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School Mathematics in the 1990s

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Foreword

The International Commission on Mathematical Instruction is planning a series of studies on topics of current interest within mathematics education. The first study was on the impact of computers and informatics on mathematics and its teaching at university and senior high school level. That study had as its centrepoint an international symposium held in Strasbourg, France in March, 1985 and attended by some seventy participants drawn from thirteen different countries.

The second study, which gave rise to this volume, has taken a slightly different form. First a discussion document, School Mathematics in the 1990s by A.G. Howson, B.F. Nebres and B.J. Wilson, was sent to all National Representatives of ICMI and circulated widely in the original English and in translation. A small, closed international seminar was then held in Kuwait in February, 1986 at which an invited group of mathematics educators, named on the title page of this book, considered issues raised in the discussion document, points made by those who had responded to that paper, and, of course, attempted to remedy its many omissions. This book is based on those discussions and has been prepared by Geoffrey Howson and Bryan Wilson. It is, as such, a compilation of views and certainly would not have the effect of drawing discussion amongst those who participated in Kuwait to an end. Its aim, however, is not to terminate discussions, but rather to provoke and stimulate them. Further serious and detailed debate will be required before sound responses to problems can be formulated. It is ICMI's hope that this book will facilitate such debate and decision-taking.

The subject is a vital one, for there are few issues which are of such concern to all countries throughout the world. Mathematics education faces a vast variety of challenges as we move towards the 1990s and it is by no means certain what responses to some particularly crucial questions will prove most appropriate. There are few certainties. Perhaps one of the few things of which we can be certain is that ICMI's approach to the problems must be based on a broader appreciation of mathematics education than is suggested by its somewhat outdated name. 'Instruction' or 'teaching' will always remain a key issue for us, but, as is emphasised in this volume, 'learning' demands equal consideration, and, what is still insufficiently recognised, this must include consideration of what mathematics students learn, and of the mathematical activities in which they engage, outside, as well as in, the classroom.

We hope also that this book will prove timely. In many countries there is a growing lack of confidence in our ability to teach mathematics successfully. Education is poorly regarded. Such a crisis of confidence must be overcome, for there is no doubt that well-informed, active citizens in the 1990s will require more mathematics and a greater comprehension of mathematics. We must react against any tendencies to see the problems of mathematics education as intractable.

Finally, I should like to acknowledge with gratitude the leading role which Geoffrey Howson has played at all stages of this study.

Jean-Pierre Kahane

August, 1986.

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The International Commission on Mathematical Instruction is deeply grateful to all these bodies and to Dr. Al-Ebrahim for their most generous assistance and encouragement. It also wishes to place on record its indebtedness to the members of the local organizing committee, Mansour G. Hussein (Chairman), Dr. Adnan Hamoui, Adnan A. Al-Abdulmuhsen, Tahseen Budair and Mahmoud Abdul-Kadir, for their most excellent and greatly appreciated work.

We should also wish to express our thanks to the Institut für Didaktik der Mathematik, Bielefeld, where Geoffrey Howson worked on part of the manuscript of this book as a visiting professor, to Elizabeth Henderson, Roseanne Glover, Charles Jackson, Doug Jones, Kurt Killion, Kenneth Shaw and Siriporn Thipkong of Athens, Georgia, who offered detailed criticisms of a draft manuscript and many of whose suggestions have been incorporated in this final version, and to Mrs. Margaret Youngs who has so carefully typed this volume.

Contents

Chapter 1

Mathematics in a Technological Society

1. A New Revolution

The world is currently in the throes of a new technological revolution which is having an impact on society at least as great as the Industrial Revolution. Moreover, the speed at which the effects of this 'Information Revolution' are being felt is considerably greater than that of its predecessor, as is its range. There are now few, if any, societies anywhere in the world that are unaffected.

It is this fact above all that underlay the decision of ICMI, the International Commission on Mathematical Instruction, to mount the study of which this present publication is the tangible outcome.

