STRENGTH OF MATERIALS

TIMOSHENKO

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PART II

Advanced Theory and Problems

BY

S. TIMOSHENKO

Professor of Theoretical and Engineering Mechanics
Stanford University

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PREFACE TO THE SECOND EDITION

In the preparation of the new edition of this volume, the general character of the book has remained unchanged; the only effort being to make it more complete and up-to-date by including new theoretical and experimental material representing recent developments in the fields of stress analysis and experimental investigation of mechanical properties of structural materials.

The most important additions to the first edition include:

1. A more complete discussion of problems dealing with bending, compression, and torsion of slender and thin-walled structures. This kind of structure finds at present a wide application in airplane constructions, and it was considered desirable to include in the new edition more problems from that field.

2. A chapter on plastic deformations dealing with bending and torsion of beams and shafts beyond the elastic limit and also with plastic flow of material in thick-walled cylinders subjected to high internal pressures.

3. A considerable amount of new material of an experimental character pertaining to the behavior of structural materials at high temperatures and to the fatigue of metals under reversal of stresses, especially in those cases where fatigue is combined with high stress concentration.

4. Important additions to be found in the portion of the book dealing with beams on elastic foundations; in the chapters on the theory of curved bars and theory of plates and shells; and in the chapter on stress concentration, in which some recent results of photoelastic tests have been included.

Since the appearance of the first edition of this book, the author's three volumes of a more advanced character, "Theory of Elasticity," "Theory of Elastic Stability," and "Theory of Plates and Shells" have been published. Reference to these books are made in various places in this volume, especially in those cases where only final results are given without a complete mathematical derivation.

It is hoped that with the additions mentioned above the book will give an up-to-date presentation of the subject of strength of materials which may be useful both to graduate students interested in engineering mechanics and to design engineers dealing with complicated problems of stress analysis.

STEPHEN P. TIMOSHENKO

PALO ALTO, CALIFORNIA
June 12, 1941

PREFACE TO THE FIRST EDITION

The second volume of THE STRENGTH OF MATERIALS is written principally for advanced students, research engineers, and designers. The writer has endeavored to prepare a book which contains the new developments that are of practical importance in the fields of strength of materials and theory of elasticity. Complete derivations of problems of practical interest are given in most cases. In only a comparatively few cases of the more complicated problems, for which solutions cannot be derived without going beyond the limit of the usual standard in engineering mathematics, the final results only are given. In such cases, the practical applications of the results are discussed, and, at the same time, references are given to the literature in which the complete derivation of the solution can be found.

In the first chapter, more complicated problems of bending of prismatical bars are considered. The important problems of bending of bars on an elastic foundation are discussed in detail and applications of the theory in investigating stresses in rails and stresses in tubes are given. The application of trigonometric series in investigating problems of bending is also discussed, and important approximate formulas for combined direct and transverse loading are derived.

In the second chapter, the theory of curved bars is developed in detail. The application of this theory to machine design is illustrated by an analysis of the stresses, for instance, in hooks, fly wheels, links of chains, piston rings, and curved pipes.

The third chapter contains the theory of bending of plates. The cases of deflection of plates to a cylindrical shape and the symmetrical bending of circular plates are discussed in detail and practical applications are given. Some data regarding the bending of rectangular plates under uniform load are also given.

In the fourth chapter are discussed problems of stress distribution in parts having the form of a generated body and symmetrically loaded. These problems are especially important for designers of vessels submitted to internal pressure and of rotating machinery. Tensile and bending stresses in thin-walled vessels, stresses in thick-walled cylinders, shrink-fit stresses, and also dynamic stresses produced in rotors and rotating discs by inertia forces and the stresses due to non-uniform heating are given attention.

The fifth chapter contains the theory of sidewise buckling of compressed members and thin plates due to elastic instability. These problems are of utmost importance in many modern structures where the cross sectional dimensions are being reduced to a minimum due to the use of stronger materials and the desire to decrease weight. In many cases, failure of an engineering structure is to be attributed to elastic instability and not to lack of strength on the part of the material.

