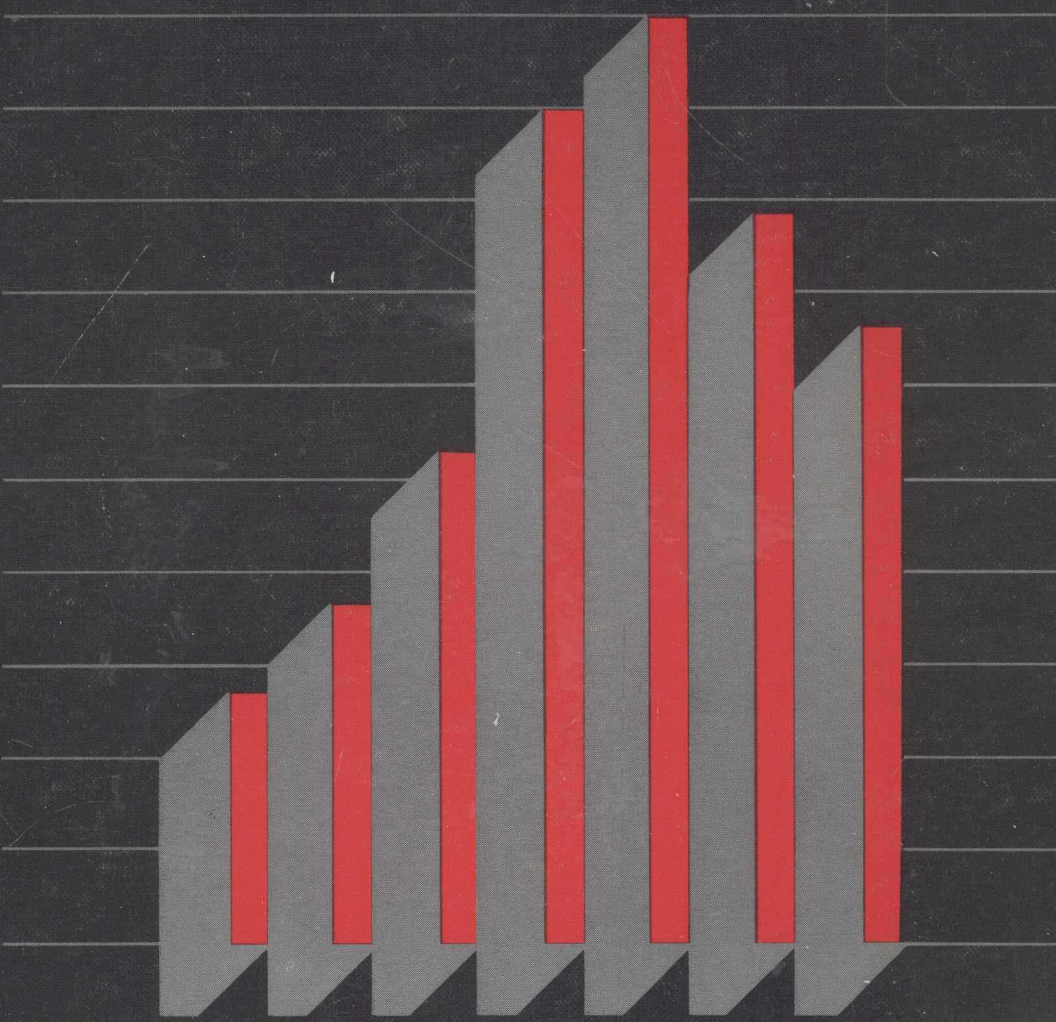
MATHEMATICS FOR BUSINESS, ECONOMICS AND MANAGEMENT

Marvin L. Bittinger/ J. Conrad Crown



MATHEMATICS FOR BUSINESS, ECONOMICS, AND MANAGEMENT

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PREFACE

The material in this book continues on from basic algebra and introduces the student to areas of finite mathematics and calculus which have applications in business, economics, and management. The basic material can be covered in two semester courses.

While most of the material in this book has been taken from *Finite Mathematics: A Modeling Approach*, second edition, by Crown-Bittinger, and *Calculus: A Modeling Approach*, second edition, by Bittinger, many sections have been rewritten after extensive class testing.

