

Lial/Miller

Intermediate Algebra / 4th



Intermediate Algebra

Fourth Edition

Margaret L. Lial

Charles D. Miller

American River College

Scott, Foresman and Company

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London, England

To the Student

The *Study Guide: Intermediate Algebra*, Fourth Edition, and the *Student Solutions Manual* are available from your local college bookstore. These books can help you study and review the course material.

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Intermediate Algebra

Fourth Edition

Preface

Ever since the first edition of *Intermediate Algebra*, we have tried to produce a book that is both *useful* to the student and *helpful* for the instructor. For the student, we have written a book with explanations and examples that are clear, direct, and to the point. The exercise sets are carefully graded in difficulty, with examples corresponding to exercises. For the instructor, we have produced a complete instructional package. The text is carefully laid out to match standard courses, with the sections only including topics that an instructor would normally expect to find in each section. The supplemental package offers those items we have found useful in our own classes: tests, complete solutions to 75 percent of the exercises, additional exercises keyed to objectives, a *Study Guide*, audiotapes, and videotapes.

KEY FEATURES


Word Problems A problem-solving approach gives students early and repeated experience in solving applied problems. A list of steps for solving word problems is first presented in Chapter 2. Throughout the text students are given practice translating English words into algebraic symbols. In this way, students see word problems early and gradually improve their problem-solving skills.

Examples More than 425 worked-out examples clearly illustrate concepts and techniques. Second color is used to identify pertinent steps within examples, as well as to highlight explanatory side comments.

Keyed Objectives Each section opens with a list of skills that students should learn in that section. These objectives are keyed to the appropriate portions of each section with symbols such as **1** or **2**.

EXERCISES

Graded Exercises The range of difficulty in the exercise sets affords students ample practice with drill problems. Then they are eased gradually through problems of increasing difficulty to problems that will challenge outstanding students. More than 4200 drill exercises and 350 word problems, keyed to examples, are included.

Calculator Exercises Calculator exercises have been included throughout the book. These optional exercises are identified with the symbol .

Review Exercises Beginning in Chapter 2, most exercise sets end with a few problems that help prepare students for the following section. For example, at the end of the section preceding the discussion of solving quadratic equations by factoring, students review the solution of equations such as $2x + 5 = 0$.

Chapter Review Exercises Extensive review exercises at the end of each chapter, more than 1000 in all, provide further opportunity for mastery of the material before students take an examination. These exercises are keyed to appropriate sections in the text.

Chapter Tests Sample tests, of a length comparable to that of actual classroom tests, have been made somewhat more difficult in this edition of the book. Nearly 300 test questions give additional practice to students.

SECOND COLOR

Second color is used pedagogically in the following ways.

- Screens set off key definitions, formulas, and procedures, helping students review.
- Color side comments within examples explain the structure of the problem.
- For clarity, the end of each example is indicated with a color symbol, ●.
- Warnings about common student errors are included in color when appropriate.

SUPPLEMENTS

Intermediate Algebra, Fourth Edition, has an extensive supplemental package that includes testing materials, solutions, and electronic media.

The **Instructor's Guide** features five tests for each chapter. One version is a multiple-choice test. In addition to the chapter tests, two forms of a final examination are given, as well as a diagnostic pretest. Answers to all tests are provided in columns that can be lined up with the answer blanks of the student tests. Answers to even-numbered exercises in the text are given in this guide.

Additional Exercises, keyed to objectives, are also available. Ten to 20 exercises for each objective in the book help students review exactly where their difficulties are.

A **Solutions Manual**, featuring complete solutions to all even-numbered exercises, is available to adopters.

A **Student Solutions Manual** has solutions to one half of the odd-numbered exercises in the text. Some students use this volume as an additional source of examples.

A **Study Guide**, in a semiprogrammed format, provides additional practice and reinforcement for students.

Professional-quality **videotapes** are available at low cost to users of this text. The tapes amplify the ideas in the book that cause students the most difficulty.

Audiotapes that cover all topics in the text are available at no charge to users of the book. Students who need help with a particular topic or who have missed class find these tapes help them master the material.

We thank the many users of the previous editions of this book who were kind enough to share their experiences with us. This revision has benefited from their comments and suggestions. (See the list of acknowledgments that follows the Preface.)

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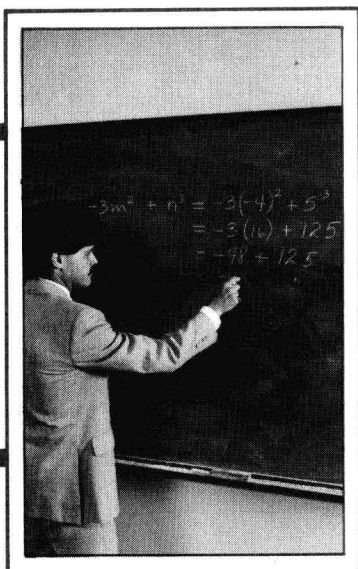
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The Real Numbers

Evaluating expressions is part of solving real-life problems with algebra.