In the sixth chapter, the irregularities in stress distribution produced by sharp variations in cross sections of bars caused by holes and grooves are considered, and the practical significance of stress concentration is discussed. The photoelastic method, which has proved very useful in investigating stress concentration, is also described. The membrane analogy in torsional problems and its application in investigating stress concentration at reentrant corners, as in rolled sections and in tubular sections, is explained. Circular shafts of variable diameter are also discussed, and an electrical analogy is used in explaining local stresses at the fillets in such shafts.

In the last chapter, the mechanical properties of materials are discussed. Attention is directed to the general principles rather than to a description of established, standardized methods of testing materials and manipulating apparatus. The results of modern investigations of the mechanical properties of single crystals and the practical significance of this information are described. Such subjects as the fatigue of metals and the strength of metals at high temperature are of decided practical interest in modern machine design. These problems are treated more particularly with reference to new developments in these fields.

In concluding, various strength theories are considered. The important subject of the relation of the theories to the method of establishing working stresses under various stress conditions is developed.

It was mentioned that the book was written partially for teaching purposes, and that it is intended also to be used for advanced courses. The writer has, in his experience, usually divided the content of the book into three courses as follows: (1) A course embodying chapters 1, 3, and 5 principally for advanced students interested in structural engineering. (2) A course covering chapters 2, 3, 4, and 6 for students whose chief interest is in machine design. (3) A course using chapter 7 as a basis and accompanied by demonstrations in the material testing laboratory. The author feels that such a course, which treats the fundamentals of mechanical properties of materials and which establishes the relation between these properties and the working stresses used under various conditions in design, is of practical importance, and more attention should be given this sort of study in our engineering curricula.

The author takes this opportunity of thanking his friends who have assisted him by suggestions, reading of manuscript and proofs, particularly Messrs. W. M. Coates and L. H. Donnell, teachers of mathematics and mechanics in the Engineering College of the University of Michigan, and Mr. F. L. Everett of the Department of Engineering Research of the University of Michigan. He is indebted also to Mr. F. C. Wilharm for the preparation of drawings, to Mrs. E. D. Webster for the typing of the manuscript, and to the D. Van Nostrand Company for their care in the publication of the book.

S. Timoshenko

Ann Arbor, Michigan
May 1, 1930

NOTATIONS

$\sigma_x, \sigma_y, \sigma_z$.... Normal stresses on planes perpendicular to x, y and z axes.
σ_n.......... Normal stress on plane perpendicular to direction n.
$\sigma_{Y.P.}$........ Normal stress at yield point.
σ_w.......... Normal working stress
τ.......... Shearing stress
$\tau_{xy}, \tau_{yz}, \tau_{zx}$.. Shearing stresses parallel to x, y and z axes on the planes perpendicular to y, z and x axes.
τ_w.......... Working stress in shear
δ.......... Total elongation, total deflection
ϵ.......... Unit elongation
$\epsilon_x, \epsilon_y, \epsilon_z$..... Unit elongations in x, y and z directions
γ.......... Unit shear, weight per unit volume
E.......... Modulus of elasticity in tension and compression
G.......... Modulus of elasticity in shear
μ.......... Poisson's ratio
Δ.......... Volume expansion
K.......... Modulus of elasticity of volume
M_t.......... Torque
M.......... Bending moment in a beam
V.......... Shearing force in a beam
A.......... Cross sectional area
I_y, I_z...... Moments of inertia of a plane figure with respect to y and z axes
k_y, k_z...... Radii of gyration corresponding to I_y, I_z
I_p.......... Polar moment of inertia
Z.......... Section modulus
C.......... Torsional rigidity
l.......... Length of a bar, span of a beam
P, Q....... Concentrated forces
t.......... Temperature, thickness

U.........Strain Energy
s..........Distance, arc length
q..........Load per unit length

