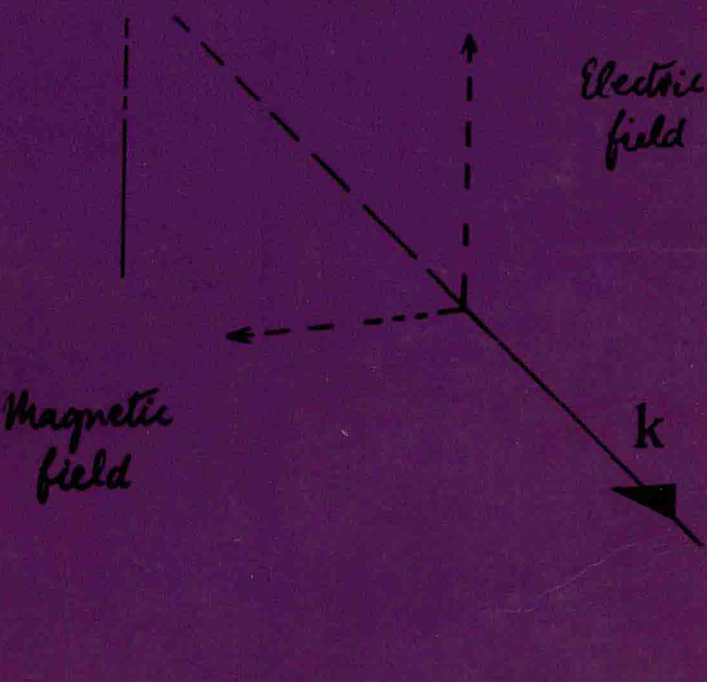
Electromagnetism

I. S. GRANT ▪ W. R. PHILLIPS



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Physical Constants

Permittivity of free space ε_0	$= 8.854 \times 10^{-12}$ farad metre^{-1}
Permeability of free space μ_0	$= 4\pi \times 10^{-7}$ henry metre^{-1}
Velocity of light in vacuo c	$= 2.998 \times 10^{8}$ metre sec^{-1}
Charge of electron e	$= 1.602 \times 10^{-19}$ coulomb
Bohr magneton μ_B	$= 9.274 \times 10^{-24}$ ampere metre^2
Mass of electron m_e	$= 9.107 \times 10^{-31}$ Kg
Mass of proton m_p	$= 1.672 \times 10^{-27}$ Kg
Planck's constant h	$= 6.626 \times 10^{-34}$ joule sec
Boltzmann's constant k	$= 1.381 \times 10^{-23}$ joule K^{-1}
Avogadro's number N_0	$= 6.022 \times 10^{23}$ mol^{-1}

Conversion factors

1 electron volt (eV)	$= 1.602 \times 10^{-19}$ joule
at 293 K, kT	$= 0.0253$ eV
1 newton	$= 10^{5}$ dynes
1 joule	$= 10^{7}$ ergs

A table of *electrical conversion factors* may be found on page 478

The Manchester Physics Series

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*Physics Department, Faculty of Science,
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Editors' Preface to the Manchester Physics Series

In devising physics syllabuses for undergraduate courses, the staff of Manchester University Physics Department have experienced great difficulty in finding suitable textbooks to recommend to students; many teachers at other universities apparently share this experience. Most books contain much more material than a student has time to assimilate and are so arranged that it is only rarely possible to select sections or chapters to define a self-contained, balanced syllabus. From this situation grew the idea of the Manchester Physics Series.

The books of the Manchester Physics Series correspond to our lecture courses with about fifty per cent additional material. To achieve this we have been very selective in the choice of topics to be included. The emphasis is on the basic physics together with some instructive, stimulating and useful applications. Since the treatment of particular topics varies greatly between different universities, we have tried to organize the material so that it is possible to select courses of different length and difficulty and to emphasize different applications. For this purpose we have encouraged authors to use flow diagrams showing the logical connection of different chapters and to put some topics into starred sections or subsections. These cover more advanced and alternative material, and are not required for the understanding of later parts of each volume.

Since the books of the Manchester Physics Series were planned as an integrated course, the series gives a balanced account of those parts of physics which it treats. The level of sophistication varies: '*Properties of Matter*' is for the first year, '*Solid State Physics*' for the third. The other volumes are intermediate, allowing considerable flexibility in use. '*Electromagnetism*', '*Optics*', '*Electronics*' and '*Atomic Physics*' start from first year level and progress to material suitable for second or even third year courses. '*Statistical Physics*' is suitable for second or third year. The books have been written in such a way that each volume is self-contained and can be used independently of the others.

Although the series has been written for undergraduates at an English university, it is equally suitable for American university courses beyond the Freshman year. Each author's preface gives detailed information about the prerequisite material for his volume.

In producing a series such as this, a policy decision must be made about units. After the widest possible consultations we decided, jointly with the authors and the publishers, to adopt SI units interpreted liberally, largely following the recommendations of the International Union of Pure and Applied Physics. Electric and magnetic quantities are expressed in SI units. (Other systems are explained in the volume on electricity and magnetism.) We did not outlaw physical units such as the electron-volt. Nor were we pedantic about factors of 10 (is 0.012 kg preferable to 12 g?), about abbreviations (while s or sec may not be equally acceptable to a computer, they should be to a scientist), and about similarly trivial matters.

Preliminary editions of these books have been tried out at Manchester University and circulated widely to teachers at other universities, so that much feedback has been provided. We are extremely grateful to the many students and colleagues, at Manchester and elsewhere, who through criticisms, suggestions and stimulating discussions helped to improve the presentation and approach of the final versions of these books. Our particular thanks go to the authors, for all the work they have done, for the many new ideas they have contributed, and for discussing patiently, and frequently accepting, our many suggestions and requests. We would also like to thank the publishers, John Wiley and Sons, who have been most helpful in every way, including the financing of the preliminary editions.

