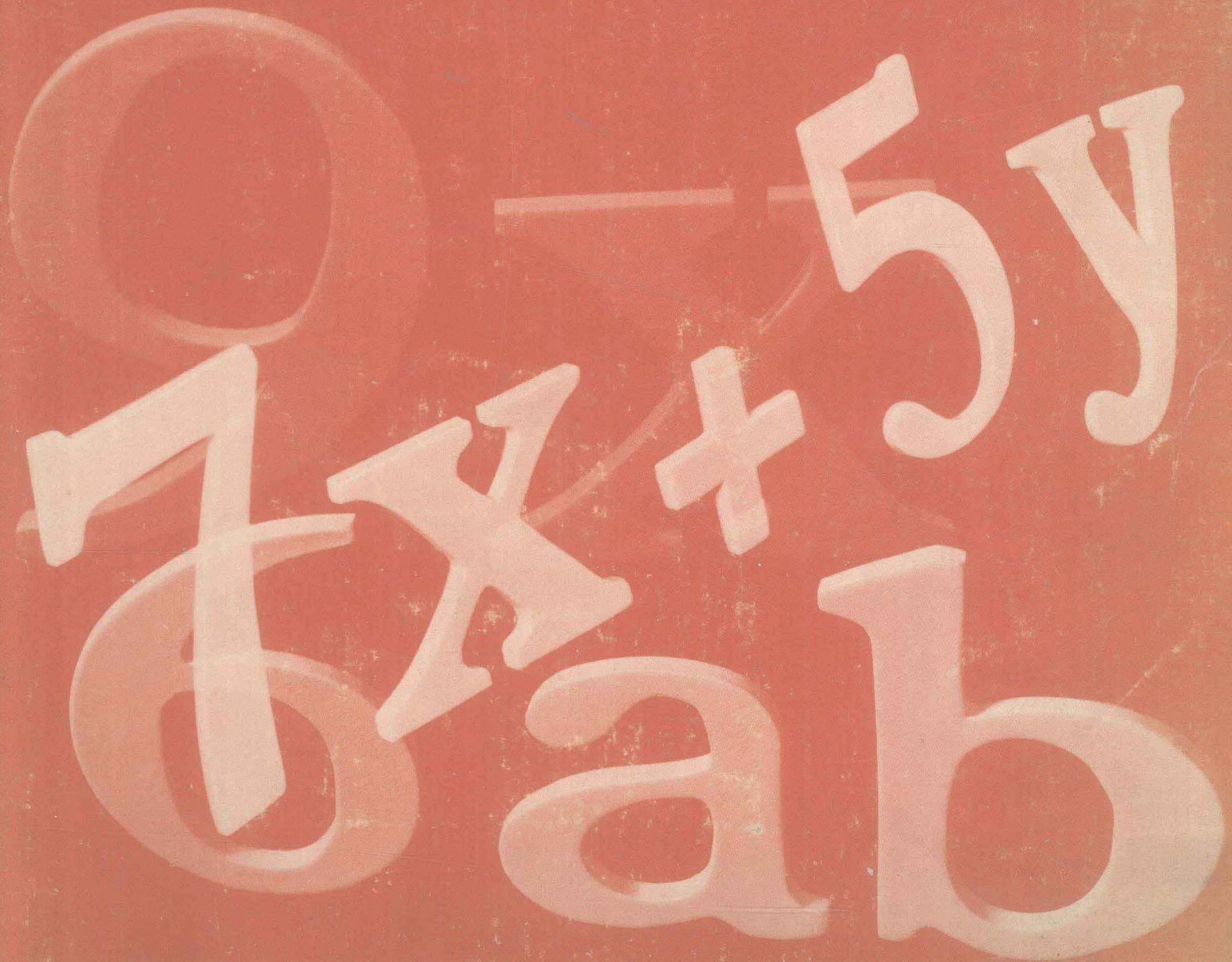


Intermediate Algebra

A Programmed Approach

Anthony J. Pettofrezzo

Lee H. Armstrong



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Preface

Intermediate Algebra is a programmed text intended for students who require an understanding of those principles and methods of algebra prerequisite for the study of college algebra, statistics, and applied mathematics of the social sciences. The book is intended for students who have a mathematical background comparable to that which may be obtained from *Elementary Algebra: A Programmed Approach* in this series. However, well-motivated students who have good arithmetical skills and a thorough understanding of the properties of real numbers may begin their study of algebra with this book.

