

K.K. Valtukh

**Marx's
Theory of
Commodity
and Surplus-
Value**

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Commodity
and Surplus-
Value**

**Formalised
Exposition**



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ОЧЕРК ТЕОРИИ ТОВАРА И ПРИВАВОЧНОЙ СТОИМОСТИ
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INTRODUCTION

The aim of this book is to present the main points of Marxist-Leninist views on the political economy of capitalism using modern mathematical models. This presentation additionally substantiates the theory scientifically and gives it new opportunities for further development.

When making increasingly extensive use of modeling as a method of cognizance the followers of Marx and Lenin rely, on the one hand, on modern developments in mathematics and technical cybernetics, and the experience of using them in the other sciences and, on the other hand, on the fundamental methodological achievements in this respect accumulated by economic theory itself. The classics of Marxism-Leninism have left us a very rich legacy of ideas in this field. *There is no room here to analyse fully the use of mathematics in the economic works of Marx and Lenin. Let us merely point to certain fundamental aspects of their experience in the formalisation of economic theory.*

There is almost no chapter in Marx's *Capital* where he has not employed formalised methods, i.e., models, for describing the subject. First, there are the specific non-mathematical formulae for economic phenomena and processes (formulae to express the various forms of value; the formula for the social exchange of matter under commodity production $C - M - C$; the general formula for capital $M - C - M'$; the formulae for the circulation of the various kinds of capital). Second, there are directly mathematical models—numerical and algebraic. With their aid, Marx in fact analysed all the major phenomena and processes of the capitalist economy studied in *Capital*. For instance, there are the formulae for commodity value ($w = c + v + m$), capital ($K = c + v$), the rate of surplus-value, the value composition of capital, the velocity of the circulation of capital, the annual rate of surplus-value, the conditions for simple and extended reproduction, capitalist costs of production, the rate of profit, prices of production, etc. From his analy-

sis of the characteristics of these models Marx derives the main laws and trends in the evolution of the capitalist economy, i.e., the laws of the changing rate of surplus-value and rate of profit, of the realisation of social capital, and the like.

Whole chapters of *Capital* are devoted to an analysis of formal dependences. For example, in Volume I of *Capital* (Chapter IX), three laws relating to the rate and mass of surplus-value are derived from an analysis of the formula for the mass of surplus-value in two specially developed modifications; Chapter XVII is based on the use of magnitudes of a partial derivative-type to examine the dependences of the relationship between the price of labour power and surplus-value on the length of the working day, intensity and productivity of labour (although Marx does not derive these partial derivatives formally).

In a number of cases, Marx made use of numerical examples and charts that are not generally expressed in the shape of formal models and, in this sense, retain the nature of illustrations. They are, however, brought to a level, the very next step beyond which would involve the building of a model. As an example, let us mention the charts used to analyse the formation of the average rate of profit and those of simple and extended reproduction. Marx also drew up a numerical chart of a four-sector economy constituting a prototype of the input-output table with more than two branches of business; in this chart, one branch of business is represented by two industrialists, the description being more general than in the usual modern input-output tables and representing the first step towards building a model of capitalist competition.¹

Lenin, of course, generalised Marx's charts of the reproduction of social capital for the case of a growth of the organic composition of capital. Let us also point to the numerical chart of an economy consisting of three branches of industry, with six producers, which Lenin used to analyse the problem of the transformation of subsistence economy into commodity economy and then into capitalist economy.² This

¹ See: Karl Marx, "Outlines of the Critique of Political Economy", Karl Marx, Frederick Engels, *Collected Works*, Vol. 28, Progress Publishers, Moscow, 1986, pp. 362-66.

² See: V. I. Lenin, "On the So-Called Market Question", *Collected Works*, Vol. 1, Progress Publishers, Moscow, 1972, pp. 79-111.

chart again leads to the verge of the creation of an appropriate model.

