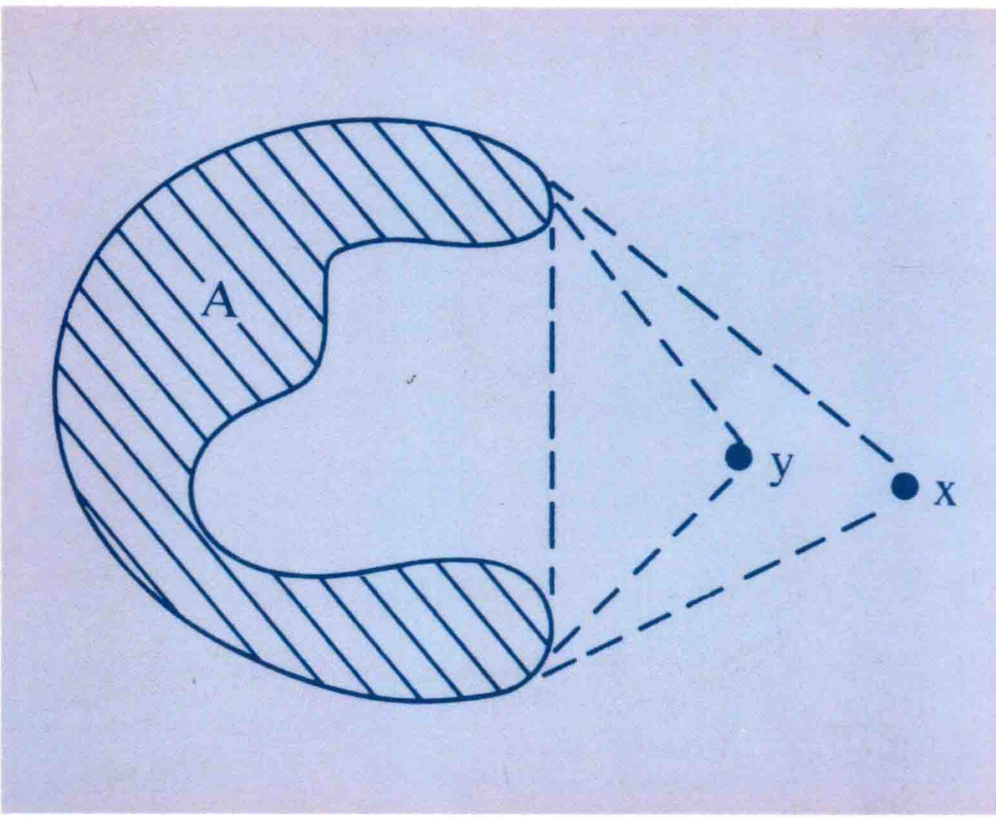
ENCYCLOPEDIA OF MATHEMATICS AND ITS APPLICATIONS 40

MATROID APPLICATIONS

Edited by NEIL WHITE



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This volume, the third in a sequence that began with *The Theory of Matroids* and *Combinatorial Geometries*, concentrates on the applications of matroid theory to a variety of topics from engineering (rigidity and scene analysis), combinatorics (graphs, lattices, codes and designs), topology and operations research (the greedy algorithm). As with its predecessors, the contributors to this volume have written their articles to form a cohesive account so that the result is a volume that will be a valuable reference for research workers.

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Matroid Applications

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PREFACE

This is the third volume of a series that began with *Theory of Matroids* and continued with *Combinatorial Geometries*. These three volumes are the culmination of more than a decade of effort on the part of the many contributors, potential contributors, referees, the publisher, and numerous other interested parties, to all of whom I am deeply grateful. To all those who waited, please accept my apologies. I trust that this volume will be found to have been worth the wait.

This volume begins with Walter Whiteley's chapter on the applications of matroid theory to the rigidity of frameworks: matroid constructions prove to be rather useful and matroid terminology provides a helpful language for the basic results of this theory. Next we have Deza's chapter on the beautiful applications of matroid theory to a special aspect of combinatorial designs, namely perfect matroid designs. In Chapter 3, Oxley considers ways of generalizing the matroid axioms to infinite ground sets, and Simões-Pereira's chapter on matroidal families of graphs discusses other ways of defining a matroid on the edge set of a graph than the usual graphic matroid method. Next, Rival and Stanford consider two questions on partition lattices. These lattices are a special case of geometric lattices and the inclusion of this chapter will provide a lattice-theoretic perspective which has been lacking in much current matroid research (but which seems alive and well in *oriented* matroids). Then we have the comprehensive survey by Brylawski and Oxley of the Tutte polynomial and Tutte–Grothendieck invariants. These express the deletion–contraction decomposition that is so important within matroid theory and some of its important applications, namely graph theory and coding theory. Björner describes the homology and shellability properties of several simplicial complexes associated with a matroid; the complexes of independent sets, of broken circuits, and of chains in the geometric lattice. This chapter and the previous one constitute a study of the deepest known matroid invariants. We conclude with an exposition by Björner and Ziegler of greedoids, a generalization of matroids that embody the greedy algorithm and hence are very useful in operations research.

