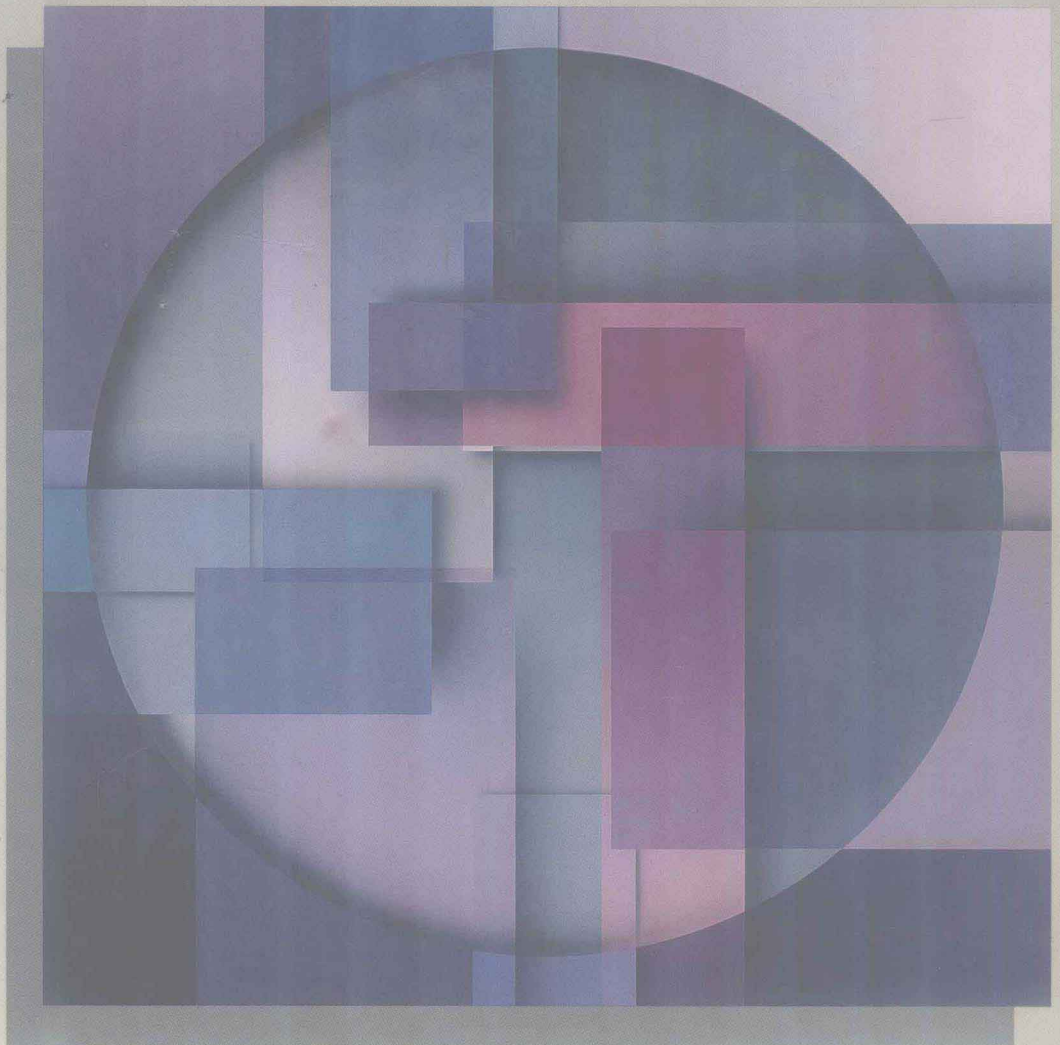
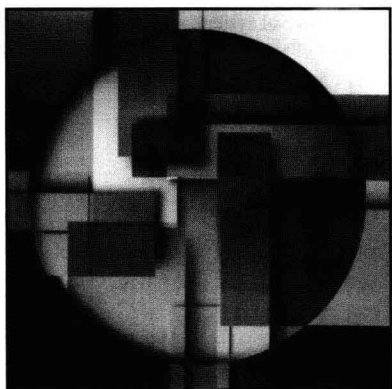