The programmed approach allows for an individualized course in which students may proceed at their own rate of learning. The material is self-contained, so that neither outside lectures nor assistance is required. In our many years of undergraduate teaching, we have found that the programmed approach offers maximum flexibility to both instructors and students.

Each page contains two columns of material. The left column consists of a series of instructions and numbered questions called *frames*. The right column contains the correct responses to the questions posed in each frame. When studying this programmed material, the student should use a card or sheet of paper to cover the right column. The correct response for a particular frame should be exposed only after the student has placed an answer in the space provided within the frame. The student's response should nearly always be correct. Failure to supply the correct response with a fair degree of regularity may result from working too rapidly or carelessly. In either case, the student should return to earlier frames of the section in which the difficulties arose.

A number of special pedagogical features should aid students as they proceed through the text.

- *Self-Tests* appear throughout each chapter at the end of related sections. These can be used by students to determine their comprehension and mastery of the material covered.
- A *Mathematics Vocabulary List*, with page references indicating where each term was first used, appears at the end of each chapter as a study aid.
- *Chapter Review Exercises* may be used to identify any aspects of the material that deserve further attention.
- *Chapter Tests*, graded *A*, *B*, and *C* according to their level of difficulty, help students evaluate their own progress through the course.

- A *Sample Final Examination* at the end of the text helps students to determine their overall mastery of the material covered in *Intermediate Algebra*.
- An *Answer Key* for the *Self-Tests*, *Chapter Review Exercises*, *Chapter Tests*, and *Sample Final Examination* appears at the back of the book.

In addition to these items meant to assist the student, an *Instructor's Guide* is provided for the benefit of the teacher. The guide includes Suggested Course Outlines, a Diagnostic Pretest to check students' preparation for the course, Teaching Suggestions, five additional Chapter Tests for each chapter, and an extensive Test Bank of Final Exam Questions.

The authors owe a debt of gratitude to the many reviewers whose comments and suggestions aided in the development of *Intermediate Algebra*. Included among the reviewers were William F. Ward of Indian River Community College, David E. Conroy of Northern Virginia Community College, Maurice E. Nott of St. Petersburg Junior College, Paul P. Miller of the Community College of Baltimore, Joe C. Prater of the University of Southern Colorado, and Richard D. Semmler of Northern Virginia Community College. A special note of thanks is due the editorial staff of Scott, Foresman and Company, especially Pam Carlson for her encouragement, Dane Tyson for his guidance, and Betty Slack for her attention to detail and seeing the manuscript through production.

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Contents

CHAPTER 1	REVIEW OF BASIC TOPICS	1
	1.1 Sets	1
	1.2 Algebra of Sets	7
	Self-Test 1	10
	1.3 Sets of Numbers	11
	1.4 Rational Numbers and Decimals	19
	1.5 Properties of Real Numbers	24
	Self-Test 2	30
	1.6 The Number Line	31
	1.7 Absolute Value	34
	Self-Test 3	35
	1.8 Operations on Signed Numbers	36
	1.9 Operations on Rational Numbers	41
	Self-Test 4	46
	Mathematics Vocabulary List	47
	Chapter Review Exercises	48
	Chapter Tests	51
 CHAPTER 2	 LINEAR EQUATIONS	 54
	2.1 Linear Equations in One Variable	54
	2.2 Applications	60
	Self-Test 1	63
	2.3 Linear Equations in Two Variables	64
	2.4 The Cartesian Coordinate System	67
	2.5 Graphs of Linear Equations	78
	Self-Test 2	84
	2.6 Intercepts of a Line	86
	2.7 Intercept Form of a Line	91
	Self-Test 3	96
	2.8 Slope of a Line	97