There are several features of the book as follows.

1. Intuitive Approach. While this word has many meanings and interpretations, its use here, for the most part, means "experience based." That is, when a concept is being taught, the learning is based on the student's prior experience or new experience given before the concept is formalized. For example, in a maximum-minimum problem a function is usually derived which is to be maximized or minimized. Instead of forging ahead with the standard calculus solution, the student is asked to stop and compute some function values. This experience provides the student with more insight into the problem. Not only does the student discover that different dimensions yield different volumes, if volume is to be maximized, but the dimensions which yield the maximum volume might even be conjectured as a result of the calculations. Provision for use of the hand calculator also provides for an intuitive approach.

2. The Hand Calculator. Exercises in this text can be done with or without a hand calculator. Most students, we find, not only have calculators but assume that calculators are *always* helpful to them. While there are many types of problems for which the calculator can reduce the work of computation, there are also many problems where there are naturally occurring fractions, and automatic conversion of all fractions to decimals may bring more distress than relief. For example, in the solution of systems of linear equations, fractions such as one-third have no exact decimal conversion and consequently conversion to decimals introduces the problem of "round-off" error. In general, we feel that calculators should *not* be used automatically but rather reserved for cases where they are necessary (or tables would be required), or where they reduce the tedium of computation.

3. Applications. Relevant and factual applications are included throughout the text to maintain interest and motivation. Problems in linear programming are of particular interest to students in business and management curricula. Problems in natural growth and decay (involving exponential and logarithmic functions) have applications in almost all areas ranging from population growth to continuously compounded interest to present value. The notions of total revenue, cost, and profit, together with their related derivatives (marginal functions) are threads which run through the text, providing continued reinforcement and unification.

4. Tests. Each chapter ends with a review. All the answers to these reviews are in the back of the book. A test on each chapter appears, classroom-ready, in the *Instructor's Manual*.

5. Exercises. Great care has been given to constructing the exercises. Many of the linear programming exercises have been designed to simplify the calculations and minimize the occurrence of fractions. The first exercises in each set are quite easy, while later ones become progressively more difficult. Most of the exercises are similar to examples worked out in that section of the text, and are arranged in matching pairs. That is, each odd-numbered exercise is very much like the one immediately following. The odd-numbered exercises have answers in the book, while the even-numbered exercises have answers in the *Instructor's Manual*.

The authors wish to acknowledge the assistance of Charles N. Kellog of Texas Tech University, Jeff Mock of Diablo Valley College, and Peter Rice of the University of Georgia, whose professional reviews were extremely valuable to the preparation of this book.

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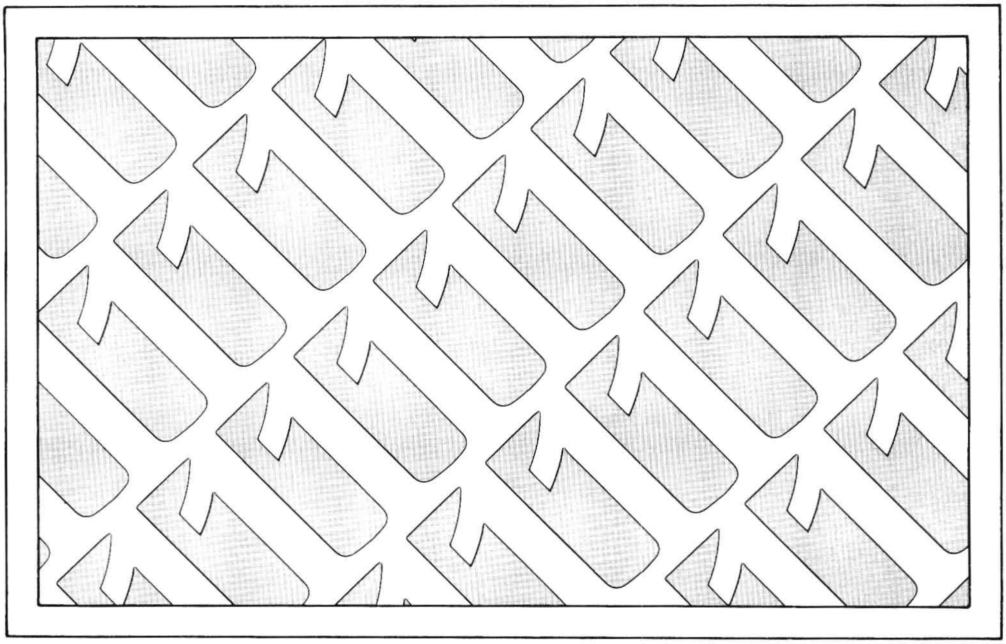