University of Florida Neil L. White

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1

Matroids and Rigid Structures

WALTER WHITELEY

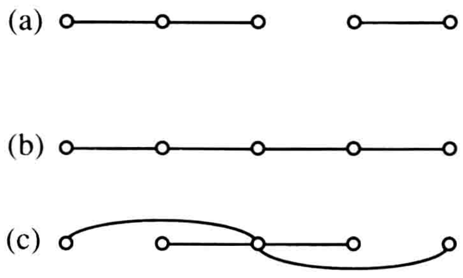
Many engineering problems lead to a system of linear equations – a represented matroid – whose rank controls critical qualitative features of the example (Sugihara, 1984; 1985; White & Whiteley, 1983). We will outline a selection of such matroids, drawn from recent work on the rigidity of spatial structures, reconstruction of polyhedral pictures, and related geometric problems.

For these situations, the combinatorial pattern of the example determines a sparse matrix pattern that has both a generic rank, for general 'independent' values of the non-zero entries, and a geometric rank, for special values for the coordinates of the points, lines, and planes of the corresponding geometric model. Increasingly, the generic rank of these examples has been studied by matroid theoretic techniques. These geometric models provide nice illustrations and applications of techniques such as matroid union, truncation, and semimodular functions. The basic unsolved problems in these examples highlight certain unsolved problems in matroid theory. Their study should also lead to new results in matroid theory.

1.1. Bar Frameworks on the Line – the Graphic Matroid

We begin with the simplest example, which will introduce the vocabulary and the basic pattern. We place a series of distinct points on a line, and specify certain *bars* – pairs of joints which are to maintain their distance – defining a *bar framework on the line.* We ask whether the entire framework is 'rigid' – i.e. does any motion of the joints along the line, preserving these distances, give all joints the same velocity, acceleration, etc.? Clearly a framework has an *underlying graph* $G = (V, E)$, with a vertex v_i for each joint p_i and an undirected edge $\{i, j\}$ for each bar $\{p_i, p_j\}$. In fact, we describe the framework as $G(\mathbf{p})$, where G is a graph without multiple edges or loops, and $\mathbf{p}$ is an assignment of points p_i to the vertices v_i. If this graph is not connected,

Figure 1.1.



then each component can move separately in the framework, and the framework is not rigid (Figure 1a). Conversely, a connected graph always leads to a rigid framework (Figure 1.1b), since each bar ensures that its two joints have the same motion on the line. This gives an informal proof of the following result.

1.1.1. Proposition. *A bar framework $G(\mathbf{p})$ on the line is rigid if and only if the underlying graph G is connected.*

To extract a matrix, we make this argument a little more formal. Assume the joints p_i move along smooth paths $p_i(t)$. The length of a bar $\|p_i(t)-p_j(t)\|$, and its square, remain constant. If we differentiate, this condition becomes

$$\frac{\mathrm{d}}{\mathrm{d}t}[p_i(t)-p_j(t)]^2=[p_i(t)-p_j(t)][p_i'(t)-p_j'(t)]=0.$$

At $t=0$, this is written $(p_i-p_j)(p_i'-p_j')=0$. If we have distinct joints on the line, so that $(p_i-p_j)\neq 0$, this simplifies to $(p_i'-p_j')=0$.

With this in mind, we define an *infinitesimal motion* of a bar framework on the line $G(\mathbf{p})$ as an assignment of a velocity u_i along the line to each joint p_i such that $u_i-u_j=0$ for each bar $\{v_i, v_j\}$. For example, consider the framework in Figure 1.1c. The four bars lead to four equations in the unknowns $\mathbf{u}=(u_1, u_2, u_3, u_4, u_5)$:

$$\begin{bmatrix}1 & 0 & -1 & 0 & 0\\ 0 & 1 & -1 & 0 & 0\\ 0 & 0 & 1 & -1 & 0\\ 0 & 0 & 1 & 0 & -1\end{bmatrix}\begin{bmatrix}u_1\\ u_2\\ u_3\\ u_4\\ u_5\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\\ 0\end{bmatrix}.$$

In general, this system of linear equations is written $R(G, \mathbf{p})\times\mathbf{u}^t=0$, where the *rigidity matrix* $R(G, \mathbf{p})$ has a row for each edge of the graph and a column for each vertex, and $\mathbf{u}^t$ is the transpose of the vector of velocities. We note that $R(G, \mathbf{p})$ is the transpose of the usual matrix representation for the graph over the reals: the rows are independent in $R(G, \mathbf{p})$ if and only if the corresponding edges are a forest (an independent set of edges in the cycle matroid of the graph).