2.9	Slope-Intercept Form	102
2.10	Point-Slope Form	107
	Self-Test 4	112
	Mathematics Vocabulary List	112
	Chapter Review Exercises	113
	Chapter Tests	118

CHAPTER 3 SYSTEMS OF LINEAR EQUATIONS 121

3.1	Systems of Linear Equations and Their Solutions	121
3.2	Graphs of Systems of Linear Equations	124
	Self-Test 1	132
3.3	Solving Systems of Equations by Substitution	134
3.4	Solutions by Addition and Subtraction	138
3.5	Use of Multiplication in the Addition Method	143
	Self-Test 2	147
3.6	Determinants	148
3.7	Solving Systems of Equations by Determinants: Cramer's Rule	151
3.8	Applications	158
	Self-Test 3	163
	Mathematics Vocabulary List	164
	Chapter Review Exercises	164
	Chapter Tests	171

CHAPTER 4 EXPONENTS, POLYNOMIALS, AND FACTORING 174

4.1	Powers and Exponents	174
4.2	Rules of Exponents	176
4.3	Zero and Negative Exponents	181
	Self-Test 1	185
4.4	Polynomials	185
4.5	Addition and Subtraction of Polynomials	189
	Self-Test 2	193
4.6	Multiplication of Polynomials	194
4.7	Factoring Common Factors in Polynomials	200
	Self-Test 3	204
4.8	Factoring Trinomials	205
4.9	Special Products and Factors	211
	Self-Test 4	215
	Mathematics Vocabulary List	215
	Chapter Review Exercises	216
	Chapter Tests	218

CHAPTER 5	RATIONAL EXPRESSIONS	221
5.1	Simplifying Rational Expressions	221
5.2	Multiplication and Division of Rational Expressions	225
	Self-Test 1	230
5.3	Addition and Subtraction of Rational Expressions	231
	Self-Test 2	238
5.4	Complex Fractions	238
5.5	Division of Polynomials	243
5.6	Synthetic Division	249
	Self-Test 3	255
5.7	Solving Equations Involving Rational Expressions	256
5.8	Applications Involving Rational Expressions	262
	Self-Test 4	268
	Mathematics Vocabulary List	269
	Chapter Review Exercises	269
	Chapter Tests	273
 CHAPTER 6	 RADICALS AND COMPLEX NUMBERS	 276
6.1	Radicals	276
6.2	Multiplication and Division of Radicals	281
6.3	Expressions Containing Radicals	287
	Self-Test 1	292
6.4	Rational Number Exponents	293
6.5	Radical Equations	297
	Self-Test 2	302
6.6	Complex Numbers	303
6.7	Addition and Subtraction of Complex Numbers	308
	Self-Test 3	311
6.8	Multiplication and Division of Complex Numbers	311
	Self-Test 4	317
	Mathematics Vocabulary List	318
	Chapter Review Exercises	318
	Chapter Tests	321
 CHAPTER 7	 QUADRATIC EQUATIONS	 324
7.1	Quadratic Equations	324
7.2	Solutions by Factoring	327
	Self-Test 1	333
7.3	Completing the Square	334
7.4	The Quadratic Formula	340

7.5	The Discriminant	344
	Self-Test 2	347
7.6	Applications	347
	Self-Test 3	350
7.7	Quadratic Equations in Two Variables	351
	Self-Test 4	360
	Mathematics Vocabulary List	361
	Chapter Review Exercises	361
	Chapter Tests	365

CHAPTER 8 INEQUALITIES 368

8.1	Order Relations	368
8.2	Linear Inequalities in One Variable	375
	Self-Test 1	379
8.3	Linear Inequalities in Two Variables	380
8.4	Graphing Linear Inequalities	383
	Self-Test 2	395
8.5	Graphs of Systems of Linear Inequalities	397
	Self-Test 3	406
8.6	Quadratic Inequalities	407
8.7	Graphs of Quadratic Inequalities	412
	Self-Test 4	415
	Mathematics Vocabulary List	416
	Chapter Review Exercises	416
	Chapter Tests	420