1.1 THE REAL NUMBERS*

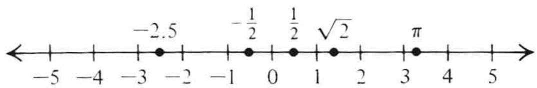
Real Numbers

In this table of stock market quotations are many kinds of numbers.

Rohr	5¼s86	7.0	75	−2	StOlin	9.20s04	14.	64	+ ¾
Ryder	11½ 90	13.	90½	......	StOInd	7⅞s07	14.	56	+1
Ryder	8⅛s92	13.	63¼	−4⅛	St OilInd	6s91	10.	59	......
StRegs	10⅝10	16.	66⅛	+3⅛	St OilInd	6s98	12.	49½	......
SanDgo	10s06	16.	61	+ ⅞	Std Oh	8⅜s07	16.	54	−2
SFeInd	6¼ 98	3.2	195	+7	StdOh	7.60s99	14.	54¼	+2
Savin	14s00	21.	67½	− ¼	Std Oh	7½s86	11.	71½	− ½
Savin	11⅜s98	21.	55½	− ⅜	St Pak	5¼s90	12.	43½	+ ¼
SCM	5½s88	8.6	64⅛	− ⅞	Stauffr	8⅛s96	14.	58	−1
Seafist	9¼s01	15.	64	+5	Storage	9s01	8.6	105	+2
SearsR	8⅝ 95	14.	62¼	− ¾	Storag	10¼00	7.6	135	+4
Sears Ro	8s06	15.	55¼	+ ¾	Storer	8½s05	9.7	87¼	− ¾
SearsR	7⅞07	14.	55⅛	+ ⅛	SunCo	10¾ 06	12.	89	......
SearsR	6⅜ 93	12.	55	− ½	Sun M	8½95D	10.	83	+2
SearsR	4¾ 83	5.6	84⅞	+ ¾	SunMn	8½s95	10.	83¾	− ¾
SecPc	8.80s85	11.	77½	+1⅜	Tandy	10s91	15.	67	−1¾
SecPc	7.70s82				Teldyne	10s04	15.	64⅞	+1½
		8.0	96 5-16	+ ½	Teldy	10 04C	16.	64	+ ¼
SheLR	15¼90	17.	91⅞	......	Teledyn	7s99	14.	48½	......
SheLR	10¾03	17.	64	−1	Telex	11¾s96	18.	67⅛	+1⅝
ShellO	14¼11	15.	92⅝	+ ¼	Telex Cp	9s96	16.	55⅝	......

**To the Instructor:* Chapters 1 and 2 of this book can be considered review and omitted by students with adequate preparation.

The numbers we use most in algebra are the *real numbers*. The real numbers are often pictured in one-to-one correspondence with the points of a line, as follows.



The positive numbers are pictured to the right of 0 and the negative numbers to the left. Zero itself is neither positive nor negative.

There are several subsets of the real numbers. They are as follows.

Natural Numbers. **The counting numbers 1, 2, 3, ...**

Whole Numbers. **The natural numbers and 0; that is, 0, 1, 2, 3, ...**

Integers. **The whole numbers and their additive inverses,**

$$0,\ 1,\ -1,\ 2,\ -2,\ 3,\ -3,\ \ldots .$$

Rational Numbers. **The integers and all quotients of integers (excluding division by 0). For example,**

$$\frac{3}{5},\ \frac{-5}{6},\ \frac{8}{1},\ 7,\ -19,\ 0,\ \frac{59}{-8},\ -\frac{8}{3}\left(\text{can also be named } \frac{-8}{3},\ \text{or } \frac{8}{-3}\right).$$

Any real number that is not rational is called *irrational*. The rational numbers and the irrational numbers can be described in several ways.

