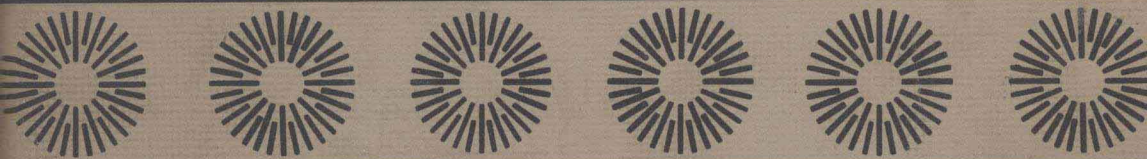


Computational Mathematics in Engineering

S. A. Hovanessian



Lexington Books

Computational Mathematics in Engineering

S.A. Hovanessian

Lexington Books

D.C. Heath and Company
Lexington, Massachusetts
Toronto

Library of Congress Cataloging in Publication Data

Hovanessian, Shahen A. 1931-

Computational mathematics in engineering.

Includes index.

1. Engineering mathematics. I. Title.

TA330.H68 519.4'02'462 76-14667

ISBN 0-669-00733-1

Copyright © 1976 by D.C. Heath and Company

All rights reserved. No part of this publication may be reproduced or transmitted in any form or by any means, electronic or mechanical, including photocopy, recording, or any information storage or retrieval system, without permission in writing from the publisher.

Published simultaneously in Canada

Printed in the United States of America

International Standard Book Number: 0-669-00733-1

Library of Congress Catalog Card Number: 76-14667

Computational Mathematics in Engineering

To the memory of my teachers
Louis A. Pipes
Professor of Engineering
and
Ivan S. Sokolnikoff
Professor of Mathematics
University of California, Los Angeles

Preface

Over the past several years, the computational methods available to engineers have greatly expanded. This, together with the fact that a greater number of engineers use digital computers, has made it desirable to compile the numerical methods applicable to the solution of engineering problems under one cover.

This text is written on a senior or graduate level for students of engineering and practicing engineers. The basics of numerical analysis and computational methods are covered in the first three chapters. These chapters consist of numerical evaluation of matrices and simultaneous equations, calculus of finite differences, and numerical solution of differential equation.

Chapters 4 and 5 cover the basics of probability theory and the least squares methods and estimates. Chapter 4 gives a brief review of probability methods and theorems which are applicable to the digital simulation of probabilistic problems. In addition, this chapter prepares the reader for the discussion of least squares methods and estimates given in the next chapter. Chapter 5 starts with a discussion of classical least squares polynomial fits and proceeds to cover recursive trackers, linear least square methods, and Kalman filtering.

Chapters 6, 7, and 8 discuss various numerical methods of special interest. Chapter 6 gives a discussion of Fourier series and transforms and follows this with computational methods and algorithms. Chapter 7 covers some of the currently used optimization methods, such as linear programming, quadratic programming, and nonlinear programming methods. Chapter 8 gives several numerical methods for the computation of characteristic values and vectors.

In the use of the book as a text, the first three chapters are basic to a knowledge of numerical methods and should be covered. Additional material from Chapters 4 through 8 may be included, depending on the orientation of the class. For example, for students of electrical engineering, the material of Chapters 4, 5, and 6 is desirable, while for students of civil and mechanical engineering, Chapters 6, 7 and 8 may be appropriate.

Each chapter contains about twenty problems, with answers to every problem given at the end of the book. A solution book will also be provided by the author to instructors upon written request.

S.A. Hovanessian

Los Angeles, California
April 1976

Contents

	List of Figures	xi
	List of Tables	xiii
	Preface	xv
Chapter 1	Numerical Evaluation of Matrices and Simultaneous Equations	1
	Some Matrix Properties	1
	Upper and Lower Triangular Matrices	3
	Choleski's Method	7
	Vectors and Matrices	9
	Solution of Simultaneous Equations	10
	Elimination Method and Pivotal Compensation	18
	Ill-conditioned Matrices	19
	Inversion of Matrices by Elimination Method	22
	Correction of the Elements of Inverse Matrix	25
	Evaluation of the Methods	26
	Problems	27
Chapter 2	Calculus of Finite Differences	33
	Difference Operators	34
	Relationship Between Difference and Differential Operators	36
	Relationship Between ∇ and D	38
	Relationship Between Δ and D	41
	Relationship Between δ and D	43
	Integration Formulas by Interpolating Polynomials	47
	Problems	51
Chapter 3	Numerical Solution of Ordinary Differential Equations	57
	Preliminary Discussion	57
	Series Expansion	62
	Runge-Kutta Methods	63
	Milne's Predictor-Corrector Methods	66

