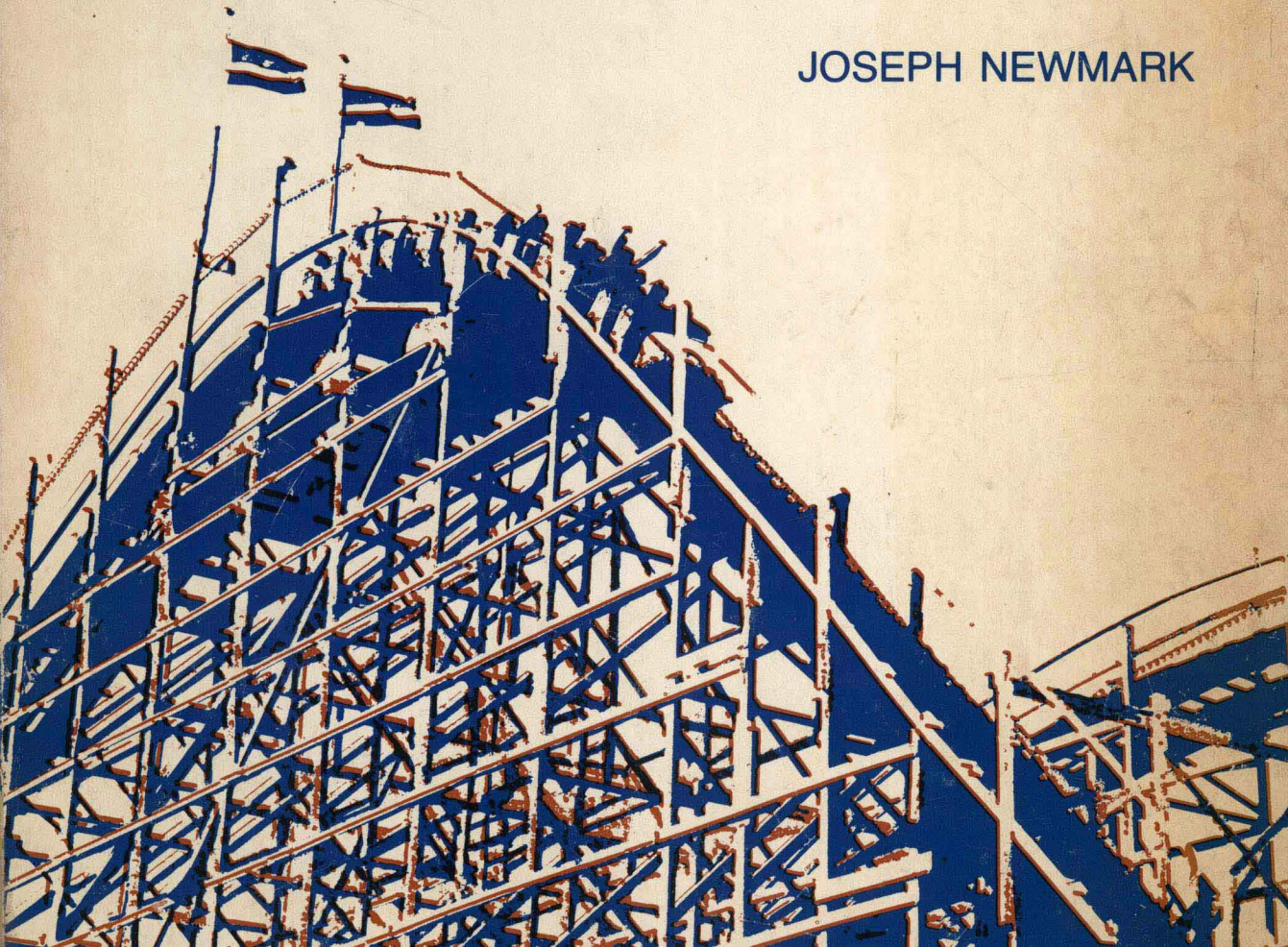


# THE USEFULNESS OF **CALCULUS**

JOSEPH NEWMARK



# THE USEFULNESS OF **CALCULUS**

FOR THE BEHAVIORAL, LIFE, AND MANAGERIAL SCIENCES

JOSEPH NEWMARK

The College of Staten Island/Brooklyn College

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# THE USEFULNESS OF CALCULUS

TO TRUDY, SHARON, ROCHELLE, AND STEPHEN

## ABOUT THE AUTHOR

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Joseph Newmark has proven himself one author who thoroughly understands the needs of students and teachers. Attesting to his success as an author and as a teacher is the enthusiastic acceptance of his two previously published textbooks, **MATHEMATICS AS A SECOND LANGUAGE** and **STATISTICS AND PROBABILITY IN MODERN LIFE**. Each is used in over 100 schools, and each text was revised and published in a second edition in 1977.

As director of the math laboratory at The College of Staten Island, where he works with students who are having trouble with math in general (and calculus in particular), he received a grant from the National Science Foundation for the production of audio-visual statistics tapes for use in the lab.

His academic career began with an NDEA fellowship and graduate work at Brooklyn College, followed by a Ph.D. at New York University. He is currently teaching “everything from arithmetic to graduate level math” at The College of Staten Island and Brooklyn College.