CHAPTER 9 FUNCTIONS 423

9.1	Relations	423
9.2	Functions	430
	Self-Test 1	438
9.3	Functional Notation	439
9.4	Graphing Polynomial Functions	443
	Self-Test 2	453
9.5	Inverse of a Function	454
9.6	The Absolute Value Function	459
	Self-Test 3	461
	Mathematics Vocabulary List	462
	Chapter Review Exercises	462
	Chapter Tests	466

CHAPTER 10 LOGARITHMS 469

10.1	Definition of a Logarithm	469
------	---------------------------	-----

10.2	Scientific Notation	473
	Self-Test 1	478
10.3	Common Logarithms	478
	Self-Test 2	486
10.4	Linear Interpolation	487
10.5	Antilogarithms	490
	Self-Test 3	494
10.6	Logarithm of a Product	494
10.7	Logarithm of a Power of a Number	498
	Self-Test 4	501
10.8	Logarithm of a Quotient	501
10.9	Computations with Logarithms	504
	Self-Test 5	507
	Mathematics Vocabulary List	508
	Chapter Review Exercises	508
	Chapter Tests	511
CHAPTER 11	SEQUENCES AND SERIES	514
11.1	Definition of a Sequence	514
11.2	Notation	517
	Self-Test 1	520
11.3	Arithmetic Sequences	521
11.4	Arithmetic Means	525
	Self-Test 2	527
11.5	Geometric Sequences	528
11.6	Geometric Means	531
	Self-Test 3	533
11.7	Finite Arithmetic Series	533
	Self-Test 4	538
11.8	Finite Geometric Series	539
11.9	Infinite Geometric Series	542
	Self-Test 5	548
	Mathematics Vocabulary List	549
	Chapter Review Exercises	550
	Chapter Tests	554
SAMPLE FINAL EXAMINATION		557
TABLE A SQUARE ROOTS 1-100		561
TABLE B COMMON LOGARITHMS		562
ANSWERS		564
INDEX		576

1

Review of Basic Topics

1.1 SETS

A **set** is a collection of objects such as numbers, persons, places, or geometric figures. Objects which belong to a set are called **members**, or **elements**, of the set. Capital letters are often used to name a particular set. For example, we may write

A = the set of days of the week;
 B = the set of vowels a, e, i, o, u.

- 1 The collection of numbers 1, 2, 3, 4, and 5 is a _____ of numbers. set
- 2 The collection of months of the year is a _____ . set

Consider the preceding sets A and B in Frames 3 through 7.

3 Wednesday is a member of set _____ .

A

4 The letter e is a member of set _____ .

B

5 Friday is a _____ of set A .

member or element

6 The letter u is a _____ of set B .

member or element

7 The letter t is _____ a member of set B .

not

To represent a set, we list the elements and enclose them within braces. For example, if C is the set of numbers 1, 2, 3, 4, and 5, we write

$$C = \{1, 2, 3, 4, 5\}.$$

If an object a is an **element** of a set S , we write $a \in S$; if a is not an element of S , we write $a \notin S$.

The symbol \in , which is the Greek letter epsilon, is read “is an element of”; the symbol \notin is read “is not an element of.”

Since 1 is an element of the above set C , we may write $1 \in C$. Similarly, $2 \in C$, $3 \in C$, $4 \in C$, and $5 \in C$.

8 The collection of vowels a, e, i, o, and u may be denoted by using braces as _____ .

$\{a, e, i, o, u\}$

9 The elements of the set $\{1, 3, 5\}$ are _____ , _____ , and _____ .

1, 3, 5

10 An object is an _____ of a set if it is contained in the list within the braces.

element

11 Represent the set of numbers 5, 10, 15, 20, and 25 by using braces. _____ .

$\{5, 10, 15, 20, 25\}$

In Frames 12 through 16 let $S = \{1, a, 2, b, 3\}$.