A *trivial infinitesimal motion* is the derivative of a rigid motion of the line – i.e. a translation with all velocities equal. These form a one-dimensional subspace of the solutions. *An infinitesimally rigid framework on the line* has only these trivial infinitesimal motions, so the rigidity matrix has rank $|V| - 1$. This rank corresponds to a spanning tree on the vertices, or a basis for the cycle matroid of the complete graph on $|V|$ vertices. This proves the following infinitesimal version of Proposition 1.1.1.

1.1.2. Proposition. *A bar framework $G(\mathbf{p})$ on the line is infinitesmially rigid if and only if the underlying graph G is connected.*

1.2. Bar Frameworks in the Plane

A *bar framework in the plane* is a graph $G = (V, E)$ and an assignment $\mathbf{p}$ of points $\mathbf{p}_i \in \mathbb{R}^2$ to the vertices v_i such that $\mathbf{p}_i \neq \mathbf{p}_j$ if $\{i, j\} \in E$. If we differentiate the condition that bars have constant length in any smooth motion, we have

$$\frac{\mathrm{d}}{\mathrm{d}t}[\mathbf{p}_i(t) - \mathbf{p}_j(t)]^2 = [\mathbf{p}_i(t) - \mathbf{p}_j(t)]\cdot[\mathbf{p}_i'(t) - \mathbf{p}_j'(t)] = 0.$$

Accordingly, an *infinitesimal motion* of plane bar framework is an assignment $\mathbf{u}$ of velocities $\mathbf{u}_i \in \mathbb{R}^2$ to the joint such that

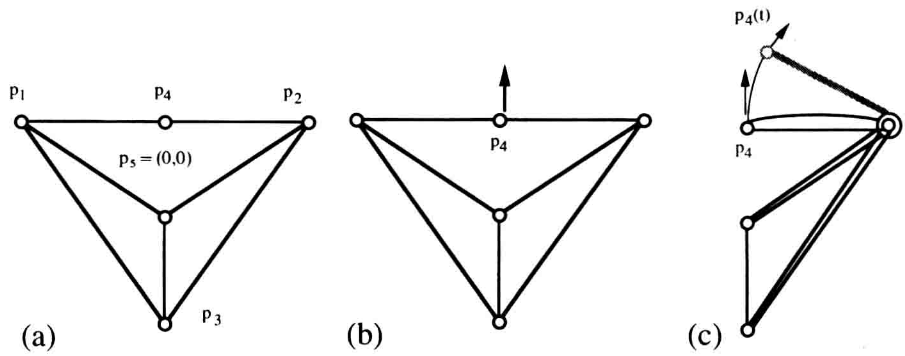
$$(\mathbf{p}_i - \mathbf{p}_j)\cdot(\mathbf{u}_i - \mathbf{u}_j) = 0 \quad \text{for each } \{i, j\} \in E.$$

A plane bar framework is *infinitesimally rigid* if all infinitesimal motions are *trivial*: $\mathbf{u}_i = \mathbf{s} + \beta(\mathbf{p}_i)^\perp$, where $\mathbf{s}$ is a fixed translation vector, $(\mathbf{x}, \mathbf{y})^\perp = (\mathbf{y}, -\mathbf{x})$ rotates the vector 90° counterclockwise, and $\beta(\mathbf{p}_i)^\perp$ represents a rotation about the origin. (These infinitesimal rotations and translations are the derivatives of smooth rigid motions of the plane.)

The system of equations for an infinitesimal motion has the form $R(G, \mathbf{p}) \times \mathbf{u}^t = 0$, where the *rigidity matrix* $R(G, \mathbf{p})$ now has a row for each edge of the graph and two columns for each vertex. The row for edge $\{i, j\}$ has the form

$$[0 \;\; 0 \;\; \ldots \;\; 0 \;\; 0 \;\; \mathbf{p}_i - \mathbf{p}_j \;\; 0 \;\; 0 \;\; \ldots \;\; 0 \;\; 0 \;\; \mathbf{p}_j - \mathbf{p}_i \;\; 0 \;\; 0 \;\; \ldots \;\; 0 \;\; 0]$$

Figure 1.2.



