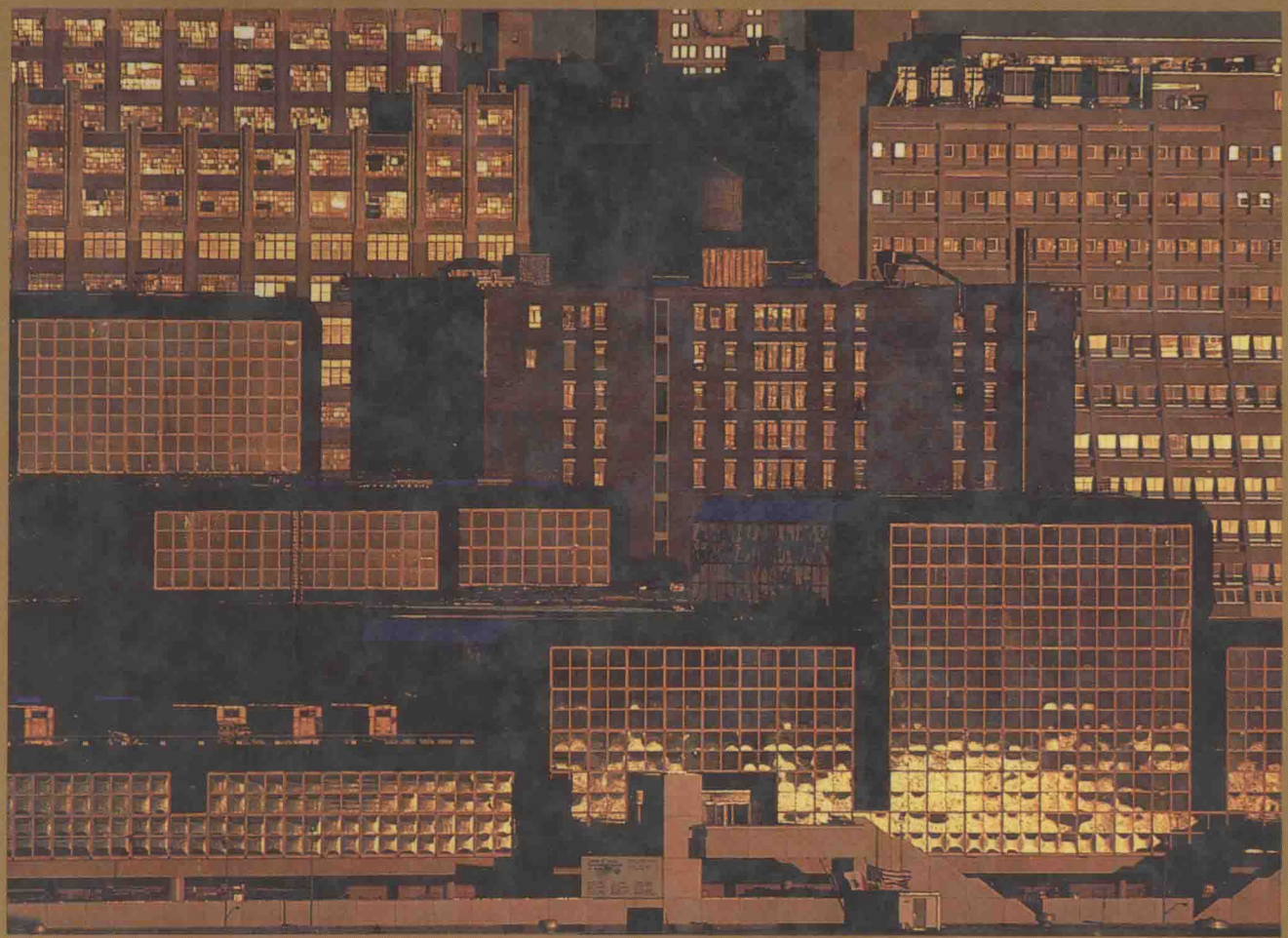
COLLEGE ALGEBRA AND TRIGONOMETRY WITH APPLICATIONS