BASIC MATHEMATICS

FOURTH EDITION

Rosanne
Proga





Basic Mathematics

Fourth Edition

Rosanne Proga



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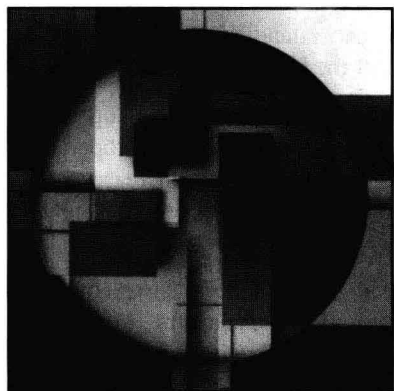
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Basic Mathematics


► *To My Parents*



Preface

Basic Mathematics, Fourth Edition is a practical guide to the fundamentals of arithmetic for students who generally prefer to avoid mathematics. The approach to the subject matter is mature, yet easily understandable, and assumes no prior knowledge of mathematics. The text explains various mathematical techniques thoroughly, because knowing *why* a technique works is an aid in remembering *how* to use it. *Basic Mathematics* provides a wide variety of word problems to reinforce each newly learned skill and to emphasize practical applications of mathematics.

Features

- ▲ **Explain It in Words** is a new feature included at the end of every chapter review. The exercises require students to express answers in writing to questions involving the application of the concepts presented in each chapter. They can be used as an individual writing exercise or as a group activity providing topics for discussion. By having students explain their use of mathematical concepts verbally or in writing, their understanding of the material will be significantly enhanced.
- ▲ **Word Problems** reinforce each newly learned skill and emphasize practical applications of mathematics. Word problems involving real-world applications have been added pertaining to whole numbers (Chapter 1), averages (Chapter 2), fractions (Chapter 3), decimals (Chapter 4), proportions (Chapter 5), percents (Chapter 6), temperature (Chapter 9), geometry (Chapter 10), and solving linear equations (Chapter 11).
- ▲ A **Quick Quiz** appears at the end of each section, enabling students to check their mastery as they progress through the text.
- ▲ **Calculator Problems** are designated by the symbol . The appendix describes how to use a pocket calculator. This book may be used with or without a calculator.
- ▲ **Chapter Summaries** highlight important definitions and calculations.

- ▲ **Review Exercises** reinforce the material learned.
- ▲ A **Chapter Test** of 20 to 25 problems has been added at the end of every chapter to provide students with an additional tool to test their understanding of the material. All answers to the chapter tests are provided at the back of the text.
- ▲ **Examples** have been added to further clarify the discussion of order of operations (Chapter 2), translating words into mathematical expressions (Chapter 2), operations with fractions (Chapter 3), rounding off decimals (Chapter 4), percent increase and decrease (Chapter 6), converting units of measurement (Chapter 9), calculating area (Chapter 10), and solving linear inequalities (Chapter 12).
- ▲ **Flexibility** is a major consideration in the organization of this text. Instructors who desire an earlier treatment of algebra, for example, may cover the chapters “Introduction to Algebra” (Chapter 11) and “Graphing” (Chapter 12) immediately after “Positive and Negative Numbers” (Chapter 7).

Highlights

- ▲ **Estimation** is presented as an important problem-solving tool in working with whole numbers (Chapter 1) and decimals (Chapter 4).
- ▲ **Mixed numbers** appear throughout the discussion of fractions to present an integrated approach to the basic operations (Chapter 3).
- ▲ **Algebraic concepts** are introduced early in the text to provide a thorough explanation of the method for solving proportions (Chapter 5). The techniques for setting up and solving equations are also used to solve problems involving percents (Chapter 6).
- ▲ **Consumer applications** are emphasized throughout the text, especially with problems involving percents and with reading graphs (Chapter 6).
- ▲ **Geometric concepts** are presented intuitively, using diagrams to enhance students’ understanding of the ideas developed (Chapter 10).
- ▲ **Number systems** and their interrelationships are presented after the discussion of signed numbers (Chapter 7) and roots of numbers (Chapter 8) so students will be better prepared for further work in mathematics.
- ▲ **Measurement** in the English and metric systems (Chapter 9) is thoroughly discussed, as well as methods for approximating conversions between the two systems, to develop the ability to “think metric.”

