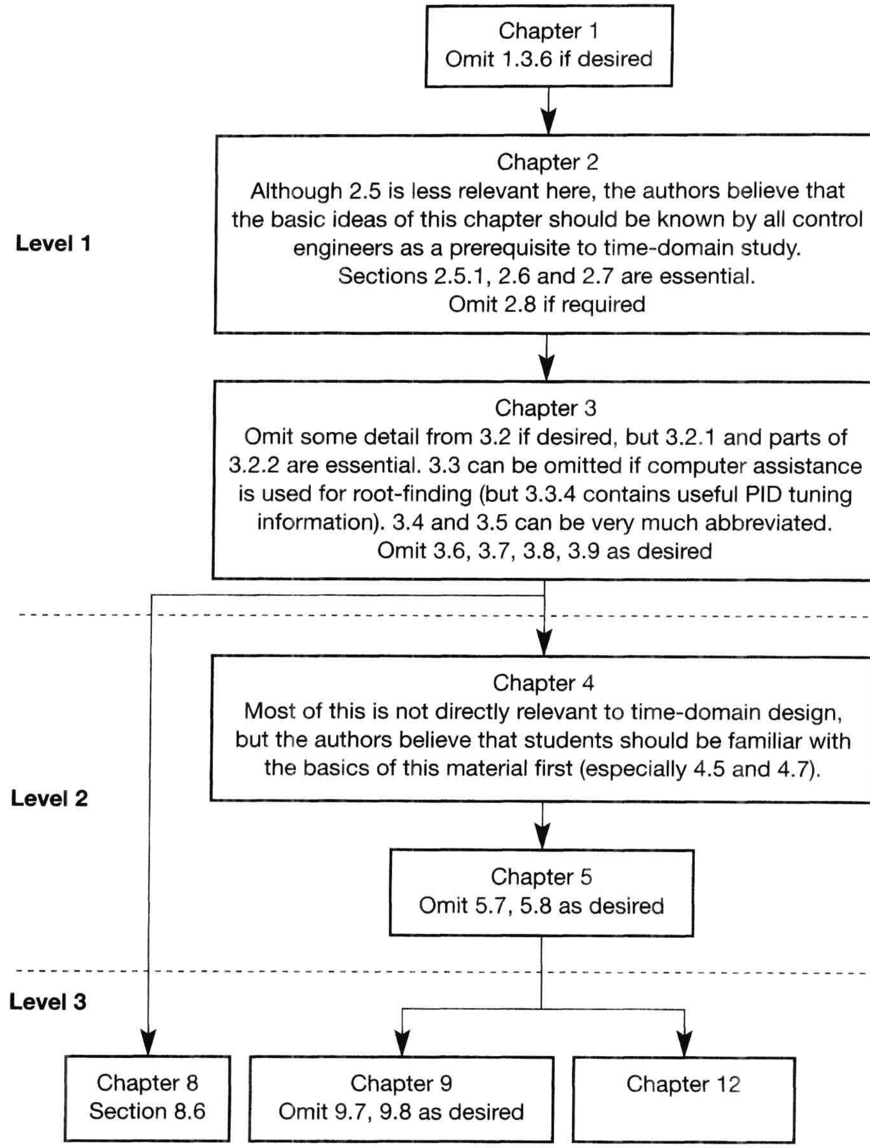

Figure P3 A mainly time-domain control course.

controllers in Chapter 7). With that proviso, Figure P4 suggests suitable routes. Note that Section 9.7 contains some useful comments on general matters of implementation, and these could usefully be extracted, even if the subject matter on which that section is based is not understood.

Finally, for those requiring a nonlinear control course, Figure P5 illustrates the possibilities. Again, it is necessary to study various basic aspects of frequency-domain and/or time-domain control (as indicated in Figure P5) depending upon the nonlinear techniques chosen for study.