Much curriculum development in school mathematics over the past thirty years has taken place in a piecemeal manner. Everyone professionally concerned with mathematics education - and many who are not, including many parents - has his or her own views on school mathematics: what should be taught, how it should be taught, how fast it can be learned, how it should be assessed .... . In order to represent as wide a range of views as practicable, while still being able to formulate guidelines sufficiently specific to be useful, a group of experienced mathematics educators were invited to meet together for six days in Kuwait in February 1986 to consider school mathematics in the 1990s. These participants, drawn from six continents, were selected as having a variety of personal viewpoints and as representing different mathematical and educational traditions. This publication is based on the discussions which took place at that Kuwait meeting.

Many of the changes over the past thirty years have resulted from curriculum developers in one educational system simply copying what is being done in another. This partly explains the similarity of mathematics curricula, particularly at secondary level, across the world. The substance of these similar curricula is referred to in this present study as the canonical mathematics curriculum.

There is no intention of prescribing a new orthodoxy. The tenor of what follows is rather in the direction of greater diversity, with mathematics curricula being developed with more attention to the cultural circumstances and employment patterns of the country concerned. Consequently, no answers are given to the issues raised. The purpose is rather to identify some of these key issues which mathematics curriculum developers will have to consider in the immediate future. For some of these issues, various alternative courses of action are suggested, and some of the consequences of each alternative are mentioned. In this way it is hoped to initiate more detailed local debate, country by country. This study offers a framework within which such debate can take place in a coherent fashion.

2. Changing Demands

Through its influence on society, modern technology is causing, and will increasingly cause, educational aims to be rethought. Mathematics itself is being directly affected, as new branches are being developed in response to technological need (see, for example, ICMI 1986a), other time-hallowed techniques are falling into disuse, and the balance of mathematical skills needed by the citizen to function effectively in daily life is changing. In addition to this direct effect, the teaching of mathematics is being affected in a variety of other ways, through the changing demands, expectations and employment patterns within society, through changing educational goals and structures, and through changing pedagogical possibilities. The canonical school mathematics curriculum and, indeed, state educational systems, developed largely in response to the demands of the Industrial Revolution; it seems self-evident that the Information Revolution will result in major changes in both schools and their curricula, as new demands are made and new opportunities provided for teaching and learning in educational systems across the world.

Of course, the impact of technology is not the same in all societies. For example, in rural societies within the Third World, technological development can bring in its wake increased demands for mathematics: subsistence and small-scale commercial farming can now require mathematically-based decision-making, as well as the ability to use machinery and to control it in response to readings on a variety of scales and measures. How does the development of such abilities fit into the school mathematics curriculum? Bearing in mind the high rate of drop-out during the primary school years in many such countries (see Figure 3.1), can the first few years of mathematics education be designe in such a way as to make children functional in the processes they will need in rural life? Further, would such a reordering of curriculum priorities be consistent with the development of skills and attitudes that would encourage pupils to continue to learn once they had left school? Such questions will be taken up again in Chapter 3.

In technologically advanced countries, on the other hand, there is evidence that some of the mathematical demands that used to be placed on the bulk of those in employment are no longer required; many traditional demands are nowadays met by the ubiquitous 'chip' (see, for example, Fitzgerald (1981) and note the report of the US Bureau of Labor Statistics (Romberg, 1984, p.6) that not only had most job openings in 1980 low skill requirements, but that this situation was likely to persist for the next quarter of a century). An area of life where such reduced demands are evident to practically every citizen of a technologically advanced country is that of shopping. No longer does a shop assistant have to be able to add up a bill, nor to subtract to calculate change, and neither does the customer have to check such arithmetic; a machine does it all. With the prospect of a cashless society, even the need to be able to count money could disappear. Yet as the flood of information available to every citizen rises both in employment and in daily life, other mathematical awarenesses are needed to handle it efficiently: ideas from statistics, probability, estimation, orders of magnitude, and understanding the assumptions that underlie a prediction or procedure. What is lessening is the need for particular skills, particularly of arithmetic; what is increasingly needed by all is an appreciation of more generalised mathematical concepts and ideas.