The Fourth Edition includes the following revisions:

- ▲ A procedure for determining whether a number is prime or composite has been added (Chapter 2).
- ▲ The discussion of least common multiple has been expanded (Chapter 2).
- ▲ Procedures for translating between words and decimal notation have been added (Chapter 4).
- ▲ The income tax problems have been updated (Chapter 6).
- ▲ The discussion of raising a signed number to a power has been expanded (Chapter 7).

- ▲ The concept of sets has been delayed until the discussion of number systems (Chapter 8).
- ▲ The geometric concepts of point, line, and plane have been introduced (Chapter 10).
- ▲ The distinction between a constant and a variable has been further clarified (Chapter 11).

Supplements

- ▲ **Instructor's Manual.** In the Instructor's Manual, the answer book for even-numbered exercises has been combined with the test bank for added convenience. (The answers to odd-numbered exercises are included in the back of the text.) The test bank includes three test forms for each chapter with problems similar to those in the book, which can be used to supplement exercises in the text or as testing tools. Answers to all tests are provided.
- ▲ **MathQuest™ Tutorial Software** is based on the interactive AUTHORWARE™ program. This tutorial runs on Microsoft WINDOWS and Macintosh platforms. Students are given questions to answer. If they give an incorrect answer, the program will respond with hints; then the student has another opportunity to answer the question. If the student answers incorrectly a second time, the program responds with the correct answer and a step-by-step solution. An on-screen, button-operated calculator can be pulled up for additional help. Password-protected teacher utilities are provided to allow access to their students' records. Individual student scorekeeping capabilities allow students to monitor their progress and score in each section of the exercises.
- ▲ **The EVR Videotape Series** teaches key topics in the text and features a professional math instructor. The videotapes are free to qualified adoptors. These tutorial tapes, produced by Educational Video Resources, can significantly improve students' comprehension and performance in algebra.
- ▲ **EXP Test for IBM PCs and Compatibles.** In this computerized test bank, the instructor can select or edit existing multiple-choice, true/false, and open-ended test items; and create and add new items for a more customized test.
- ▲ **ExamBuilder**, a computerized test bank for the Macintosh, is a simple testing program that allows instructors to view and edit existing tests, as well as create new test items. Questions can be stored by objective and tests can be created using multiple-choice, true/false, fill-in-the-blank, essay, and/or matching formats. Questions can be scrambled to avoid duplicate testing and graphs can be generated and printed. Demonstration disks are available.

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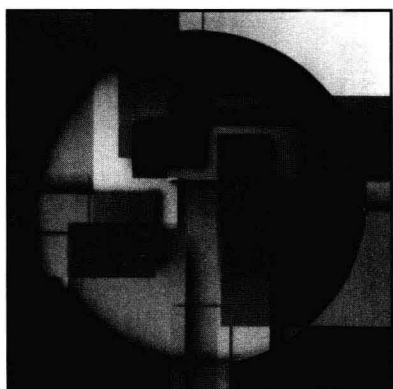
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Rosanne Proga

Basic Mathematics



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Chapter 1

Whole Numbers

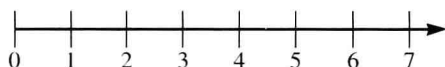
1.1 The Base Ten System

When we begin to study any new field of knowledge, we first need to learn the vocabulary that is used to communicate the fundamental ideas of the subject. Thus, in order to study mathematics, we must become familiar with the language of mathematics, whose alphabet consists not only of letters, but also of numbers.

