Computational Mathematics in Engineering

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Computational Mathematics in Engineering

To the memory of my teachers

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and

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Preface

Over the past several years, the computational methods available to engineers have greatly expanded. This, together with the fact that a greater number of engineers use digital computers, has made it desirable to compile the numerical methods applicable to the solution of engineering problems under one cover.

This text is written on a senior or graduate level for students of engineering and practicing engineers. The basics of numerical analysis and computational methods are covered in the first three chapters. These chapters consist of numerical evaluation of matrices and simultaneous equations, calculus of finite differences, and numerical solution of differential equation.

Chapters 4 and 5 cover the basics of probability theory and the least squares methods and estimates. Chapter 4 gives a brief review of probability methods and theorems which are applicable to the digital simulation of probabilistic problems. In addition, this chapter prepares the reader for the discussion of least squares methods and estimates given in the next chapter. Chapter 5 starts with a discussion of classical least squares polynomial fits and proceeds to cover recursive trackers, linear least square methods, and Kalman filtering.

Chapters 6, 7, and 8 discuss various numerical methods of special interest. Chapter 6 gives a discussion of Fourier series and transforms and follows this with computational methods and algorithms. Chapter 7 covers some of the currently used optimization methods, such as linear programming, quadratic programming, and nonlinear programming methods. Chapter 8 gives several numerical methods for the computation of characteristic values and vectors.

In the use of the book as a text, the first three chapters are basic to a knowledge of numerical methods and should be covered. Additional material from Chapters 4 through 8 may be included, depending on the orientation of the class. For example, for students of electrical engineering, the material of Chapters 4, 5, and 6 is desirable, while for students of civil and mechanical engineering, Chapters 6, 7 and 8 may be appropriate.

Each chapter contains about twenty problems, with answers to every problem given at the end of the book. A solution book will also be provided by the author to instructors upon written request.

S.A. Hovanessian

Los Angeles, California
April 1976

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1 Numerical Evaluation of Matrices and Simultaneous Equations

Matrices form an important branch of mathematics, as they are the logical extension of vectors. Vectors can be considered as one-dimensional matrices while matrices, in their usual representation, are considered as two dimensional. Higher dimension (more than two) matrices are replaced by tensors. Thus, vectors and matrices can be considered one- and two-dimensional tensors, respectively.

In applied mathematics matrices are encountered in the solution of simultaneous equations, numerical solution of boundary value differential equation, numerical solution of partial differential equations, and in a number of problems in electrical circuits and mechanical vibrations. The formulation of many engineering problems for numerical solution results in a set of simultaneous linear equations which can be solved using matrix methods and digital computers.

This chapter discusses some useful properties of matrices and will give several numerical methods for the evaluation of matrices and simultaneous linear equations.

Some Matrix Properties

1. Multiplication of some row i of an nth order matrix by a constant α is equivalent to premultiplication of the matrix by

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} & & & i & & & \end{array} \\
\begin{array}{c} \\ \\ \\ i \\ \\ \\ \\ \end{array} &
\begin{pmatrix}
1 & & & & & & \\
 & \ddots & & \vdots & & & \\
 & & 1 & & & & \\
\cdots & \cdots & \cdots & \alpha & & & \\
 & & & & 1 & & \\
 & & & & & \ddots & \\
 & & & & & & 1
\end{pmatrix}
\end{array}
\qquad (1.1)
$$

where the above matrix is an nth order unity matrix with the element of the ith row, ith column replaced by α. As an example, consider the multiplication of the second row of the third-order matrix

$$\begin{pmatrix} a & b & c \\ x & y & z \\ u & v & w \end{pmatrix} \tag{1.2}$$

by α. This can be written as

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a & b & c \\ x & y & z \\ u & v & w \end{pmatrix} = \begin{pmatrix} a & b & c \\ \alpha x & \alpha y & \alpha z \\ u & v & w \end{pmatrix} \tag{1.3}$$

2. Consider an nth order matrix. Adding to the elements of some row (i) numbers proportional to the elements of some preceding row (j) is equivalent to the premultiplication of the matrix by

$$\begin{array}{cc} & \begin{array}{cccccc} & & j & & & \end{array} \\ i & \begin{pmatrix} 1 & & \vdots & & & \\ & \ddots & & & & \\ & & 1 & & & \\ & & \vdots & \ddots & & \\ \cdots & & \alpha & \cdots & 1 & \\ & & & & & \ddots \\ & & & & & & 1 \end{pmatrix} \end{array} \tag{1.4}$$

where the above matrix represents an nth order unity matrix with the element of the ith row and jth column replaced by the proportionality constant α. As an example, consider the third-order matrix of equation (1.2). Let it be required to multiply the elements of the second row of this matrix by α and add these to the elements of the third row. This can be accomplished as follows:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \alpha & 1 \end{pmatrix}\begin{pmatrix} a & b & c \\ x & y & z \\ u & v & w \end{pmatrix} = \begin{pmatrix} a & b & c \\ x & y & z \\ y+\alpha x & v+\alpha y & w+\alpha z \end{pmatrix} \tag{1.5}$$