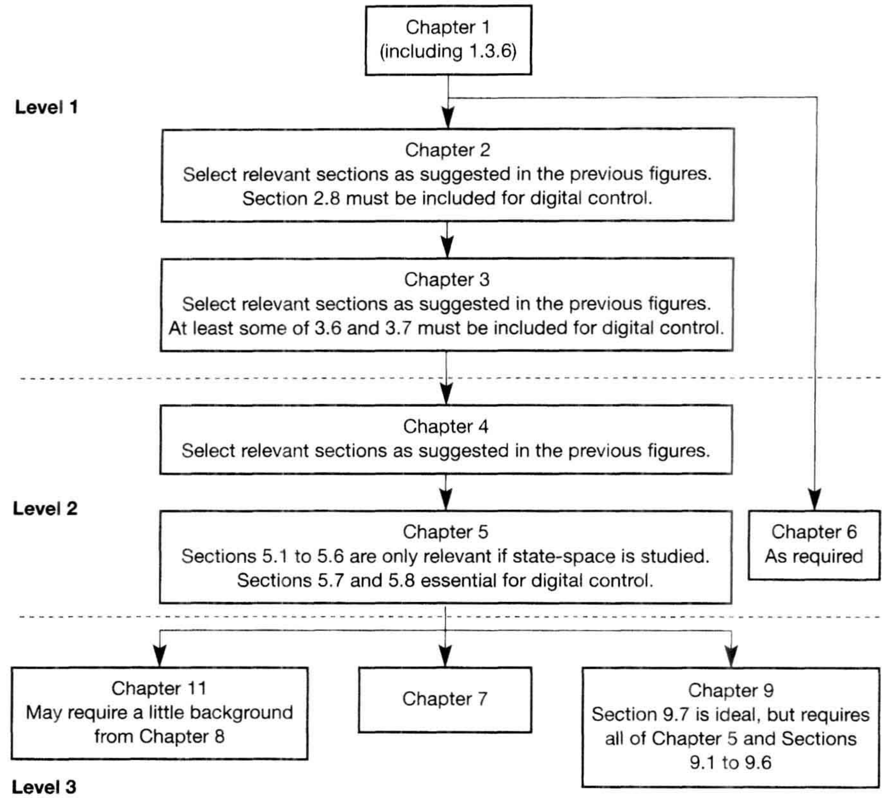

Figure P4 A digitally biased control course.

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The production of a tome such as this is impossible without the contributions of many people. Those to be thanked for helping to shape our views and for their helpful suggestions include our own teachers, all our former industrial colleagues and our present industrial contacts and academic colleagues (in the case of Ken Dutton, this includes thanking Bill Barraclough, who taught some parts of Ken's control engineering degree in the early 1970s!). Not to be forgotten are several generations of students (undergraduate and postgraduate), from whose questionings we never cease to learn, and whose comments have helped in organizing the presentation of much of the material. A special thanks goes to those academics at other institutions who have tested some of our material on their own students and provided invaluable feedback.

We are also grateful to the School of Engineering of Sheffield Hallam University for providing the necessary computing and printing facilities to allow production of the draft. The staff of Addison Wesley Longman in the UK have also been extremely helpful whenever required, and the helpful comments of the referees of the proposal and the draft were also very gratefully received (and many were acted upon!).

Lastly, but certainly not least, we heartily thank all our families and friends who endured several hundred hours of our time being diverted away from them and from our normal activities, towards the writing of this book. Without their