In this chapter, we will consider the numbers most commonly encountered—the **whole numbers**. These include 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, . . . The symbol . . . indicates that it is impossible to list all the whole numbers. We express this idea by saying that the set of whole numbers is *infinite*.

The whole numbers can be represented by a diagram called a number line, shown in Figure 1.1. Notice that larger numbers are located to the right of smaller numbers. The arrow on the number line indicates that there is an infinite number of whole numbers.

Figure 1.1



The number system we most frequently use is called the **base ten system**. In this system, ten different digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) are used to express all numbers. Since computers basically consist of “off” and “on” switches, machine language is based on the number system of base two, which uses only the digits 0 and 1. The location of each digit, or its **place value**, determines what number that digit expresses.

► **DEFINITION** The *place value* of a digit is the name of the location of that digit.

The place value of each digit increases as we move from right to left. Each place value is ten times greater than the one to the right of it. The rightmost location is the ones place, the position immediately to the left of it is the tens place, the next place is the hundreds place, and so on, as shown in Table 1.1.

TABLE 1.1
Place Values

Periods														
Trillions			Billions			Millions			Thousands			Ones		
Hundred Trillions	100,000,000,000,000													
Ten Trillions	10,000,000,000,000													
Trillions	1,000,000,000,000													
Hundred Billions	100,000,000,000													
Ten Billions	10,000,000,000													
Billions	1,000,000,000													
Hundred Millions	100,000,000													
Ten Millions	10,000,000													
Millions	1,000,000													
Hundred Thousands	100,000													
Ten Thousands	10,000													
Thousands	1,000													
Hundreds	100													
Tens	10													
Ones	1													

We name numbers according to the location of their digits. For example, the number one thousand has a 1 in the thousands place followed by three zeros: 1,000. After every group of three digits, moving from right to left, we place a comma to make it easier for us to read the number. Each of these three-digit groups is called a **period**. A chart indicating the place values of the first 15 locations included in the first five periods is shown in Table 1.1. The place value names the location of the digit 1 in each numerical representation given. Notice that we place a comma after the trillions, billions, millions, and thousands place. The comma is sometimes omitted after the thousands place in numbers such as 5937 that have only four digits.

The place value of each digit in the number

8,416,923

can be determined using the chart as illustrated, beginning with the ones place and moving from right to left.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
8	4	1	6	9	2	3

Once we have identified the place value of each digit, we can write the number in words as follows.

8,416,923 in words becomes
eight million, four hundred sixteen thousand, nine hundred twenty-three

The word “and” is not used in the names of whole numbers.

Now let us look at some examples that illustrate how place value is used to assign names to numbers.

EXAMPLE 1 For each of the following numbers, give the place value of each digit and rewrite the number in words.

(a) 325

3: hundreds place

2: tens place

5: ones place

three hundred twenty-five

Hundreds	Tens	Ones
3	2	5

(b) 8,016

8: thousands place

0: hundreds place

1: tens place

6: ones place

eight thousand, sixteen

Thousands	Hundreds	Tens	Ones
8	0	1	6

(c) 423,701

4: hundred thousands place

2: ten thousands place

3: thousands place

7: hundreds place

0: tens place

1: ones place

four hundred twenty-three thousand, seven hundred one

Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
4	2	3	7	0	1

(d) 61,004,072

6: ten millions place

1: millions place

0: hundred thousands place

0: ten thousands place

4: thousands place

0: hundreds place

7: tens place

2: ones place

sixty-one million, four thousand, seventy-two

Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
6	1	0	0	4	0	7	2

(e) 7,325,018,640

7: billions place

3: hundred millions place

2: ten millions place

5: millions place

0: hundred thousands place

1: ten thousands place

8: thousands place

6: hundreds place

4: tens place

0: ones place

seven billion, three hundred twenty-five million, eighteen thousand, six hundred forty

Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
7	3	2	5	0	1	8	6	4	0