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Authors' Preface

This book is based on lectures on classical electromagnetism given at Manchester University. The level of difficulty is suitable for honours physics students at a British University or physics majors at an American University. A-level or high school physics and calculus are assumed, and the reader is expected to have some elementary knowledge of vectors. Electromagnetism is often one of the first branches of physics in which students find that they really need to make use of vector calculus. Until one is used to them, vectors are difficult, and we have accordingly treated them rather cautiously to begin with. Brief descriptions of the properties of the differential vector operators are given at their first appearance. These descriptions are not intended to be a substitute for a proper mathematical text, but to remind the reader what div, grad and curl are all about, and to set them in the context of electromagnetism. The distinction between macroscopic and microscopic electric and magnetic fields is fully discussed at an early stage in the book. It is our experience that students do get confused about the field **E** and **D**, or **B** and **H**. We think that the best way to help them overcome their difficulties is to give a proper explanation of the origin of these fields in terms of microscopic charge distributions or circulating currents.

The logical arrangement of the chapters is summarized in a flow diagram on the inside of the front cover. Provided that one is prepared to accept Kirchhoff's rules and the expressions for the e.m.f.'s across components before discussing the laws on which they are based, the A.C. theory in Chapters 7 and 8 does not require any prior knowledge of the earlier chapters. Chapters 7 and 8 can therefore be used at the beginning of a course on electromagnetism. Sections of the book which are starred may be omitted at a first reading, since they do not contain material needed in order to understand later chapters.

We should like to thank the many colleagues and students who have helped with suggestions and criticisms during the preparation of this book; any errors which remain are our own responsibility. It is also a pleasure to thank Mrs. Margaret King and Miss Elizabeth Rich for their rapid and accurate typing of the manuscript.

May, 1974. I. S. GRANT
Manchester, England. W. R. PHILLIPS

Contents

★ Starred sections may be omitted as they are not required later in the book.

CHAPTER 1

Force and energy in electrostatics

The only laws of force which are known with great precision are the two laws describing the gravitational forces between different masses and the electrical forces between different charges. When two masses or two charges are stationary, then in either case the force between them is inversely proportional to the square of their separation. These inverse square laws were discovered long ago: Newton's law of gravitation was proposed in 1665, and Coulomb's law of electrostatics in 1785. This chapter is concerned with the application of Coulomb's law to systems containing any number of stationary charges. Before studying this topic in detail, it is worth pausing for a moment to consider the consequences of the law in the whole of physics.

In order to make full use of our knowledge of a law of force, we must have a theory of mechanics, that is to say, a theory which describes the behaviour of an object under the action of a known force. Large objects which are moving at speeds small compared to the speed of light obey very closely the laws of classical Newtonian mechanics. For example, these laws and the gravitational force law together lead to accurate predictions of planetary motion. But classical mechanics does not apply at all to observations made on particles of atomic scale or on very fast-moving objects. Their behaviour can only be understood in terms of the ideas of quantum theory and of the special theory of relativity. These two theories have changed the framework of discussion in physics, and have made possible the spectacular advances of the twentieth century.

It is remarkable that while mechanics has undergone drastic amendment, Coulomb's law has stood unchanged. Although the behaviour of atoms does not fit into the framework of the old mechanics, when the Coulomb force is used with the theories of relativity and quantum mechanics, atomic interactions are explained with great precision in every instance when an accurate comparison has been made between experiment and theory. In principle, atomic physics and solid state physics, and for that matter the whole of chemistry, can be derived from Coulomb's law. It is not feasible to derive everything in this way, but it should be borne in mind that atoms make up the world around us, and that its rich variety and complexity are governed by electrical forces.

1.1 ELECTRIC CHARGE

Most of this book applies electromagnetism to large-scale objects, where the atomic origin of the electrical forces is not immediately apparent. However, to emphasize this origin, we shall begin by consideration of atomic systems. The simplest atom of all is the hydrogen atom, which consists of a single proton with a single electron moving around it. The hydrogen atom is stable because the proton and the electron attract one another. In contrast, two electrons repel one another, and tend to fly apart, and similarly the force between two protons is repulsive*. These phenomena are described by saying that there are two different kinds of *electric charge*, and that like charges repel one another, whereas unlike charges are attracted together. The charge carried by the proton is called *positive*, and the charge carried by the electron *negative*.

The magnitude and direction of the force between two stationary particles, each carrying electric charge, is given by Coulomb's law. The law summarizes four facts:

(i) Like charges repel, unlike charges attract.
(ii) The force acts along the line joining the two particles.
(iii) The force is proportional to the magnitude of each charge.
(iv) The force is inversely proportional to the square of the distance between the particles.

The mathematical statement of Coulomb's law is:

$$\mathbf{F}_{21} \propto \frac{q_2 q_1}{r_{21}^3}\mathbf{r}_{21}. \tag{1.1}$$

The vector $\mathbf{F}_{21}$ in Figure 1.1 represents the force on particle 1 (carrying a charge q_1) exerted by the particle 2 (carrying charge q_2). The line from q_2 to q_1 is represented by the vector $\mathbf{r}_{21}$, of length r_{21}: since the unit vector along the direction $\mathbf{r}_{21}$ can be written $\mathbf{r}_{21}/r_{21}$, Equation (1.1) is an *inverse square law* of force, although r_{21}^3 appears in the denominator. Notice that the equation automatically

* If the protons are separated by a distance less than 10^{-14} m, they are affected by the very short range nuclear forces. Unlike gravitational and electrical forces, nuclear forces are not known precisely. However, in the study of atoms one does not need to know anything about nuclear forces beyond the fact that they are strong enough to bind together the constituent parts of the atomic nucleus.