12 True or false: $2 \in S$. _____

true

13 True or false: $0 \notin S$. _____

true

14 True or false: $4 \in S$. _____

false

15 Since 3 is an element of set S , we may write _____.

$3 \in S$

16 Since the letter d is not an element of set S , we may write _____.

$d \notin S$

Another method of defining a set involves the use of **set-builder notation**:

$\{x \mid \text{_____}\}$,

which is read “the set of all elements x such that _____.”

The common property of each element x is described in the space _____ . For example,

$\{a, e, i, o, u\} = \{x \mid x \text{ is a vowel in the English alphabet}\};$

$\{1, 2, 3, 4, 5\} = \{x \mid x \text{ is a counting number less than } 6\}.$

17 Consider the set

$W = \{x \mid x \text{ is a three-letter word}\}.$

The word “man” is a member of set W since “man” is a three-letter word. Is the word “house” a member of W ? _____

no

18 Is the word “cow” a member of the set W in Frame 17? _____

yes

19 Represent the set

$\{x \mid x \text{ is a season of the year}\}$

by listing the members within braces.

$\{\text{winter, spring, summer, fall}\}$

20 Represent the set

$\{x \mid x \text{ is a vowel in the word “mathematics”}\}$

by listing the members within braces. _____

$\{a, e, i\}$

21 Use set-builder notation to describe the set

$\{1, 2, 3, 4\}.$

$\{x \mid x \text{ is a counting number less than } 5\}$

Consider the sets

$$A = \{2, 3, 1, 4\} \quad \text{and} \quad B = \{4, 3, 1, 2\}.$$

Every element of set A is also an element of set B , and every element of set B is also an element of set A . In such a case, sets A and B are **equal** and we write $A = B$; when A is not equal to B , we write $A \neq B$.

22 Is $\{a, b, c\} = \{c, a, b\}$? _____

yes

23 Consider $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Since $3 \in A$, but $3 \notin B$, A _____ B .

\neq

24 Let $A = \{x | x \text{ is a country in North America that borders the United States}\}$, and let $B = \{\text{Canada, Mexico}\}$. Replace \square by $=$ or \neq to make the statement $A \square B$ true. _____

$=$

Sets may have a limited or finite number of elements. Such sets are called **finite sets**. For example, the set

$$\{a, b, c, d, e\}$$

has five elements and is a finite set; the set $\{1, 2, 3\}$ has three elements and is a finite set.

25 The set of letters in the English alphabet

$$\{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$$

has twenty-six elements and is an example of a _____ .

finite set

26 The set of months of the year is a finite set since it has a finite number of elements. This set has _____ elements.

twelve

27 Although the number of people in the world is large, the set of people in the world is a _____ .

finite set

28 The set $\{1, 2, 3, 4\}$ is a finite set. The set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is a finite set. The set of the first one thousand counting numbers is a finite set. Is the set of all counting numbers 1, 2, 3, 4, and so on a finite set? _____

no

Some sets, such as the set of all counting numbers, may have an unlimited number of elements. Such sets are called **infinite sets**.

29 Is the set of counting numbers greater than 100 an infinite set?

yes

30 The set of all counting numbers may be represented by

$\{1, 2, 3, 4, \dots\}$.

Three dots are used in this manner to list the elements of an infinite set whenever it is understood what the missing elements are.

The set $\{1, 2, 3, 4\}$ is a _____, but the set

$\{1, 2, 3, 4, \dots\}$ is an _____.

finite set, infinite set

31 True or false: $\{1, 3, 5, 7\}$ is a finite set. _____

true

32 True or false: $\{1, 3, 5, 7, \dots\}$ is an infinite set. _____

true

Every element of the set $\{a, c\}$ is an element of the set $\{a, b, c, d\}$. In such a case we say that the first set is a *subset* of the second set.