CONTENTS

CHAPTER I

SPECIAL PROBLEMS IN BENDING OF BEAMS

1. Beams on Elastic Foundation.—Let us consider a prismatical beam supported along its entire length by a continuous elastic foundation, such that when the beam is deflected, the intensity of the continuously distributed reaction at every section is proportional to the deflection at that section.[1] Under such conditions the reaction per unit length of the bar can be represented by the expression ky, in which y is the deflection and k is a constant usually called the *modulus of the foundation.* This constant denotes the reaction per unit length, when the deflection is equal to unity. The simple assumption that the continuous reaction of the foundation is proportional to the deflection is a satisfactory approximation in many practical cases. For instance, in the case of railway tracks, the solution obtained on this assumption is in good agreement with actual measurements.[2] In studying the deflection curve of the beam we use the differential equation:[3]

$$EI_z \frac{d^4y}{dx^4} = q, \qquad (a)$$

in which q denotes the intensity of the load acting on the beam.

[1] The beam is inbedded in a material capable of exerting downward as well as upward forces on it.

[2] See S. Timoshenko and B. F. Langer, Trans. A. S. M. E., Vol. 54, p. 277, 1932. The theory of bending of a bar on elastic foundation has been developed by E. Winkler, Die Lehre v. d. Elastizität u. Festigkeit, Prag, 1867, p. 182. See also A. Zimmermann, Die Berechnung des Eisenbahn-Oberbaues, Berlin, 1888. Further development of the theory will be found in the following publications: Hayashi, Theorie des Trägers auf elastischer Unterlage, Berlin, 1921; Wieghardt, Zeitschrift für angewandte Math. u. Mech., Vol. 2 (1922); K. v. Sanden and Schleicher, Beton und Eisen, 1926, Heft 5; Pasternak, Beton u. Eisen, 1926, Heft 9 and 10; W. Prager, Zeitschrift f. angewandte Math. u. Mech., Vol. 7, 1927, p. 354; M. A. Biot, Journal Appl. Mech., Vol. 4, p. 1A, 1937.

[3] See "Strength of Materials," Part I, p. 137.

For an unloaded portion the only force on the beam is the continuously distributed reaction from the side of the foundation of intensity ky. Hence $q = -ky$ and equation (a) becomes

$$EI_z \frac{d^4y}{dx^4} = -ky. \tag{1}$$

Using the notation

$$\sqrt[4]{\frac{k}{4EI_z}} = \beta, \tag{2}$$

the general solution of eq. (1) can be represented as follows:

$$y = e^{\beta x}(A \cos \beta x + B \sin \beta x) + e^{-\beta x}(C \cos \beta x + D \sin \beta x). \tag{b}$$

This can easily be verified by substituting (b) in eq. (1). In particular cases the arbitrary constants A, B, C, and D of the solution must be determined from the known conditions at certain points.

Let us consider, as an example, the case of a single concentrated load acting on an infinitely long beam (Fig. 1), taking the origin of coordinates at the point of application of the force. From the condition of symmetry, only that part of the beam to the right of the load need be considered (Fig. 1, b). In applying the general solution (b) to this case, the arbitrary constants must first be found. It is reasonable to assume that at points infinitely distant from the force P the deflection and the curvature are equal to zero. This condition can be fulfilled only if the constants A and B in eq. (b) are taken equal to zero. Hence the deflection curve for the right portion of the beam becomes

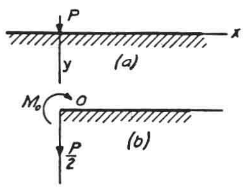

FIG. 1.

$$y = e^{-\beta x}(C \cos \beta x + D \sin \beta x). \tag{c}$$

The two remaining constants of integration C and D must be found from the conditions at the origin, $x = 0$. At this point, the deflection curve must have a horizontal tangent; therefore

$$\left(\frac{dy}{dx}\right)_{x=0} = 0,$$

or substituting expression (c) for y

$$e^{-\beta x}(C \cos \beta x + D \sin \beta x + C \sin \beta x - D \cos \beta x)_{x=0} = 0$$