	Numerov's Method	69
	Boundary Value Problems	70
	Solution by Direct Substitution	74
	Simulation of Servo Systems	76
	Stability of Solution of Difference Formulations	79
	Discussion of the Methods	82
	Problems	83
Chapter 4	Probability Theory and Monte Carlo Simulations	93
	Basic Ideas of Probability Theory	93
	Bernoulli's Problem	95
	First and Second Moments	95
	Expected Values	98
	Properties of Expected Values	100
	Normal Distribution	102
	Exponential Distribution	104
	Goodness-of-fit Tests	105
	Tolerance Limits	111
	Confidence Limits	113
	Generation of Random Numbers	113
	Monte Carlo Simulations	114
	Problems	115
Chapter 5	Least-Squares Methods and Estimates	127
	Method of Least Squares	127
	Recursive Trackers	131
	The α - β - δ Tracker	134
	Linear Least Squares Estimates	136
	Gauss-Markov Theorem	138
	Recursive Estimators	139
	Recursive Estimators with a priori Knowledge	141
	Kalman's Theorem	142
	Numerical Example of Kalman Filtering	143
	Problems	147
Chapter 6	Spectral Analysis and the Fast Fourier Transform	155
	Fourier Series	155
	Complex Fourier Series and Transforms	158
	Numerical Evaluation of Fourier Transform	161
	Cooley-Tukey Method	165
	Analytic Derivation of Cooley-Tukey Relations	171

	Aliasing Effect	172
	Digital Filtering	173
	Discussion of the Methods	178
	Problems	179
Chapter 7	Computer Optimization Methods	181
	Optimization with Constraints	181
	Lagrangian Multipliers Method	183
	Linear Programming Method	186
	Lagrangian Multipliers and Quadratic Program- ming	191
	Quadratic Programming in Simplex Formulation	192
	Nonlinear Constraints	196
	Problems	199
Chapter 8	Calculation of Characteristics Values and Vectors	203
	General Background	203
	First Characteristic Value and Vector (Power Method)	205
	Improving the Convergence of the Iteration Process	207
	Second Characteristic Value and Vector	210
	Second Characteristic Value and Vector of Sym- metric Matrices	213
	Method of A.M. Danilevsky	217
	Jacobi's Method for Characteristic Values and Vectors	221
	Discussion of the Methods	225
	Problems	226
	Answers to Problems	231
	Index	247
	About the Author	253

List of Figures

1-1	Geometric Representation of the Three-Dimensional Vector \mathbf{X}	9
2-1	The Function $y = f(x)$	35
3-1	Function y vs. x with Increments of h	58
3-2	Servo System of the Example Problem	77
3-3	Input to Servo System	78
3-4	Possible Output of the Servo System	78
4-1	Orthogonal Vectors \mathbf{A} and \mathbf{B} of Equation (4.43)	102
4-2	Graphs of Normal Distribution	103
4-3	Gaussian Probability Paper	104
4-4	Graphs of Exponential Distribution	106
4-5	Exponential Distribution on Semilogarithmic Paper	110
4-6	Servo System Monte Carlo Simulation	115
4-7	Effect of Input Noise on the Output	116
5-1	Error Between Data Points and the Fitted Curve	128
5-2	Least-Squares Fit of the Example Problem	130
5-3	Critical and Underdamped Behavior of α - β Tracker	134
5-4	Geometric Interpretation of Linear Estimate	140
5-5	Actual and Filtered Values of Variables of Example Problem	148
5-6	The Values of α - β - γ Computed for the Example Problem	149
6-1	Example of Fourier Series	157
6-2	Discontinuous Function $f(x)$ vs. x	158
6-3	Continuous Function $f(x)$ with Discontinuous Derivative $f'(x)$	158
6-4	$F(t)$ for the Example Problem	160
6-5	Solution of the Example Problem	160
6-6	Discrete Representation of $F(t)$ and $G(\omega)$	162
6-7	Powers of W with $N = 4$	164
6-8	The Function $f(t)$ vs. Time, Example 1	166
6-9	Computation Tree for $N = 4$	169
6-10	Relation Representing the Result of Cooley-Tukey Computation, ($N = 8$)	172

6-11	Misrepresentation of a High-Frequency Wave Form by a Low-Frequency Sampling	173
6-12	System Response to Input $f(t)$	173
6-13	The Function $f(t)$ as a Series of Impulses	174
6-14	Bandpass Filter Passing Frequencies Between Ω_1 and Ω_2	176
6-15	Time Function $f(t)$ vs. t	177
7-1	Graphical Presentation of Linear Programming Solution	190
7-2	Constraints and the Solution of the Example	197