L. MURPHY JOHNSON
ARNOLD R. STEFFENSEN

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L. MURPHY JOHNSON
ARNOLD R. STEFFENSEN
Northern Arizona University

Scott, Foresman and Company
Glenview, Illinois
Boston
London

For the Student

To help you study and understand the course material, a Solutions and Study Guide, *by Joseph Mutter, is available from your college bookstore. This book provides complete, step-by-step solutions to more than half of the odd-numbered exercises in the text, detailed chapter summaries, and practice chapter tests with complete solutions.*

To Barbara, Barbara, Becky, Cindy, and Pam

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Cover photo © 1986 Lee Marshall, New York

Library of Congress Cataloging-in-Publication Data

Johnson, L. Murphy (Lee Murphy)
College algebra and trigonometry with applications.

Includes index.
1. Algebra. 2. Trigonometry. I. Steffensen, Arnold R. II. Title.
QA154.2.J56 1988 512.9 87-13917
ISBN 0-673-18355-6

Printed in the United States of America.
1 2 3 4 5 6—RRC—92 91 90 89 88 87

Preface

College Algebra and Trigonometry with Applications is designed to provide comprehensive coverage of the usual topics in algebra and trigonometry needed by students for later courses in mathematics, engineering, business, statistics, or the natural sciences. Students with two years of high school algebra or its equivalent should have the necessary prerequisite skills. The text is organized for maximum instructional flexibility. More than enough material is included for a one-semester or two-quarter course.

Chapter 1 provides a review of basic algebra, which some classes may cover quickly or skip altogether. Chapters 2 through 5 present the major topics in college algebra including equations and inequalities, functions and their graphs, the theory of polynomials, and polynomial, rational, exponential, and logarithmic functions. Chapters 6 through 8 provide a thorough treatment of trigonometry; the trigonometric functions are introduced via right triangles, followed by the unit circle approach. For added flexibility, topics in analytic geometry are presented separately in Chapter 9. The text concludes with a variety of other algebraic topics including systems of equations and inequalities, matrices, determinants, sequences, series, and probability. Applications provide practical motivation throughout the book.

FEATURES

The text is written informally; explanations are carefully worded to ensure student comprehension. Second color is used pedagogically to highlight important steps and emphasize methods and terminology. The many figures and graphs are labeled for easy reference and employ color to clarify the concepts presented. Cautions warn students of common mistakes and special problems, while Notes provide additional explanations or other pertinent information.

Examples

The text contains over 650 carefully selected examples with detailed step-by-step solutions and helpful side annotations.

Exercises

There are over 4600 exercises in the text. The exercise sets are carefully graded and begin with paired routine problems that are followed by a variety of challenging extension problems and numerous applications. A set of For Review exercises is included at the end of most exercise sets to help students review previously covered material or prepare for the next section. A collection of review exercises concludes each chapter. Answers to odd-numbered section exercises and to all For Review and Chapter Review exercises are included at the back of the book.

Applications

To demonstrate the usefulness and practicality of mathematics, applications have been given special attention in this text. Over 700 relevant applied problems from such diverse areas as business, engineering, geology, physics, chemistry, medicine, forestry, and agriculture are included in the chapter introductions, examples, and exercises.

Calculators

Calculators are discussed at appropriate places throughout the text, and illustrations are included for both Algebraic Logic and Reverse Polish Notation. Calculator exercises are not specifically marked, however, since students should learn when to use and when not to use calculators. Appendices on the use of logarithmic and trigonometric function tables are provided for instructors who prefer that their students learn these techniques.

SUPPLEMENTS

For the Instructor

The **Instructor's Guide** contains a Placement Test, four different but equivalent tests for each chapter, two final examinations, an extensive bank of additional problems, and answers to all test items and even-numbered text exercises. As an alternative to the tests in the Instructor's Guide, the **Computer-Assisted Testing System (CATS)** can be used with Apple and IBM computers to construct and print tests. More than 50 **overhead transparencies** featuring key figures from the text are also provided for classroom lectures and presentations.

For the Student

The **Solutions and Study Guide** contains complete, step-by-step solutions to more than half of the odd-numbered text exercises, detailed chapter summaries, and practice chapter tests with complete solutions.