from which

$$C = D.$$

Equation (c) therefore becomes

$$y = Ce^{-\beta x}(\cos \beta x + \sin \beta x). \tag{d}$$

The consecutive derivatives of this equation are

$$\frac{dy}{dx} = - 2\beta Ce^{-\beta x} \sin \beta x,$$

$$\frac{d^2y}{dx^2} = 2\beta^2 Ce^{-\beta x}(\sin \beta x - \cos \beta x), \tag{e}$$

$$\frac{d^3y}{dx^3} = 4\beta^3 Ce^{-\beta x} \cos \beta x. \tag{f}$$

The constant C can now be determined from the fact that at $x = 0$ the shearing force for the right part of the beam (Fig. 1, b) is equal to $-(P/2)$. The minus sign follows from our convention for signs of shearing forces (see p. 72, Part I). Then

$$(V)_{x=0} = \left(\frac{dM}{dx}\right)_{x=0} = -EI_z\left(\frac{d^3y}{dx^3}\right)_{x=0} = -\frac{P}{2},$$

or using eq. (f)

$$EI_z \cdot 4\beta^3 C = \frac{P}{2},$$

from which

$$C = \frac{P}{8\beta^3 EI_z}.$$

Substituting this in eqs. (d) and (e), we obtain the following equations for the deflection and bending moment curves:

$$y = \frac{P}{8\beta^3 EI_z} e^{-\beta x}(\cos \beta x + \sin \beta x)$$

$$= \frac{P\beta}{2k} e^{-\beta x}(\cos \beta x + \sin \beta x), \quad (3)$$

$$M = -EI_z \frac{d^2y}{dx^2} = -\frac{P}{4\beta} e^{-\beta x}(\sin \beta x - \cos \beta x). \quad (4)$$

Both expressions (3) and (4) have, when plotted, a wave form with gradually diminishing amplitudes. The length a of these waves is given by the period of the functions $\cos \beta x$ and $\sin \beta x$, i.e.,

$$a = \frac{2\pi}{\beta} = 2\pi \sqrt[4]{\frac{4EI_z}{k}}. \quad (5)$$

To simplify the determination of the deflection, the bending moment, and the shearing force the numerical table below is given, in which the following notations are used:

$$\left.\begin{aligned} \varphi &= e^{-\beta x}(\cos \beta x + \sin \beta x); \\ \psi &= -e^{-\beta x}(\sin \beta x - \cos \beta x); \\ \theta &= e^{-\beta x} \cos \beta x; \qquad \zeta = e^{-\beta x} \sin \beta x \end{aligned}\right\} \quad (6)$$

In Fig. 2 the functions φ and ψ are shown graphically.

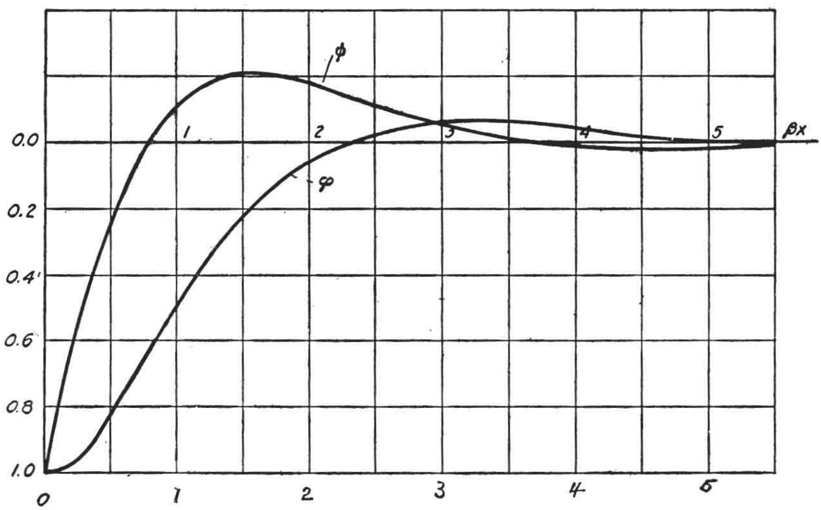

FIG. 2.