The *Rational Numbers* are

1. **Those numbers that can be named with fractional notation a/b, where a and b are integers and $b \neq 0$ (definition).**
2. **Those numbers for which decimal notation either ends or repeats.**

All of the following numbers are rational.

Example 1 $\frac{7}{16} = 0.4375$ (This is an ending or terminating decimal.)

Example 2 $-\frac{9}{7} = -1.285714285714\ldots = -1.\overline{285714}$ (The bar indicates the repeating part.)

Example 3 $\frac{4}{11} = 0.363636\ldots = 0.\overline{36}$ (Repeating decimal)

Irrational Numbers are

1. **Those real numbers that are not rational (definition).**
2. **Those real numbers that cannot be named with fractional notation a/b, where a and b are integers and $b \neq 0$.**
3. **Those real numbers for which decimal notation does not end and does not repeat.**

There are many irrational numbers. For example, $\sqrt{2}$ is irrational. We can find rational numbers a b for which $(a/b) \cdot (a/b)$ is close to 2, but we cannot find such a number for which $(a/b) \cdot (a/b)$ is *exactly* 2.

Unless a whole number is a perfect square its square root is irrational. For example, $\sqrt{4}$ and $\sqrt{49}$ are rational, but all of the following are irrational:

$$\sqrt{5},\ -\sqrt{12},\ \sqrt{37}.$$

There are also many irrational numbers that cannot be obtained by taking square roots. The number π is an example. Decimal notation for π does not end and does not repeat.

All of the following are irrational.

Example 4 $\pi = 3.1415926535\ldots$ (Numeral does not repeat. $\frac{22}{7}$ and 3.14 are only rational approximations to the irrational number π.)

Example 5 $-7.2020020002000020000002\ldots$ (Numeral does not repeat.)

Example 6 $\sqrt[3]{2} = 1.25992105\ldots$ (Numeral does not repeat.)

In Example 5, there is a pattern, but it is not a repeating pattern.

Algebra and Properties of Real Numbers

In arithmetic we use numbers, performing calculations to obtain certain answers. In algebra, we use arithmetic symbolism, but in addition we use symbols such as a, b, c, x, y, and z to represent unknown numbers. We do calculations and manipulations of symbols, based upon properties of numbers, which we review now. Algebra is thus an extension of arithmetic and a more powerful tool for solving problems. We will study even more powerful mathematical tools later in the book.

Addition

Assuming that addition of nonnegative real numbers is familiar, let us review how the definition of addition is extended to include the negative numbers. Recall first that the absolute value of a nonnegative number is that number itself. To get the absolute value of a negative, change its sign (make it positive). The absolute value of a number x is denoted $|x|$. Thus, $|5| = 5$, $|0| = 0$, and $|-2.7| = 2.7$.

To add

1. **Two negative numbers, add their absolute values (the sum is negative).**
2. **A negative and a positive number, find the difference of their absolute values. The result will have to have the sign of the number with the larger absolute value. If the absolute values are the same, the sum is 0.**

For example,

$$-7 + (-8) = -15, \quad -\frac{5}{6} + \left(-\frac{7}{12}\right) = -\frac{17}{12}, \quad 9 + (-6) = 3,$$
$$-7 + 3 = -4, \quad 9.7 + (-5.2) = 4.5, \quad -\pi + \pi = 0,$$
$$-\frac{7}{6} + \frac{3}{6} = -\frac{2}{3}, \quad -\sqrt{2} + (-8\sqrt{2}) = -9\sqrt{2}.$$

Properties of Real Numbers under Addition

The following are the fundamental properties of real numbers under addition. These are properties upon which algebraic manipulations are based, especially when symbols are used.