# PREFACE

This text has been designed to fit the needs of liberal arts, life science, and managerial science students, who must frequently demonstrate a knowledge of the language and methods of the calculus. It is for this reason that the basic ideas of the calculus (limits, derivatives, integrals) are developed in an intuitive manner. Limits are discussed without the use of epsilons and deltas. Derivatives are calculated without the  $\Delta$ -notation. Many of the exercises and examples illustrate how the calculus can be applied in business situations, social science situations, and so on. The *Index of Applications* lists many wide-ranging situations to which the ideas of the calculus can be applied.

Since the concepts of the calculus are assuming an ever-increasing role in the social sciences, a special effort has been made in this text to make some of these ideas available to students who are not prepared for elaborate symbolism or complex calculations. Although the mathematical content is complete and correct, the language is rather elementary and understandable. This makes the text comprehensible to students of varying backgrounds, especially those in the Open Admissions Program or those with little mathematical background. Words, phrases, and any modes of expression that students find difficult to comprehend have been avoided. In frequent *comments*, various points that my teaching experience has shown are often misunderstood or missed completely by students are carefully discussed. These ideas are often taken for granted or slurred over completely in other texts.

Numerous teaching and learning aids have been included in this text. Among these are (1) *Newspaper and magazine clippings*. Each chapter starts off with a newspaper or magazine article that presents the ideas discussed in the chapter in an applied context. This is intended to motivate the reader by showing how the ideas mentioned in the chapter are applied in everyday situations. (2) *Chapter objectives*. Each chapter is introduced by a chapter objectives section, which pinpoints the main ideas of the chapter. (3) *Concluding materials*. Each chapter concludes with a summary, a study guide, and a list of formulas to remember. (4) *Mastery tests*. Each chapter contains two sets of mastery tests designed to measure the student's comprehension of the material covered in the chapter, as well as to present a few challenging problems to the more demanding student. (5) *Course outlines*. A suggested course outline is provided for both one-semester and two-semester courses. (6) *Historical notes*. Many chapters contain brief

historical notes of interest on mathematicians who made contributions to the development of the calculus.

Because exercises provide an extremely important and integral part of any text, they have been provided in abundance. There are almost 800 exercises and about 250 illustrative examples with detailed step-by-step explanations. The Mastery Tests contain an additional 200 exercises.

A Solutions Manual providing detailed solutions to all problems is available from the publisher.

## SUGGESTED COURSE OUTLINE

The following outline indicates how this text can be used in schools that offer one-semester introductory calculus courses for liberal arts students that meet about 40 times during the semester for 40 minutes each session. The text can also be used in schools that offer two-semester introductory calculus courses.

The instructor should have no difficulty in organizing a course from the wide range of topics covered.

### ONE-SEMESTER COURSE

Text material	Approximate amount of time
Chapter 1	4 lessons
Chapter 2 (skip Sec. 2.7)	4 lessons
Chapter 3 (skip Sec. 3.8)	6 lessons
Chapter 4 (skip Sec. 4.8)	7 lessons
Chapter 5 (skip Sec. 5.7)	8 lessons
Chapter 6	5 lessons
	<u>34*</u>

\*The remaining time can be devoted to exams and review.

### TWO-SEMESTER COURSES

#### SEMESTER 1

Text material	Approximate amount of time
Chapter 1	4 lessons
Chapter 2	5 lessons
Chapter 3	8 lessons
Chapter 4	9 lessons
Chapter 5 (through Sec. 6)	6 lessons
	<u>32*</u>

\*The remaining time can be devoted to exams and review.



## SEMESTER 2

Text material	Approximate amount of time
Chapter 5 (Sec. 7)	2 lessons
Chapter 6	5 lessons
Chapter 7	8 lessons
Chapter 8	9 lessons
	<hr/> 24*

\*The remaining time can be devoted to exams and review.

# TO THE STUDENT

This is a “nonmathematical” text on elementary calculus. The only background needed is high school algebra. Even a knowledge of trigonometry is not needed. Chapter 1 contains a review of the ideas of graphs and functions needed, if you feel that your math background is a bit rusty. This chapter can be deleted if you feel that you are adequately prepared. Additional review is provided in the chapters where it is relevant.

The examples are plentiful and are chosen from real-life situations. Also, the ideas of the calculus are applied to a variety of subject areas. In frequent *comments*, appropriate explanations of the “why’s” of the calculus are given.

Occasionally a section or exercise is starred (\*). This means that it is slightly more difficult and may require some additional time and thought.

Each chapter concludes with a summary, formulas to remember, and a study guide containing all of the key words introduced in the chapter. In addition, there are Mastery Tests that will help you in preparing for exams. They will be more useful if you take them after you have studied and solved the exercises in the chapter. Answers to many of the exercises are provided at the end of the book.

I hope that you will find reading and using this book an enjoyable and rewarding experience. Good luck!

Joseph Newmark

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Finally, and most importantly, I wish to thank my wife Trudy and our children, Sharon, Rochelle, and Stephen, for their understanding and patience as they endured the enormous strain associated with completing the project. Without their encouragement this project could not have been undertaken.

J.N.

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