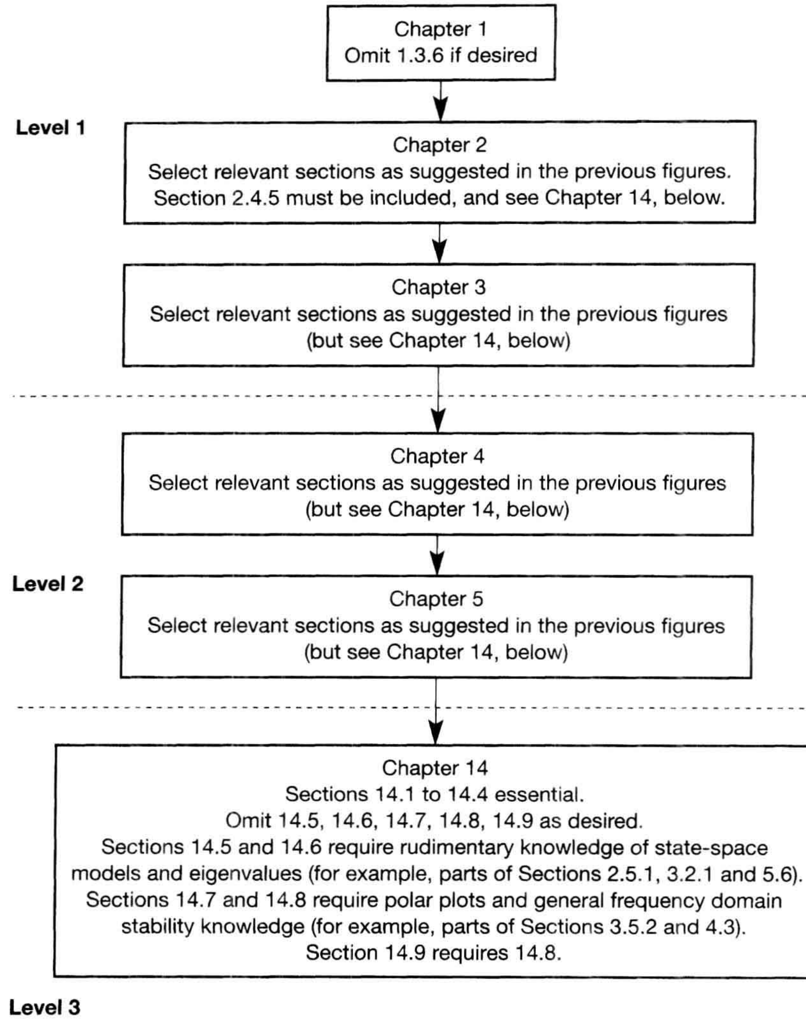

Figure P5 A nonlinear control course.

continued patience and understanding, the writing of the book really would have been impossible.

The authors would be very happy to receive any comments or suggestions. These can be sent to Ken Dutton via the email address: k.dutton@shu.ac.uk

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1 Introduction

1.1 PREVIEW
1.2 CONTROL ENGINEERING – TERMINOLOGY
1.3 A STROLL THROUGH THE FIELD OF CONTROL
1.4 CONCLUSIONS
1.5 PROBLEMS

1.1 Preview

Each chapter in this book begins with a section like this. Usually (especially in the later chapters) it tells the reader what he or she needs to know before reading the chapter, in order to be able to understand it fully. Generally, once the reader has read Chapter 1, further chapters will only need knowledge gained earlier in the text. These 'preview' sections also indicate what the reader should expect to learn by studying the chapter. In addition, they list any mathematical techniques which will be introduced for the first time during the chapter.

Sometimes, the preface to a book is there because people expect there to be a preface. In this book, the preface also contains much useful information about the structure and use of the book. It shows that major topics have been divided between several chapters. When this occurs, reference is made to the next chapter in which the topic is developed.

In this chapter the reader will learn:

- the fundamental concepts and terminology of control engineering – some of which are necessary to understand fully the remaining items in this list!
- that *models* in the form of mathematical equations and block diagrams can often be found, which adequately describe how a real-world 'system' (yet to be defined) behaves
- that these models might be useful in designing a control system – which will alter the behaviour of the real-world system, in some desired manner
- how the performance of control systems can be specified
- that there are very many different aspects of control engineering that can be studied.

NEW MATHEMATICS FOR THIS CHAPTER

Many control engineering texts have early chapters which introduce all the mathematical techniques needed in the book. This text is not like that. Instead, we do not introduce any mathematical tool until it is needed in a control engineering context. In this way, the reader only covers the mathematics actually required in order to understand his or her chosen set of topics; and covers it at the time it is used. Sometimes, the new mathematics is introduced in the text, where it is needed. At other times, for more complicated or involved aspects of mathematics, an appendix is used so as not to upset the flow of the text too much.

There is very little mathematics in this introductory chapter, but we do mention one or two mathematical ideas, and use one or two equations. To understand this chapter, the reader need only be familiar with the basic ideas of algebra (forming and manipulating equations) and calculus (simple integration and differentiation). Matrix algebra is mentioned in passing, but not actually used. If required, Appendix 1 gives a unique slant on matrix algebra from a control engineering viewpoint. However, it might be better to delay study of that until it is really needed.