ACKNOWLEDGMENTS

We extend our sincere gratitude to those who helped develop this book by reviewing all or part of the manuscript. We have implemented many of your suggestions to the great benefit of the text.

Richard D. Armstrong, St. Louis Community College
Edgar M. Chandler, Phoenix College
Ben Cornelius, Oregon Institute of Technology
Mary Coughlin, University of Toledo
Michael B. Curry, Pima Community College
John A. Dersch, Jr., Grand Rapids Junior College
Karen R. Fawcett, University of Southern Mississippi
Linda Holden, Indiana University
Charles M. Lindsay, Coe College
Carolyn F. Neptune, Johnson County Community College
Charles V. Peele, Marshall University
William Radulovich, Florida Community College
James V. Stewart, Oregon Institute of Technology
Eric Tellenbach, Citrus Community College

We extend special appreciation to Joseph Mutter for reviewing the entire manuscript and offering numerous suggestions for improvements, for writing the supplements, and for checking the problems. Thanks go to Diana Denlinger Vanlandingham and Gail Dickerson for typing the manuscript and supplements.

To everyone at Scott, Foresman and Company we are greatly indebted. Special thanks go to Jack Pritchard, Steve Quigley, Terry McGinnis, Sarah Joseph, and Ellen Pettengell.

Finally, we are deeply indebted to our families and in particular to our wives, Barbara and Barbara, who have given us unceasing support, time, and encouragement over the years.

L. Murphy Johnson

Arnold R. Steffensen

To the Student

During the past several years we have taught college algebra and trigonometry to more than 1500 students having a variety of career choices. Some were taking mathematics to satisfy graduation requirements, while others were preparing for more advanced courses in mathematics, science, business, or engineering. Regardless of your educational goals, this text has been written with you, the student, in mind. The material is introduced gradually, building from basic to more advanced skills. We have tried to demonstrate the relevance and usefulness of mathematics throughout the text by including practical everyday applications. As you begin this course, keep in mind these guidelines that are both necessary and helpful.

GENERAL GUIDELINES

1. Mastering algebra and trigonometry requires motivation and dedication. Just as an athlete does not improve without commitment to his or her goal, an algebra student must be prepared to work hard and spend time studying.
2. Algebra and trigonometry are not learned simply by watching, listening, or reading; *they are learned by doing*. Use your pencil and practice. When your thoughts are organized and written in a neat and orderly way, you have taken a giant step toward success. Be complete and write out all details. The following are samples of two students' work on an applied problem. Can you tell which one was more successful in the course?

Student A

Let n = number of units produced during the month

C = total cost per month

$C = 10n^2 - 100n - 2000$

$10{,}000 = 10n^2 - 100n - 2000$

$0 = 10n^2 - 100n - 12{,}000$

$0 = n^2 - 10n - 1200$

$0 = (n-40)(n+30)$

$n - 40 = 0$ or $n + 30 = 0$

$n = 40$ or $n = -30$

Thus, 40 units were produced.

Student F

$10(10{,}000)^2 - 100(10{,}000) - 2000$

$=$

$n =$ units

$n = 10n^2 - 100n - 2000$

$0 = 10n^2 - 101n - 2000$

3. A calculator is useful in any course in algebra or trigonometry. Become familiar with your calculator by consulting your owner's manual. Use the calculator as a time-saving device for work with decimals or complicated functions, but do not become so dependent that you use it for simple calculations that can be done mentally. Learn when to use and when not to use your calculator. See "A Word About Calculators" for more information about how calculators can be used with this text.