3. Consider an nth order matrix. Addition to the elements of some row (i) numbers proportional to the elements of some following row (j) is equivalent to premultiplication of the matrix by

$$\begin{array}{cc} \begin{matrix} & & & j & & \end{matrix} & \\ \begin{pmatrix} 1 & & & \vdots & & \\ & \ddots & & \vdots & & \\ & & 1 \cdots & \alpha \cdots & \cdots & \\ & & \ddots & \vdots & & \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 1 \end{pmatrix} & \begin{matrix} \\ \\ i \\ \\ \\ \\ \\ \end{matrix} \end{array} \tag{1.6}$$

where the above matrix represents an nth order unity matrix with the element in row (i) column (j) replaced by the proportionality constant α. As an example, consider the multiplication of the third row of the matrix of equation (1.2) by α and the subsequent addition of this row to the second row. This can be accomplished by matrix multiplication

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \alpha \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a & b & c \\ x & y & z \\ u & v & w \end{pmatrix} = \begin{pmatrix} a & b & c \\ x+\alpha u & y+\alpha v & z+\alpha w \\ u & v & w \end{pmatrix} \tag{1.7}$$

Upper and Lower Triangular Matrices

Consider the following third-order triangular matrices

$$\begin{pmatrix} c_{11} & 0 & 0 \\ c_{21} & c_{22} & 0 \\ c_{31} & c_{32} & c_{33} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ 0 & b_{22} & b_{23} \\ 0 & 0 & b_{33} \end{pmatrix} \tag{1.8}$$

lower triangular matrix — upper triangular matrix

where the lower triangular matrix is defined as a matrix with zero elements above the diagonal and the upper triangular matrix is defined as a matrix with zero elements below the diagonal. With these definitions, we consider the following methods based on the utilization of triangular matrices. These methods, for ease of understanding, are described using third-order matrices. The methods, of course, can be generalized to higher order matrices.

Theorem: On the condition that the leading submatrices of the matrix

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \tag{1.9}$$

are nonsingular, i.e.,

$$a_{11} \neq 0, \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \neq 0, \ldots, [\mathbf{A}] \neq 0 \tag{1.10}$$

matrix **A** may be represented by the product of lower and upper triangular matrices. In equation (1.10), brackets represent determinants of the leading submatrices of (1.9). These submatrices start with the element of row 1 column 1 and proceed to the second- and third-order matrices along the diagonal of the original matrix **A**.

Denoting the lower and upper triangular matrices of **A** by **L** and **U**, respectively, we have

$$\mathbf{A} = \mathbf{LU}$$

$$\mathbf{A} = (a_{ij}) \quad \mathbf{L} = (l_{ij}) \quad \mathbf{U} = (u_{ij}) \tag{1.11}$$

$$a_{ij} = \sum_{k=1}^{\min(i,j)} l_{ik}\, u_{kj}$$

In the above notation a_{ij} represents the element of row i column j of matrix **A**. The lowercase letters are used to denote matrix elements and uppercase letters are used to denote matrices. Note that we will have n^2

equations in $n^2 + n$ unknowns u_{ij} and l_{ij}. Our latitude lies in the specifying of the diagonal coefficients l_{ii} or u_{ii} as seen in the example below.

Example: For a 3×3 matrix we have

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} \tag{1.12}$$

where the **A** matrix is represented as the product of the lower and the upper triangular matrices **L** and **U**. Performing the matrix product of (1.12) and equating the resulting matrices term by term, we get the set of equations

$$\begin{aligned} a_{11} &= l_{11}\, u_{11} & a_{21} &= l_{21}\, u_{11} \\ a_{12} &= l_{11}\, u_{12} & a_{22} &= l_{21}\, u_{12} + l_{22}\, u_{22} \\ a_{13} &= l_{11}\, u_{13} & a_{23} &= \ \ldots \end{aligned} \tag{1.13}$$

By selecting values of l_{11}, l_{22} and l_{33}, the rest of the values of the triangular matrices can be computed.

Furthermore, the inverse of lower and upper triangular matrices have the same form. For example, the inverse of a lower triangular matrix will also be a lower triangular matrix, as shown below for a third-order matrix

$$\begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} x_{11} & 0 & 0 \\ x_{21} & x_{22} & 0 \\ x_{31} & x_{32} & x_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1.14}$$

Assuming that we already have calculated the elements $l_{11}, l_{21}, \ldots$ of the lower triangular matrix, we can perform the above matrix multiplication and obtain a set of equations for the calculation of the elements of the inverse matrix $x_{11}, x_{21}, x_{22}, \ldots$ This results in

$$\left\{ \begin{aligned} & l_{11}x_{11} = 1 \\ & l_{21}x_{11} + l_{22}x_{21} = 0 \\ & l_{31}x_{11} + l_{32}x_{21} + l_{33}x_{31} = 0 \end{aligned} \right. \tag{1.15}$$

$$\left\{ \begin{aligned} & l_{22}x_{22} = 1 \\ & l_{32}x_{22} + l_{33}x_{32} = 0 \qquad (1.16) \\ & l_{33}x_{33} = 1 \qquad (1.17) \end{aligned} \right.$$