Such reducing demands on particular mathematical skills of many citizens, within societies in which mathematics itself is playing an ever increasing role, is a paradox. One result is that there is an employed élite on whom greater, and changing, mathematical demands are placed and who, as a result, are coming to enjoy increasing power. As in other aspects of life, there seems to be growing polarisation in the mathematical demands of employment. These factors must affect curriculum design, pointing to the need for greater diversification of curricula. There is now also the serious and growing problem of long-term unemployment to be considered. How should this and other factors affect the goals of 'mathematics for all'?

Technology, too, affects our ideas of what comprises 'useful knowledge'. Does the long-division algorithm remain in this category in a world in which there is ready access to calculators? Yet the concept of division retains its importance, and even the algorithm itself may still be 'useful' in a propadeutical sense to the minority of students who may later wish to factorise polynomials (unless they in their turn make use of computer software for symbolic manipulation). It must be borne in mind also that there are other varieties of 'usefulness' as well as immediate practical use (such as many specific skills concerned with sport, art, crafts, ...). There is knowledge of non-immediate practical use, where the learner is not yet in a position to need it (e.g. much of school 'commercial arithmetic', concerned with taxes, budgets, investments and the like); there is knowledge of more general, unspecific use (e.g. literacy and numeracy, enabling the learner to do a vast range of practical things); there is career-orientated individual usefulness (e.g. the calculus for potential engineers); there is usefulness for society (e.g. operational research, which enables certain kinds of important problems to be solved, though not everyone in the society needs to know how). All these facets of usefulness must be considered in formulating an answer to the question: What is likely to comprise 'useful' mathematical knowledge in the 1990s?

Students' attitudes to schools and schooling, and their conception of what comprises desirable knowledge and understanding, are also affected by a technological environment. This is yet another important development to which a response must be made.

Despite the considerable and growing impact of technology on schools, our experience of all previous phases of educational change leads us to believe that teachers will continue to play a central role in students' education. However, the nature of that role is changing, and will continue to do so. Technology challenges the present role that most teachers have of being the chief source of knowledge for their students. Children learn a great deal about their world from television and other media, while there is much wider variety of, and easier access to, print materials than during the teacher's own childhood. This change of role from source of information to manager of the curriculum will be a difficult one for the present generation of teachers, and they will need much help and support in making it. Technology can and will affect classroom practice: but in what ways? Here we have a choice of approach. Either we can respond to the latest developments - how do we use the micro? - or we can seek to influence the designers and producers by setting educational goals for future technology. What kind of classroom do we envisage for the 1990s? What technological developments would we wish to take place? How can we help fashion the development of technological hardware for educational purposes? How may the teachers of the 1990s be prepared for new, and to some extent unknown, developments through both pre-service and in-service training? In later chapters we will consider these, among other matters, in greater detail.

3. Mathematics Education in and for Society

It will be clear that technological developments, and responses to them and to the social problems which accompany them, inform much of this study. The general question therefore arises as to whether the teaching of mathematics should be more explicitly related to some of the social issues characteristic of our modern world. The particular issues would vary from one country to another, but could include matters like the statistical relationship between smoking and lung cancer; the proportions of the national budget spent on defence, on health and on education; the development of a critical attitude to statistics put out both by commercial and by official sources; the quantitative gulf which separates the rich and poor countries. The discussion of this question illustrates the format which is used throughout this study.

Can we perceive a new social role for mathematics education in a world in which technology plays a dominant role?

Alternative 1. Mathematics is neutral, and is best taught in isolation from contentious social issues.

Consequences:

1. Teachers will continue to feel comfortable in keeping strictly within the confines of their own subject specialism.
2. The teaching of mathematics will continue to be given high priority by all governments, who see it as a crucial tool for economic and technological advance.
3. Mathematics, in the eyes of the public at large, will continue to retain its aura of mystique and purity, above the common concerns of mankind.
4. Mathematics education will make no direct contribution to the urgent social issues of this generation.

Alternative 2. Since mathematics underpins both technology in all its manifold forms, and the policies that determine how it is used,its teaching should deliberately be related to these issues.