There are now, of course, much greater opportunities for systematic use of mathematics than at the time of the classics of Marxism-Leninism. These opportunities first of all result from the development of mathematics itself, especially from the achievements in the field of set theory, probability theory, modern algebra, and the theory of vector spaces, as well as from their application in describing numerous subjects in various sciences. They also result from the mastering of mathematics by economists and the involvement of mathematicians in economic studies. These processes have taken some time to develop. At the same time, it should be emphasised that the legacy of the classics of Marxism-Leninism relating to the methodology in the field of the employment of formal techniques retains its great importance. The main thing here is that, using relatively simple techniques, Marx and Lenin formalised not only the description of the technological component of the economy, but also that of the social relations constituting its form; moreover, the dynamic and probabilistic properties of the economy, generated by law-governed technological progress, are taken into account.

The legacy of the classics of Marxism-Leninism contains, above all, the principles of scientific abstraction, of constructing pure subjects, which is a necessary condition for creating productive models, especially theoretical ones. The attentive reader of *Capital* cannot but notice the thoroughness with which Marx, every time, points to specific features of the real economy from which he abstracts. He demonstrates in particular that, if they were taken into account, they would not change the conclusions of the corresponding step in the study; when further considering problems lying closer to the surface, in the substantial analysis, Marx consequently makes use of the factors he initially disregarded to specify and develops his previous conclusions on this basis, in accordance with the general methodology of ascent from the abstract to the concrete.

To illustrate this let us merely point to the transition from the formula for the rate of surplus-value to the formula for the annual rate of surplus-value—to that of the rate of profit without regard for the velocity of the circulation of capital—to that of the annual rate of profit—to that of the average rate of profit and the price of production—to that of

the rate of profit with regard not only for productive capital but also to capital of circulation —to that of the rate of profit with regard not only to industrial but also to merchant's capital—to the formalised description of the division of average profit into interest and the profit of enterprise—to the formalised description of the transformation of a part of surplus-value into ground rent, with the appropriate specification of the description of the average rate of profit, etc.

It is of profound general methodological importance that Marx was able to present, in a formalised manner, such fundamental points of his socio-economic theory as the concept of the exploitation of the proletariat by the class of capitalists, the general capitalist production development trends (growth of the organic composition of capital and its effect on the general rate of profit), with the ensuing conclusions relating to the class struggle of the proletariat. To create theoretical socio-economic models to express the fundamental causes and inherent laws of the totality of phenomena observed as they develop is the only way to build up a methodologically sound theory. Such models cannot be replaced by empirical ones for formulating a theory. The latter models reflect only the surface of the phenomena, although these do have a certain role to play in science.

The formalised description of the subject at the level of its laws has opened up the natural way to relevantly expressing its probabilistic and dynamic nature. Marx repeatedly indicated directly that the economic laws of capitalism operate through a chaos of chance occurrences as long-term trends, as blindly acting law of mean numbers. The building of theoretical models to reflect the average result of numerous random fluctuations permits deterministic formalisation techniques (both relatively simple and the most developed) to be employed to describe the probabilistic environment.

As the models are built as theoretical ones, expressing the typical properties of the subject, their use means that the national economic nature of economic laws can be reflected. When considering private capital and formally expressing its specific features, Marx disregarded any branch or other local peculiarities; when examining the departments and branches (studying social reproduction, the overall rate of profit, ground rent) he took them as being interconnected with the other branches, and presented their system as a national economic whole. This saves his models from the inev-

itable shortcomings engendered when local subjects and individual parts of the economy are described separately.

The methodological achievements of the classics of Marxism-Leninism in the use of modeling techniques are especially important because they employed them to build the actual *theory* of social production.