SPECIFIC GUIDELINES

1. As you begin to study each section, look through the material for a preview of what is coming.
2. Return to the beginning of the section and study the text and examples carefully. The side comments in color will help you if something is not clear.
3. Periodically you will encounter a CAUTION or a NOTE. The CAUTIONs warn you of common mistakes and special problems to avoid. The NOTEs provide pertinent information or additional explanations.
4. After you have completed the material in the section, check your mastery of the skills and apply what you have learned by working the exercises assigned by your instructor. Answers to the odd-numbered problems are at the back of the text. Complete worked-out solutions to selected odd-numbered problems are also available in the *Solutions and Study Guide*.
5. Exercises marked For Review, located at the end of most exercise sets, help keep previously covered materials fresh in your mind and often prepare you for the next section.
6. After you have completed a chapter, review each section and work the CHAPTER REVIEW EXERCISES. Answers to all these exercises are at the back of the text. To help you prepare for tests, additional chapter review material is also included in the *Solutions and Study Guide*.

If you follow these suggestions and work closely with your instructor, you will greatly improve your chances for success in the course.

A Word About Calculators

It is assumed that most students will have a hand calculator in this course. Although it is not absolutely essential, your work will be easier if you use a calculator. The major difference between the types of calculators is in the way they perform various operations. Perhaps more desirable at this level, since the order of operations is the same as in algebra, is the type that uses Algebraic Logic (ALG). The alternative system, Reverse Polish Notation (RPN), is preferred by many mathematicians and professionals, however. Each system, with its advantages and disadvantages, will perform the calculations necessary in this course. Throughout the text we illustrate both systems using ALG for Algebraic Logic and RPN for Reverse Polish Notation. As an example, we show the sequence of steps used in each system to compute

$$\frac{(2)(4.5) - (1.3)^2}{5\sqrt{3}}.$$

		Display
ALG:	2 [×] 4.5 [−] 1.3 [x^2] [=] [÷] 5 [÷] 3 [$\sqrt{}$] [=] →	0.8440861
RPN:	2 [ENTER] 4.5 [×] 1.3 [x^2] [−] 5 [÷] 3 [$\sqrt{}$] [÷] →	0.8440861

Notice that RPN calculators use an [ENTER] key instead of the [=] key found on ALG calculators. This is an essential difference between the two operating systems. Other variations in the types of keys are strictly notational. For example, to change the sign of a number (for entering negative numbers), some calculators have a [+/−] key, while others have a [CHS] key. Also, one calculator uses the [STO] key to place a number in memory, while another has an [M] key. We will try to point out some of the differences that arise as we consider various computations. However, since it is impossible to mention all of these differences, the best advice is to read your owner's manual.

With calculators, slight variations in accuracy due to rounding differences are bound to occur. Most of these will appear in the seventh or eighth decimal place and should not be of much concern. Throughout the text we have not rounded results until the final step, holding calculated values in memory. Even with this agreement, small variations due to individual calculator differences may arise. Don't panic if your calculator gives an answer that disagrees slightly with what we have shown.

Finally, keep in mind that a calculator is a tool for doing complicated computations; it does not think and only reacts to your input. Do not become so dependent on your calculator that you reach for it to make simple computations that can be made mentally. You must learn when a calculator should and should not be used and when your results are reasonable and appropriate.

Contents

CHAPTER

1 REVIEW OF FUNDAMENTAL CONCEPTS

A knowledge of algebra not only gives us a foundation for the study of more advanced mathematics but also provides the tools for solving many applied problems in business, science, and engineering. Consider the following applications.

BUSINESS ▶

A payroll office administrator needs a formula for calculating the new salary of employees who have received an 8% raise.