Set A is a **subset** of set B if and only if every element of A is also an element of B . When A is a subset of B , we write $A \subseteq B$; when A is not a subset of B , we write $A \not\subseteq B$.

In Frames 33 through 35 consider the sets

$A = \{1, 3, 4\}$ and $B = \{1, 2, 3, 4, 5\}$.

33 Note that $1 \in B$, $3 \in B$, and $4 \in B$. Since every element of A is also an element of B , set A is a _____ of set B .

subset

34 Since A is a subset of B , we may write _____.

$A \subseteq B$

35 Since $2 \in B$ but $2 \notin A$, not every element of set B is in set A . Therefore, set B is _____ a subset of A .

not

36 True or false: The finite set $\{1, 2, 3, 4\}$ is a subset of the infinite set $\{1, 2, 3, 4, \dots\}$. _____

true

37 True or false: $A \subseteq A$ for every set A (every set is a subset of itself).

_____ true

38 True or false: The set of dogs is a subset of the set of animals.

_____ true

39 True or false: The set of all oak trees is a subset of the set of all trees.

_____ true

A finite set may have no elements. For example, the following are finite sets with no elements.

The set of counting numbers between 2 and 3

The set of triangles with five sides

A set that contains no elements is called the **null set** or **empty set**. The null set is denoted either by $\{ \}$ or by the special symbol \emptyset .

40 The set of counting numbers less than 1 is the _____ .

null set or empty set

41 True or false: The set of people born on the sun is the null set.

_____ true

42 The null set is a set with _____ elements.

zero

43 The null set is denoted either by _____ or by the special symbol _____ .

$\{ \}$, \emptyset

44 Since no state is larger than Alaska, the set of states in the United States larger than Alaska is the _____ .

null set or empty set

45 The null set is a subset of every set and every set is a subset of itself. The subsets of $\{3\}$ are $\{ \}$ and $\{3\}$. The subsets of $\{7\}$ are _____ and _____ .

$\{ \}$, $\{7\}$

46 The subsets of $\{k\}$ are _____ and _____ .

$\{ \}$, $\{k\}$

47 The subsets of $\{a, b\}$ are $\{ \}$, $\{a\}$, $\{b\}$, and $\{a, b\}$. The subsets of $\{5, 8\}$ are _____ , _____ , _____ , and _____ .

$\{ \}$, $\{5\}$, $\{8\}$, $\{5, 8\}$

The set of all objects under study is called the **universal set** and is represented by the symbol U .

The universal set serves as a reference set for a particular discussion, and every set considered must be a subset of U .

48 If the universal set is the set of days of the week, then

_____ = {Sunday, Monday, Tuesday, Wednesday,
Thursday, Friday, Saturday}. U

49 Let the set of letters of the English alphabet be the universal set U .

May we consider the set of vowels in our discussion? _____ yes

May we consider the set of consonants in our discussion? _____ yes

50 Describe the set V of vowels: $V =$ _____ . {a, e, i, o, u}

51 Describe the set C of consonants: $C =$ _____
_____ {b, c, d, f, g, h, j, k, l, m,
n, p, q, r, s, t, v, w, x, y, z}

In Frames 49 through 51 every element of the set U of letters is either in set V or in set C . Furthermore, no letter is both a vowel and a consonant. Each of the sets V and C is the *complement* of the other.

The **complement** of a set A is the set of elements in the universal set U that are not in set A . The complement of set A is denoted by \overline{A} .

52 If $U = \{1, 2, 3, 4, 5\}$ and $A = \{2, 3\}$, then the elements in U that are not in A are _____, _____, and _____. Therefore, $\overline{A} =$ _____. 1, 4, 5, {1, 4, 5}

53 If $U = \{a, b, c, d, e\}$, the complement of $\{a, d, e\}$ is _____ . {b, c}

54 If U is the set of counting numbers and A is the set of odd counting numbers, then \overline{A} is the set of _____ counting numbers. even