Consequences:

1. It is very difficult to do. Indeed, many - if not most - mathematics teachers will not see it as part of their duty to touch on social and contentious issues.
2. Governments are likely to respond adversely. This has already happened in some countries which have attempted to include a 'social responsibility' component in the teaching of physics, or to introduce 'Peace Studies' into schools.
3. Student motivation is likely to be enhanced.
4. Mathematics educators might make a direct professional contribution to some of the major issues facing human society.

(See, for example, Chapter 1 of Christiansen et al (1986) for a more detailed discussion of possibilities and of difficulties.)

It is hoped that the use of this 'alternatives and consequences' format in subsequent chapters will facilitate constructive discussion of the issues concerned. In considering them in the light of their own circumstances, teachers, students, curriculum developers, and other mathematics educators working in different educational systems will - and should - sometimes come to different conclusions. Often they may consider that a mix of two or more of the alternatives set out is the most appropriate response. Such 'mixes' are not usually included explicitly among the alternatives, since to describe the extremes helps to clarify what it is that is being mixed. As we have already indicated, our aim is not to promulgate any particular doctrine, rather we seek to identify key issues on which decisions will have to be made, and to suggest possible responses together with probable consequences.

Chapter 2

Mathematics and General Educational Goals

1. Mathematics in the School Curriculum

Before trying to subdivide the problem area, it seems useful to consider some general issues that arise from the specific social contexts in which mathematics is taught, and the fact that mathematics is but one component of school life.

School systems are a relatively new phenomenon in historical terms, having only developed during the last hundred years or so. Before then, there were schools in some societies, but these tended to live independent lives united only by their religious underpinning, be it, for example, Christian or Moslem. Education, however, took place before there were schools, and much of what children learn is still learned outside school. Yet much of the socialisation that previously took place in and around the home now takes place in school, and it has now become the prerogative of the school to teach certain limited and specific skills and areas of knowledge. This range of knowledge and skills comprises the formal school curriculum, and contained within it is much of the mathematics that children learn.

How did mathematics come to achieve its central place in the school curriculum? Originally school systems offered education principally at what is now called primary (or elementary) level, and the secular curriculum was almost wholly devoted to the 'Three R's': Reading, Writing, Arithmetic. As the beginnings of secondary level education emerged, this curriculum was expanded to include language (both native and foreign), mathematics, science (physics, chemistry, biology), history and geography, and, later, art and sport. The basic, academic curriculum was proposed in England by the pioneering Chemist, Joseph Priestley, in 1760, and it is astonishing to realise that it is still the standard pattern across the world over 200 years later. The surprise is even greater when one considers what does not appear in it: medicine, economics, politics, technology, individual and group psychology (i.e. understanding people).... The persistence of this curriculum structure aptly illustrates the essential conservatism of the educational process, a conservatism equally evident when we consider the history of the mathematics curriculum.

The familiar school mathematics curriculum was developed in a particular historical and cultural context, that of Western Europe in the aftermath of the Industrial Revolution. Those who framed it only had a minority of society in mind, for at that time only a small élite sector had access to a substantial number of years of schooling. In recent decades, what was once provided for a few has now been made available to -indeed, forced upon - all. Furthermore, this same curriculum has been exported, and to a large extent voluntarily retained, by other countries across the world. The result is an astonishing uniformity of school mathematics curricula world-wide.

It is still true that, faced with a standard school mathematics textbook from an unspecified country, even internationally experienced mathematics educators find it almost impossible to say what part of the world it comes from without recourse to the essentially non-mathematical clues of language and of place-names.

If such uniformity was the result of the 'universality' of mathematics itself, it might be justified. Yet few would argue that mathematics holds its central place in school education simply for its own sake. Its justification as usually stated is to a large extent its 'usefulness', in employment and the future daily lives of students as citizens. Seen in this light, such uniformity is strange. Employment opportunities vary widely, making very different demands on both the nature and levels of mathematical skills and understanding, while the societies concerned span the range from subsistence living to high-technology urban life. There is urgent need for each country's curriculum developers to make a more radical assessment in order to determine a mathematics curriculum appropriate to the needs of their nation in the 1990s.