In order to distinguish theoretical models from empirical ones, the very concept of theory should be discussed as a certain stage in the cognizance of external objects. Science widely accepts the concept of theory in the form imparted to it by Albert Einstein. Theory is characterised, first, by the attribute of external justification, second, by internal perfection, the former, i.e., accordance with the properties of the facts that science already knows, being a general requirement that science must fulfil at all the stages of acquiring knowledge. Theory as such is specifically characterised by the fact that it reduces the whole set of well-known regularities to a substantially smaller number of their causes (inner laws); theory is the more perfect the smaller the number of assumptions with which the empirically given properties of an object can be explained. This concept of theory, being a formal one, provides a profound answer to the essential definition of theoretical knowledge within the system of stages of cognizance: this is knowledge of the essence of phenomena, which explains their surface and allows specific features of phenomena not yet observed, including the results of human practice, to be forecast.

Since theory explains and forecasts the set of the object's empirically discovered properties from its relatively small number of unobservable, internal properties, the work on building a theory logically consists only in, first, formulating some system of assumptions; second, analysing the properties of this system, deriving as an advanced system of conclusions as possible from the assumptions (for actual verification and practical use). That is why the mathematical form of the building, development, and exposition of a theory is, in principle, suited to its concept. The level of mathematical formalisation of a theory depends, of course, on the development level of mathematics itself, on the degree to which all the logical achievements of human thinking have been mastered. Mathematics seems, in its most general, abstract sections, to develop, in fact, as specific logical techniques created by science as suitable forms for building a theory. (This is but one aspect of the development of mathema-

tics which is, at the same time, abstraction of the quantitative side of various external phenomena and creates techniques for carrying out applied calculations.)

In striving to create sufficiently elaborated theories, economic science cannot but make use of the entire set of opportunities offered by modern mathematics. At the same time, special research justifies the assertion that economic theory cannot, today, be built up only in the form of a system of mathematically formulated axioms and theorems. Moreover, modern mathematics fails to provide the techniques required precisely for the most general and profound results of economic theory.

The most advanced sector of economic theory is the theory of the capitalist mode of production which, at the same time, contains the fundamental methodological ideas of economic theory as a whole. This theory answers the general concept to the highest degree: all the major properties of its subject are derived from one (certainly internally complex) assumption about the commodity as the cell of the bourgeois economy, i.e., they are represented as the development of the internal contradiction between the use-value and value of the commodity. Lenin saw this as a model of any theory in general.

A formalised presentation of the political economy of capitalism using mathematics broadly is, in our opinion, of special scientific importance. By giving a more sophisticated form to this theory, it opens up broad scope for its further development on the basis of the potent, multi-faceted, sophisticated, logical techniques of mathematics and computers. The path to the application of theory (in particular, for forecasting purposes) is substantially shortened when it is represented by models. Theory is directly involved in the flow of mathematically described research, which is of great importance for its comprehension by both economists and scientists engaged in other fields. This is also of importance for its creative mastering by university students in both economic and other studies. Lastly, it offers further opportunities for Marxists to succeed in the ideological struggle against bourgeois economic doctrines; in particular, it deprives bourgeois economists of the opportunity to play on the difference in the mathematical level of the two opposing theories.

Recently, the flow of mathematical economic literature has been swollen rapidly in the West by works devoted to

the fundamentals of Marxist-Leninist economic theory, i.e., the concepts of value and surplus-value; a number of books¹ and many articles have been published. Generally we are dealing with a quite distinctly established special sector of theoretical economic literature. This can be seen as a reaction to the increased interest in Marxism demonstrated in scientific and student circles owing to the obvious bankruptcy of all attempts by bourgeois economists to propose a theory of prices and cycle that corresponds to reality.

This literature requires thorough critical analysis by Marxists. The difference in the authors' outlooks and the disputes among them must not be ignored. Some of them, for example P. Samuelson, J. Steedman, are obviously anti-Marxists, whereas others try to understand Marx's economic theory and treat it as science (for instance, W. Baumol, G. Abraham-Frois and E. Berreby, Y. Fujimori). Various intermediate positions are taken by M. Morishima, G. Maarek, etc.