1.2.1. Example. Consider the frameworks in Figure 1.2. The framework of Figure 1.2a gives the rigidity matrix

$$\begin{array}{c} \{1,3\} \\ \{1,4\} \\ \{1,5\} \\ \{2,3\} \\ \{2,4\} \\ \{2,5\} \\ \{3,5\} \end{array} \begin{bmatrix} x_1-x_3 & y_1-y_3 & 0 & 0 & x_3-x_1 & y_3-y_1 & 0 & 0 & 0 & 0 \\ \frac{1}{2}(x_1-x_2) & \frac{1}{2}(y_1-y_2) & 0 & 0 & 0 & 0 & \frac{1}{2}(x_2-x_1) & \frac{1}{2}(y_2-y_1) & 0 & 0 \\ x_1 & y_1 & 0 & 0 & 0 & 0 & 0 & 0 & -x_1 & -y_1 \\ 0 & 0 & x_2-x_3 & y_2-y_3 & x_3-x_2 & x_3-y_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2}(x_2-x_1) & \frac{1}{2}(y_2-y_1) & 0 & 0 & \frac{1}{2}(x_1-x^2) & \frac{1}{2}(y_1-y^2) & 0 & 0 \\ 0 & 0 & x_2 & y_2 & 0 & 0 & 0 & 0 & -x_2 & -y_2 \\ 0 & 0 & 0 & 0 & x_3 & y_3 & 0 & 0 & -x_3 & -y_3 \end{bmatrix}$$

The rows of this matrix are dependent and have rank 6. This leaves a $(10-6=4)$-dimensional space of infinitesimal motions, including the non-trivial motion shown in Figure 1.2b, which assigns zero velocity to all joints but $\mathbf{p}_4$, and gives $\mathbf{p}_4$ a velocity perpendicular to the bars at $\mathbf{p}_4$. Thus the framework is not infinitesimally rigid.

The infinitesimal motion is not the derivative of some smooth path for the vertices. The framework is *rigid* – all smooth paths, or even continuous paths, give frameworks congruent to the original framework. Figure 1.2c gives a similar framework which has the same infinitesimal motions, but is not rigid.

These examples show that there is a difference in the plane between rigid frameworks and infinitesimally rigid frameworks. A *non-rigid* plane framework will have an analytic path of positions $\mathbf{p}(t) = (\ldots, \mathbf{p}_i(t), \ldots)$, with all bar lengths of $\mathbf{p}(t)$ the same as bars in $\mathbf{p}(0)$, but $\mathbf{p}(t)$ not congruent to $\mathbf{p}(0)$, for all $0 < t < 1$ (Figure 1.2c). The first non-zero derivative of this path will be a non-trivial infinitesemal motion. However, the converse is false: many infinitesimal motions are not the derivative of an analytic path (recall Figure 1.2b). For any framework, the independence of the rows of the rigidity matrix induces a matroid on the edges of the graph. If 'rigidity' in a particular plane framework were used to define an independence structure on the edges of a graph, this need not be a matroid (see Exercise 1.6). Therefore, we will restrict ourselves, throughout this chapter, to the simpler concepts of infinitesimal motions and infinitesimal rigidity.

The space of trivial plane infinitesimal motions has dimension 3, for frameworks with at least two distinct joints. This space can be generated by two translations in distinct directions and a rotation about any fixed point. Thus an infinitesimally rigid framework with more than two joints will have an $|E|$ by $2|V|$ rigidity matrix of rank $2|V|-3$. Our basic problem is to determine which graphs G allow this matrix to have rank $2|V|-3$ for at least some plane frameworks $G(\mathbf{p})$.

The independence structure of the rows of the rigidity matrix defines a matroid on the edges of the complete graph on the vertices. This matroid depends on the positions of the joints. If we vary the positions there are 'generic' positions that give a maximal collection of independent sets (for example, positions where the coordinates are algebraically independent real numbers). At these positions we have the *generic rigidity matroid for $|V|$ vertices in the plane*.

1.2.2. Example. Consider the framework in Figure 1.3a. With vertices as indicated we have the rigidity matrix

$$\begin{array}{r} (a_1, b_1) \\ (a_1, b_2) \\ (a_1, b_3) \\ (a_2, b_1) \\ (a_2, b_2) \\ (a_2, b_3) \\ (a_2, b_1) \\ (a_3, b_2) \\ (a_3, b_3) \end{array} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 1 & 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -2 & -1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

The graph of the framework has $|E| = 2|V| - 3$, so the framework is infinitesimally rigid if and only if the rows are independent. This independence can be checked by deleting the final three columns and seeing that the determinant of the 9×9 submatrix is non-zero. This framework is infinitesimally rigid and the graph is generically rigid, and generically independent.

Consider any realization with distinct joints $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3$ on a unit circle centred at the origin (Figure 1.3b). This has a non-trivial 'in–out' infinitesimal motion (Figure 1.3c):

for joints $\mathbf{a}_i$ take the velocity $\mathbf{a}_i' = \mathbf{a}_i$;

for joints $\mathbf{b}_j$ take the velocity $\mathbf{b}_j' = -\mathbf{b}_j$.

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