1.1 Basic Terms

- 1 Write sets.
- 2 Use number lines.
- 3 Find additive inverses.
- 4 Use absolute value.
- 5 Know the common sets of numbers.

Algebra depends on symbols, and many of the symbols used in this book are introduced in this first chapter. A summary of all the symbols used is given in the Appendix.

1 A basic term used in algebra is **set**, a collection of objects. The objects in a set are the **elements** or **members** of the set. In algebra, the elements in a set are usually numbers, with braces, { }, used to enclose the elements. For example, 2 is an element of the set {1, 2, 3}.

A set can be defined either by listing or by describing its elements. For example,

$$S = \{\text{Oregon, Ohio, Oklahoma}\}$$

defines the set S by *listing* its elements. The same set might be *described* by saying that set S is the set of all states in the United States whose names begin with the letter "O."

Set S above has a limited or **finite** number of elements. Some sets contain an unlimited or **infinite** number of elements, such as

$$N = \{1, 2, 3, 4, 5, 6, \dots\},$$

where the three dots show that the list continues in the same pattern. Set N is called the set of **natural numbers**, or **counting numbers**. A set containing no elements, such as the set of natural numbers less than 1, is called the **empty set**, or **null set**, written \emptyset .

To write the fact that 2 is an element of the set $\{0, 1, 2, 3\}$, use the symbol \in :

$$2 \in \{0, 1, 2, 3\}.$$

The number 2 is also an element of set N above, or

$$2 \in N.$$

To show that 0 is *not* an element of set N , draw a slash through the symbol \in :

$$0 \notin N.$$

Two sets are **equal** if they contain exactly the same elements. For example,

$$\{1, 2\} = \{2, 1\},$$

because the sets contain the same elements. (The order doesn't matter.) Also, $\{1, 2\} \neq \{0, 1, 2\}$ (\neq means "is not equal to") since one set contains the element 0 while the other does not.

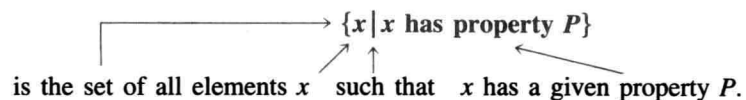
In algebra, letters called **variables** are often used to represent numbers. Variables also can be used to define sets of numbers. For example,

$$\{x \mid x \text{ is a natural number between 3 and 15}\}$$

(read "the set of all elements x such that x is a natural number between 3 and 15") defines the set

$$\{4, 5, 6, 7, \dots, 14\}.$$

The notation $\{x \mid x \text{ is a natural number between 3 and 15}\}$ is an example of **set-builder notation**. That is,



EXAMPLE 1 List the elements in each set.

(a) $\{x \mid x \text{ is a natural number less than 4}\}$

The natural numbers less than 4 are 1, 2, and 3. The given set is

$$\{1, 2, 3\}.$$

(b) $\{y \mid y \text{ is one of the first five even natural numbers}\} = \{2, 4, 6, 8, 10\}$

(c) $\{z \mid z \text{ is a natural number at least } 7\}$

The set of natural numbers at least 7 is an infinite set; write it with three dots as

$$\{7, 8, 9, 10, \dots\}. \quad \bullet$$

Set A is a **subset** of set B if every element of A is also an element of B . The symbol \subseteq is used for subset, so $A \subseteq B$ says that A is a subset of B . A sketch of a set A that is a subset of a set B is shown in Figure 1.1.

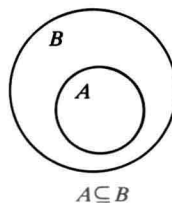


Figure 1.1

EXAMPLE 2

Write *true* or *false* for each statement.

(a) $\{1, 2, 3\} \subseteq \{1, 2, 3, 4, 5\}$

Every element of the set $\{1, 2, 3\}$ is also an element of the set $\{1, 2, 3, 4, 5\}$. Because of this, $\{1, 2, 3\}$ is a subset of $\{1, 2, 3, 4, 5\}$ and the given statement is true.

(b) $\{0, 1, 2\} \subseteq \{1, 2, 3, 4, 5\}$

The number 0 is an element of $\{0, 1, 2\}$ but not of $\{1, 2, 3, 4, 5\}$. The statement is false, or

$$\{0, 1, 2\} \not\subseteq \{1, 2, 3, 4, 5\}$$

($\not\subseteq$ indicates “is not a subset of”).

(c) $\{0, 1, 2\} \subseteq \{0, 1, 2\}$

Every element of the first set is an element of the second set, so the given statement is true. \bullet

Do not confuse the symbol \in , “is an element of,” with the symbol \subseteq , “is a subset of.” The symbol \in is used only between an element and a set, while \subseteq is used only between two sets. For example,

$$5 \in \{3, 4, 5, 7, 9\},$$

but

$$\{5\} \subseteq \{3, 4, 5, 7, 9\}.$$