In this literature, most attention has been attracted by the works by P. Samuelson, M. Morishima, J. Steedman, and other authors who employ modern mathematical techniques in an attempt to refute Marx's economic theory, and the fundamentals of his theory of society as a whole. Outwardly this is usually offered in the form of attempts to expose Marx's theory strictly, to fix its assumptions and conclusions, and to demonstrate that they do not correspond to one another. In fact, the "exposition" either directly contradicts Marx's theory (and reality together with it) or reduces this theory to a particular, almost non-realistic case. In some or other way, the above authors try to remove Marx's economic theory concerning value and surplus-value from economics.

All these attempts to expose or refute Marx's economic theory demonstrate, in fact, that the latter is too much for them. The use of mathematical models merely lays this fact bare. Mathematics is a strict science and Marxian economic

¹ Michio Morishima, *Marx's Economics. A Dual Theory of Value and Growth*, London, 1973; Khoshimura Shinzaburo, *Theory of Capital Reproduction and Accumulation*, London, 1975; Jan Steedman, *Marx after Sraffa*, London, 1977; Gilbert Abraham-Frois and Edmond Berreby, *Theory of Value, Prices and Accumulation. A Mathematical Integration of Marx, von Neumann and Sraffa*, Cambridge, 1979; Gérard Maarek, *An Introduction to Karl Marx's "Das Kapital". A Study in Formalization*, Oxford, 1979; Y. Fujimori, *Modern Analysis of Value Theory*, Berlin—Heidelberg—New York, 1982; etc.

theory is the same. That is why they are in deep internal mutual correspondence. If mathematical techniques are really used scientifically, it is impossible to refute the theory; on the contrary, its might can be repeatedly demonstrated, and it can be developed further. If, nevertheless, a certain author strives to refute Marxism using mathematical methods, he has to choose one of two ways: (1) instead of mathematical models identical to the subject of study and providing sufficient grounds for studying relevant problems, to propose inadequate models that ignore precisely those characteristics of the subject without an understanding of which the theory cannot be constructed; (2) build more or less adequate models, but interpret them at variance with their own properties.

Yet in either case, mathematical presentation hampers the bourgeois authors: their logical errors show through much more clearly and are uncovered much more quickly than if they had chosen a less formalised exposition. In fact, extensive use of mathematics can only serve to convince any objective, unprejudiced researcher of the correctness of Marxism and the invalid nature of criticisms of it. Once again Lenin's words have been substantiated: "The development of science is providing more and more material that proves that Marx was right".¹

All attempts, without exception, to refute the fundamental concepts of the theory of value and surplus-value mathematically are reduced to the above two variants of scientifically incorrect points. I have published papers refuting the main lines of mathematical "critiques" of Marx: attempts to demonstrate the existence of some contradiction between the theory of value and that of prices of production,² attempts to demonstrate that social value can be a negative or indefinite magnitude, etc.³

In this work I shall not return to the dispute with Marx's "critics". The main thing that can be offered to the unprejudiced reader is a positive exposition of Marx's theory using modern mathematical techniques. At the same time, the book will demonstrate that the laws of Marxian political

¹ V. I. Lenin, "The Collapse of the Second International", *Collected Works*, Vol. 21, 1974, p. 222.

² See: *Social Sciences*, USSR Academy of Sciences, Vol. XI, No. 4, 1980, pp. 179-97, and Vol. XIV, No. 2, 1983, pp. 211-16.

³ See: *Social Sciences*, USSR Academy of Sciences, Vol. XIV, No. 1, 1983.

économie are supported by mass statistics with great precision, the actual verification being, in any true science, the key criterion for evaluating theoretical conceptions.

The theory of value and surplus-value not only explains the properties of capitalism. This inevitably suggests the conclusion that capitalism must be replaced by a new, communist system, where every member of society is free from exploitation and other forms of oppression, where all the conditions are created for complete satisfaction of material and intellectual requirements, free comprehensive development of the personality of everyone. It is these conclusions that are the reason that the bourgeois apologists strive to undermine the fundamentals of Marx's economic theory. We shall focus on the demonstration of validity precisely of these fundamentals of the theory.

* * *

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