Commutative. **For any real numbers a and b, $a + b = b + a$. (The *order* in which numbers are added does not affect the sum.)**

Associative. **For any real numbers a, b, and c, $a + (b + c) = (a + b) + c$. (When only additions are involved, parentheses for *grouping* purposes may be placed as we please without affecting the sum.)**

Identity. **There exists a unique real number 0, such that for any real number a, $a + 0 = 0 + a = a$. (Adding 0 to any number gives that same number as the sum.)**

Inverses. **For every real number a, there exists a unique number, denoted $-a$, for which $-a + a = a + (-a) = 0$.**

CAUTION! It is common to read an expression such as $-x$ as "negative x." This can be confusing, indeed incorrect, because $-x$ may be positive, negative, or zero, depending on the value of x. The symbol $-$, used in this way, indicates an *additive inverse*; somewhat unfortunately the same symbol may also indicate a negative number, as in -5, or it may indicate subtraction, as in $8 - x$.

CAUTION! An initial $-$ sign, as in $-x$, or $-(x^2 - 3x + 2)$ should always be interpreted as meaning "the additive inverse of." The entire expression may be positive, negative, or zero, depending upon the value of the part of the expression that follows the $-$ sign. Taking the additive inverse is sometimes called "changing the sign."

Find the additive inverse for each of the following expressions.

Example 7 $-x$, when $x = 3$ $\quad -(3) = -3$ (Negative 3)

Example 8 $-x$, when $x = -8$ $\quad -(-8) = 8$

Example 9 $-x$, when $x = 0$ $\quad -(0) = 0$

Example 10 $-(x^2 + 8x + 2)$, when $x = 4$ $\quad -(4^2 + 8 \cdot 4 + 2) = -50$
when $x = -4$ $\quad -((-4)^2 + 8(-4) + 2) = 14$

It can be easily shown that $-1 \cdot x = -x$ for any number x. That is, multiplying a number x by negative 1 results in the additive inverse of x. This can be proved as a theorem but we will not do that here.

THEOREM **For any real number x, $-1 \cdot x = -x$. (Negative one times any number is its additive inverse.)**

Multiplication

Assuming that multiplication of nonnegative real numbers is familiar, let us review how the definition of multiplication is extended to include the negative numbers.

To multiply

1. **Two negative numbers, multiply their absolute values (the product is positive).**

2. A positive number and a negative number, multiply their absolute values and take the additive inverse of the result (the product is negative).

For example,

$$5 \cdot (-4) = -20, \quad 1.6 \cdot (-3.8) = -6.08, \quad -8 \cdot (-6) = 48,$$
$$-\tfrac{2}{3} \cdot (-\tfrac{4}{5}) = \tfrac{8}{15}, \quad -3 \cdot (-2) \cdot (-7) = -42.$$

We now list the properties of real numbers under multiplication. Recall that ab is an abbreviation for $a \cdot b$.

Commutative. **For any real numbers a and b, $ab = ba$. (The *order* in which numbers are multiplied does not affect the product.)**

Associative. **For any real numbers a, b, and c, $a(bc) = (ab)c$. (When only multiplications are involved, parentheses for grouping purposes may be placed as we please without affecting the product.)**

Identity. **There exists a unique number 1, such that for any real number a, $a \cdot 1 = 1 \cdot a = a$. (Multiplying any number by 1 gives that same number as the product.)**

Inverses. **For each nonzero real number a, there exists a unique number $\dfrac{1}{a}$ or a^{-1}, for which $a\left(\dfrac{1}{a}\right) = \left(\dfrac{1}{a}\right)a = 1$.**

Multiplicative inverses are also called *reciprocals*.

Example 11 The multiplicative inverse, or reciprocal, of 8 is $\frac{1}{8}$.

Example 12 The multiplicative inverse of $-\frac{4}{5}$ is $-\frac{5}{4}$.

Example 13 The reciprocal of 0.32 is 3.125.

There is a very important property, or law, that connects addition and multiplication, as follows.

Distributive. **For any real numbers a, b, and c,**

$$a(b + c) = ab + ac.^*$$

(This is called the distributive law of multiplication over addition.)

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* The expression $ab + ac$ means $(a \cdot b) + (a \cdot c)$. By agreement, we can omit parentheses around multiplications. According to this agreement, multiplications are to be done before additions or subtractions.