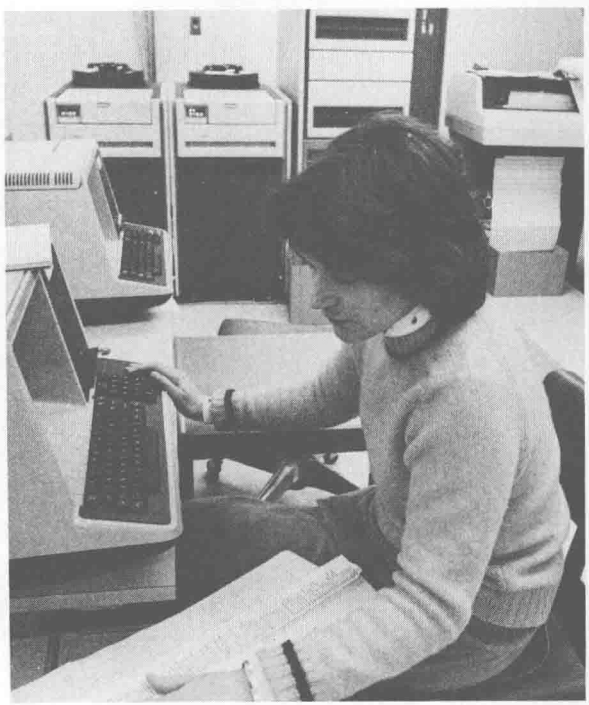
Let x = employee's previous salary,

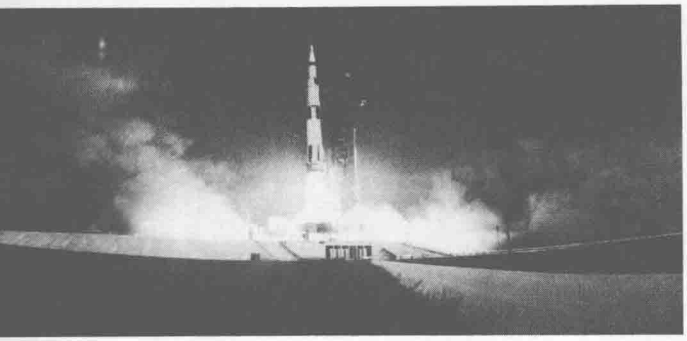
$0.08x$ = employee's raise.

The new salary is found by adding the raise to the old salary.

$$\text{new salary} = x + 0.08x = 1.08x$$

For example, if an employee's old salary x was \$32,000 per year, the new salary $1.08x$ would be $(1.08)(\$32{,}000) = \$34{,}560$.





◀ ENGINEERING

The height in feet of a rocket t seconds after firing is given by the expression $-16t^2 + 180t$. Find the height 4 seconds into its flight.

$$\begin{aligned} \text{height} &= -16t^2 + 180t \\ \text{height at 4 sec} &= -16(4)^2 + 180(4) \\ &= -256 + 720 \\ &= 464 \text{ ft} \end{aligned}$$

In this chapter we review number systems and their basic properties, which serve as a foundation for our work.

1.1 The Real Number System

Because sets of numbers are fundamental to our study of algebra, we begin by reviewing them briefly. Remember that a **set** is a collection of objects called **elements.** The elements of a set are listed within braces, { }. The most basic sets of numbers are given here.

$N = \{1, 2, 3, \ldots\}$ **Natural (counting) numbers**

$W = \{0, 1, 2, 3, \ldots\}$ **Whole numbers**

$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$ **Integers**

$Q = \left\{\frac{a}{b} \text{ such that } a \text{ and } b \text{ are integers with } b \neq 0\right\}$ **Rational numbers**

$P = \{x \text{ such that } x \text{ is not rational}\}$ **Irrational numbers**

$R = \{x \text{ such that } x \text{ is rational or irrational}\}$ **Real numbers**

In addition to these sets of numbers, we often have occasion to refer to the set of *negative integers,* $\{\ldots, -3, -2, -1\}$, and the set of *positive integers,* $\{1, 2, 3, \ldots\}$. Notice that the three dots indicate that the pattern continues.

The set of **rational numbers** includes the set of integers together with all quotients of integers. Every rational number can be written as a fraction or a decimal. For example, $\frac{3}{8}$ can be written as 0.375 (dividing 3 by 8) and $\frac{3}{11}$ as 0.2727... (dividing 3 by 11). The decimal 0.375 is called a **terminating decimal** because the sequence of digits comes to an end, while 0.2727... is a **repeating decimal** because the block of digits 27

repeats indefinitely. Repeating decimals are often written with a bar over the block of digits that repeats. For example,

$$\frac{3}{11} = 0.\overline{27} \quad \text{and} \quad \frac{1}{3} = 0.\overline{3}.$$

Every rational number has a decimal form that either terminates or repeats. This property is sometimes used to define the set of rational numbers.

