

ROBERT E. MCGRATH

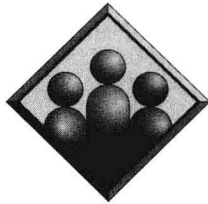
UNDERSTANDING STATISTICS

A RESEARCH PERSPECTIVE



UNDERSTANDING STATISTICS

A Research Perspective



Robert E. McGrath

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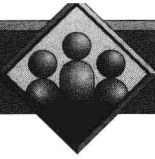
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UNDERSTANDING STATISTICS

To Deborah
I would spend my days
in conversation with you



Preface

However much we detail a transmitted description, it will always be unnecessarily precise for some and fragmentary for others. (Lem, 1968/1983, p. 84)

TO THE READER

Many students interested in the behavioral sciences consider the required statistics course the most dreaded part of their curriculum. I think this perception is unfortunate, because I have found the study of statistics both fascinating and practically useful: fascinating because I am often struck by the logical beauty of the ideas that underlie statistical methods; practically useful because an understanding of statistical concepts such as probability and variability can help one think more effectively about both personal and social issues.

In writing this book I have attempted in several ways to address those fears. For one thing, I tried to focus as much on the *why* of statistics as on the *how*. It seems to me too many students finish their statistics course with a list of statistical methods they have studied, but little grasp of when and why they would be useful. I hope by the time you finish this course you will not only know how to use various statistics, but also when and why you would use them.

I also think it is important to provide a context for using statistics. Throughout this book I have emphasized practical issues that will emerge in the use of statistics in a research setting. Although this text is not meant to provide a full introduction to both research and statistical methods, enough research is thrown into the mix that I hope it gives life to the statistics.

I chose the aphorism at the beginning of this Preface because I think it is particularly appropriate to topics as complex as statistics. There is no way I can present this material so that it is equally accessible to all. I expect there will be times when you will find the material covered here difficult. I urge you to stick with it until it makes sense, and I hope that your efforts will be rewarded by seeing some of that logical beauty I mentioned above.

TO THE INSTRUCTOR

I originally decided to write this book because I could not find a well-written text that provided the level of understanding about statistics that I thought makes it most meaningful to the student. This textbook was written under the guiding principle that it has become less important for a student to know how to compute a *t* test or correlation coefficient than it is for a student to understand sampling distributions and hypothesis testing. While the former provides the student with information which may be useful in a particular research study, the latter has implications for almost any research project. In particular, understanding the

concepts underlying statistics will help the student decide which statistics are appropriate for which research contexts.

In the course of working on the text, I have also tried to be innovative in the structure of the text. These innovations include the following:

- The text was written with flexibility in mind. Many chapters include supplements containing material which, while not essential or even necessary to understand the topic of the chapter, may enhance some students' understanding of statistics and research design. Instructors may also find some of the supplementary materials worth requiring, if the instructor wishes to emphasize certain topics addressed only in passing in the chapters themselves. In my own teaching of statistics I routinely require students to read the material on research design in the supplement to Chapter 3, since I like to cover research design issues in more detail than is standard for a statistics course.
- The first 12 chapters follow a traditional model for textbook chapters: each addresses a general topic, and includes all information relevant to the topic. After Chapter 12, the format is somewhat unusual. Each chapter is devoted to a single hypothesis testing statistic, and chapters are written so they can be read relatively independently of each other. This makes it easy for the instructor to eliminate material. Sections on confidence intervals are also clearly marked, and can be removed from the curriculum quite easily.
- Each hypothesis testing chapter follows the same sequence of topics: introduction, statistical hypotheses, testing the null hypothesis, making a decision about the null hypothesis, conditions for using the test, directional versus nondirectional hypotheses, power and sample size issues, and confidence intervals. This provides a common framework the student can use to organize information about hypothesis tests.
- I find most books provide at best an abstract discussion of power analysis. This model of instruction no doubt contributes to the disregard for power evident in behavioral research today. I have attempted to develop a practical approach to power analysis, presenting it in the context of sample size estimation for each statistic.
- Exercises were developed to reinforce basic principles. For example, almost every chapter has at least one question that asks about the type of study and the nature of the variables used, concepts that are raised in the first three chapters of the text.
- The final chapter is extremely integrative. It provides a model for choosing the correct statistics that draws upon information introduced from the first chapter on. The chapter also includes a table listing characteristics of each statistic that in my experience students tend to find particularly difficult to remember.

There are few references to computers in this text. This omission is not meant to imply that I do not consider it worthwhile to train students in the use of statistical software. To the contrary, I think this is an essential component of becoming a competent researcher in the behavioral sciences. However, many instructors do not find they have the time or resources for addressing computer use during the course. The text is written so a knowledge of computers is unnecessary (although in several places the discussion was slanted so it would be consistent with common standards among statistical software packages). For instructors who are able to integrate computer methods into their course, a supplementary text is planned.

GOALS

If I have been successful in achieving my goals with this text, upon completing the course the student should have a basic understanding of the following topics:

- the role statistics play in the research process.
- the types of situations in which each of the descriptive statistics are appropriate.
- the types of situations in which each of the inferential statistics are appropriate.
- the logic of hypothesis testing and interval estimation.
- the role samples, populations, and sampling distributions play in statistics.

These are the concepts I believe are most important to master if the student is to apply what has been learned in a flexible manner at a later point.

GENERAL ORGANIZATION

The text is divided into six sections. The first focuses on basic concepts that provide the context for subsequent discussions. This includes the introduction of some basic statistical concepts as well as research design issues common to all studies.

The second and third sections focus on descriptive statistics. Part 2 deals with statistics used to describe patterns in the data associated with a single variable, Part 3 with descriptive statistics used to describe relationships among two variables. This structure parallels the discussion of scientific hypotheses in Chapter 3, where a distinction is drawn between hypotheses about variable patterns and hypotheses about variable relationships.

Part 4 provides the context for the subsequent discussion of inferential statistics. Probability and sampling distributions, the logic of hypothesis testing, and interval estimation are outlined. The fifth and sixth sections focus on inferential statistics. Part 5 deals with statistics used to draw conclusions about a single variable in the population, Part 6 with statistics used to draw inferences about relationships among two or three variables in the population. With several exceptions, each chapter in Parts 5 and 6 focuses on a single statistic. This allows the instructor to modify the order of presentation or skip certain statistics with relative ease. In Parts 5 and 6 parametric statistics are discussed first, followed by non-parametric statistics. The final chapter attempts an integration of the material covered in the text by offering a model for determining the right statistics.

Two supplements are provided to accompany this text—a Study Guide for students (0-673-99424-4) and a Test Bank for instructors (0-673-55843-6). Both are authored by me. Addison Wesley Longman also publishes StatTutor, a student tutorial program available for both IBM and MacIntosh systems.

FINAL COMMENTS

There are several people whose contributions to this work deserve note. Christopher Capuano, Ph.D., John Herr, R.P., David Book, R.P., and Deborah Bernstein, Psy.D., provided important input into some of the examples used. I would also like to thank the reviewers whose invaluable feedback made this a much stronger work: Dr. Gordon Allen,

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Robert E. McGrath



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