2. Mathematics for All

The movement towards 'mathematics for all' has given rise to major problems in mathematics education. It has happened so quickly that the full consequences have still to be evaluated and appreciated. New policies will be required for the 1990's. Much thought is already being given to what they should be (see, for example, Damerow et al (1986), Keitel (1985)).

Should mathematics remain at the heart of the school curriculum for all?

Alternative 1. No; 'real mathematics' cannot be taught to everybody.

Consequences:

1. Mathematics will no longer occupy a privileged position as part of the core of general education.

2. An academic élite will study 'real mathematics'.

3. The majority of students will meet only 'useful mathematics', and this in timetable slots labelled physics, technical education, economics, etc.

4. Problems of course selection for different students arise.

Alternative 2. Yes; mathematics must be planned so that it can be effectively taught to all.

Consequences:

1. Mathematics will appear to retain its central place in the school curriculum.
2. This new school mathematics might differ substantially from that traditionally taught.
3. The gap between school mathematics and higher mathematics will grow.
4. Equality of opportunity for all students will be seen to be preserved.

Alternative 3. Yes; but it is accepted that although taught to all it will not be understood by all.

Consequences:

1. Mathematics will retain its status both in the school curriculum and in popular estimation.
2. All students who are capable of understanding the mathematics in the curriculum will have the opportunity to do so.
3. Many students will, as in the past, experience frustration and discouragement.
4. Much teacher time will be wasted in teaching a style of mathematics to students who have effectively given up on the subject.

Alternative 4. Yes; but students will be taught different types of mathematics, or will be taught the same mathematics at different rates, according to their levels of 'ability' or standards of 'attainment'.

Consequences:

1. Mathematics will retain its place in the core of the school curriculum.
2. All students will have opportunity to study a style of mathematics appropriate to them as individuals.
3. Problems of course selection for different students arise.
4. Curriculum planning and the provision of appropriate resources becomes very much more difficult and expensive, at all levels, than with a uniform curriculum.

3. Differentiation of Students and Curricula

Of the alternatives given in the previous section, it is the last, (4), which currently would appear to attract most support. But how can effective differentiation of students and curricula be achieved? To what degree is differentiation desirable/possible?

The range of achievement among different individuals exposed to the same opportunities is greater in mathematics than in almost any other aspect of intellectual endeavour, with the possible exception of music. Such self-evident variation in mathematical capability within a population leads naturally to demands that different mathematical diets be available at school. Yet nearly all attempts to introduce effective differentiation of curricula have failed. Various possible strategies for such differentiation will be considered later, in the hope that they might point the way forward to some resolution of this major current problem in mathematics education.

This lack of success in introducing effective differentiation of mathematics curricula is particularly acute in the developing countries, where the waste of financial and human resources represented by failure at school can be least afforded. This results in increasing failure rates in public examinations as the proportion of the nation's children who go to secondary school rapidly increases, while the school mathematics curriculum remains essentially unchanged from that which was planned for a small academic élite in earlier years.

The reasons for this state of affairs are almost all social and political. While there would be widespread agreement among educational planners to the proposition that differentiation is needed, decisions as to which child follows which curriculum are taken individually, and every individual pupil - and parent - insists on doing the course that opens the way to the greatest opportunities. This multitude of individual decisions is founded on a sense of equal opportunity, which is as strongly developed in a young nation as in those older countries in which egalitarianism has been an increasingly powerful force in recent years. Yet the demand that educational provision shall be equitable for all may well be in conflict with the other widely-held belief that education should allow each individual to realise his or her maximum potential.

Attempts to differentiate by institution have been no more successful than schemes to differentiate within institutions; politicians will rarely risk the unpopularity of having second-class schools established within their constituencies. Can a way be found to meet the widely differing mathematical needs of students at school, while circumventing these social and political objections to overt differentiation? Can it be done without continuing to pay the unacceptable price of widespread 'failure'? The issue is fundamental to many of the decisions that have to be taken by curriculum developers. It will be discussed further in the next chapter in the