Numbers that are not rational, that is, that *cannot* be written as a quotient of integers, are called **irrational numbers.** An irrational number cannot be written as a terminating or repeating decimal. One of the best known irrational numbers is π, the ratio of the circumference of any circle to its diameter. Numbers such as $\sqrt{2}$ and $\sqrt{26}$, square roots of positive integers that are not perfect squares, are also irrational.

The Set of Real Numbers

The set of **real numbers** consists of the rational numbers together with the irrational numbers. The relationships among the sets of numbers we have discussed is displayed in Figure 1.

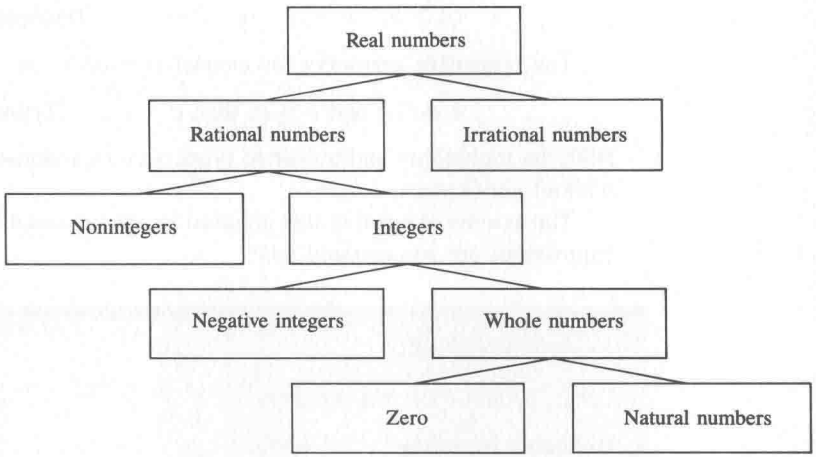


Figure 1 The Real Number System

An excellent means of displaying numbers and showing some of their important properties is a **number line,** as shown in Figure 2. The **origin** is labeled zero and unit lengths in both directions are marked off. Points to the right of zero are identified with positive numbers, while points to the left of zero correspond to negative numbers.

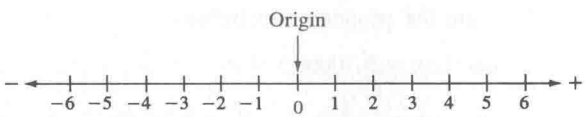


Figure 2 Number Line

Every real number can be identified with exactly one point on a number line, and every point on a number line corresponds to exactly one real number. Figure 3 shows a number line with points corresponding to several real numbers plotted on it.

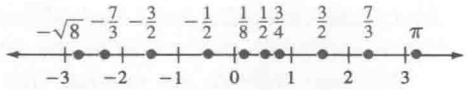


Figure 3 Points on a Number Line

Numbers that are **equal** ($a = b$) correspond to the same point on the number line. If a is to the left of b, we say a is **less than** b and write $a < b$. We also say that b is **greater than** a and write $b > a$. More formally, $a < b$ or $b > a$ if $b - a$ is a positive number ($b - a > 0$).

If a and b are any real numbers the **trichotomy** property states that exactly one of the following holds.

$$a > b, \quad a = b, \quad \text{or} \quad a < b \qquad \textbf{Trichotomy property}$$

The **transitive property** for inequalities states that

$$\text{if } a < b \text{ and } b < c, \text{ then } a < c. \qquad \textbf{Transitive property}$$

Both the trichotomy and transitive properties are **axioms,** properties that are accepted without verification.

The axioms of equality that are used in solving equations and simplifying algebraic expressions are summarized next.

Axioms of Equality

Let a, b, and c be real numbers.

Reflexive property	$a = a$
Symmetric property	If $a = b$, then $b = a$.
Transitive property	If $a = b$ and $b = c$, then $a = c$.
Substitution property	If $a = b$, then either may replace the other in any statement without affecting the truth of the statement.

EXAMPLE 1 State the property illustrated.

(a) If $x = 5$, then $5 = x$. Symmetric property

(b) $-2 < -\frac{1}{2}$ and $-\frac{1}{2} < 3$ implies $-2 < 3$. Transitive property for $<$