List of Tables

2-1	Difference Operators of $y = x^2$	37
2-2	Relationship Between Backward Difference Operator ∇ and Differential Operator D	41
2-3	Relationship Between Forward Difference Operator Δ and Differential Operator D	43
2-4	Relationship Between Central Difference Operator δ and Differential Operator D	46
3-1	Solutions of the Example Problem	61
3-2	Results of Predictor-Corrector Method	68
4-1	Data and Calculation for Example Problem	99
4-2	Kolmogorov-Smirnov Numerics for Goodness-of-Fit Test	108
4-3	Data Points for Mean Time Between Failures	109
4-4	Kolmogorov-Smirnov Test Applied to Monte Carlo Results	112
6-1	Solution of the Example Problem No. 1	167
6-2	The Inverse Transform	168
6-3	The Order of the G Matrix ($N = 4$)	170
6-4	The Order of the G Matrix ($N = 8$)	170
7-1	Necessary and Sufficient Conditions for an Extremum	186
7-2	Linear Programming Formulation	195
7-3	Linear Programming Formulation	197
7-4	Solution of Separable Programming Problem	198

$$\begin{pmatrix} c_{11} & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & b_{23} \\ 0 & 0 & b_{33} \end{pmatrix} \quad (1.8)$$

lower triangular matrix
upper triangular matrix

where the lower triangular matrix is defined as a matrix with zero elements above the diagonal and the upper triangular matrix is defined as a matrix with zero elements below the diagonal. With these definitions, we consider the following methods based on the utilization of triangular matrices. These methods, for ease of understanding, are described using third-order matrices. The methods, of course, can be generalized to higher order matrices.

Theorem: On the condition that the leading submatrices of the matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \quad (1.9)$$

are nonsingular, i.e.,

$$a_{11} \neq 0, \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \neq 0, \dots, [\mathbf{A}] \neq 0 \quad (1.10)$$

matrix \mathbf{A} may be represented by the product of lower and upper triangular matrices. In equation (1.10), brackets represent determinants of the leading submatrices of (1.9). These submatrices start with the element of row 1 column 1 and proceed to the second- and third-order matrices along the diagonal of the original matrix \mathbf{A} .

Denoting the lower and upper triangular matrices of \mathbf{A} by \mathbf{L} and \mathbf{U} , respectively, we have

$$\begin{aligned} \mathbf{A} &= \mathbf{L}\mathbf{U} \\ \mathbf{A} &= (a_{ij}) \quad \mathbf{L} = (l_{ij}) \quad \mathbf{U} = (u_{ij}) \quad (1.11) \\ a_{ij} &= \sum_{k=1}^{\min(i,j)} l_{ik} u_{kj} \end{aligned}$$

In the above notation a_{ij} represents the element of row i column j of matrix \mathbf{A} . The lowercase letters are used to denote matrix elements and uppercase letters are used to denote matrices. Note that we will have n^2

equations in $n^2 + n$ unknowns u_{ij} and l_{ij} . Our latitude lies in the specifying of the diagonal coefficients l_{ii} or u_{ii} as seen in the example below.

Example: For a 3×3 matrix we have

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} \quad (1.12)$$

where the **A** matrix is represented as the product of the lower and the upper triangular matrices **L** and **U**. Performing the matrix product of (1.12) and equating the resulting matrices term by term, we get the set of equations

$$\begin{aligned} a_{11} &= l_{11} u_{11} & a_{21} &= l_{21} u_{11} \\ a_{12} &= l_{11} u_{12} & a_{22} &= l_{21} u_{12} + l_{22} u_{22} \\ a_{13} &= l_{11} u_{13} & a_{23} &= \dots \end{aligned} \quad (1.13)$$

By selecting values of l_{11} , l_{22} and l_{33} , the rest of the values of the triangular matrices can be computed.

Furthermore, the inverse of lower and upper triangular matrices have the same form. For example, the inverse of a lower triangular matrix will also be a lower triangular matrix, as shown below for a third-order matrix

$$\begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} x_{11} & 0 & 0 \\ x_{21} & x_{22} & 0 \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.14)$$

Assuming that we already have calculated the elements l_{11} , l_{21} , ... of the lower triangular matrix, we can perform the above matrix multiplication and obtain a set of equations for the calculation of the elements of the inverse matrix x_{11} , x_{21} , x_{22} , ... This results in

$$\begin{cases} l_{11}x_{11} = 1 \\ l_{21}x_{11} + l_{22}x_{21} = 0 \\ l_{31}x_{11} + l_{32}x_{21} + l_{33}x_{31} = 0 \end{cases} \quad (1.15)$$

$$\begin{cases} l_{22}x_{22} = 1 \\ l_{32}x_{22} + l_{33}x_{32} = 0 \end{cases} \quad (1.16)$$

$$l_{33}x_{33} = 1 \quad (1.17)$$