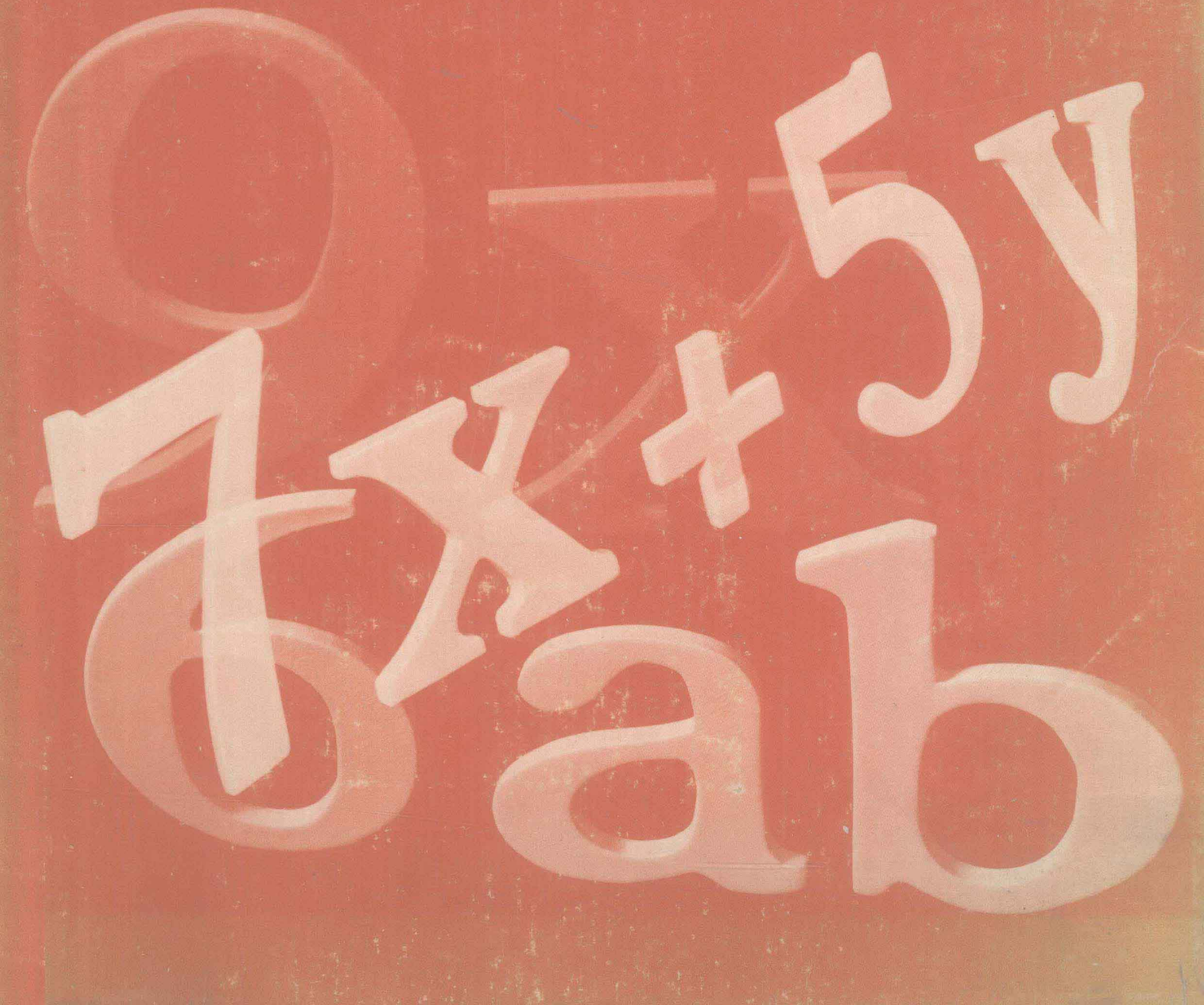
Intermediate Algebra

A Programmed Approach

Anthony J. Pettofrezzo Lee H. Armstrong



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Anthony J. Pettofrezzo
Lee H. Armstrong

University of Central Florida

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Preface

Intermediate Algebra is a programmed text intended for students who require an understanding of those principles and methods of algebra prerequisite for the study of college algebra, statistics, and applied mathematics of the social sciences. The book is intended for students who have a mathematical background comparable to that which may be obtained from *Elementary Algebra: A Programmed Approach* in this series. However, well-motivated students who have good arithmetical skills and a thorough understanding of the properties of real numbers may begin their study of algebra with this book.

The programmed approach allows for an individualized course in which students may proceed at their own rate of learning. The material is self-contained, so that neither outside lectures nor assistance is required. In our many years of undergraduate teaching, we have found that the programmed approach offers maximum flexibility to both instructors and students.

Each page contains two columns of material. The left column consists of a series of instructions and numbered questions called *frames*. The right column contains the correct responses to the questions posed in each frame. When studying this programmed material, the student should use a card or sheet of paper to cover the right column. The correct response for a particular frame should be exposed only after the student has placed an answer in the space provided within the frame. The student's response should nearly always be correct. Failure to supply the correct response with a fair degree of regularity may result from working too rapidly or carelessly. In either case, the student should return to earlier frames of the section in which the difficulties arose.

A number of special pedagogical features should aid students as they proceed through the text.

- *Self-Tests* appear throughout each chapter at the end of related sections. These can be used by students to determine their comprehension and mastery of the material covered.
- A *Mathematics Vocabulary List,* with page references indicating where each term was first used, appears at the end of each chapter as a study aid.
- *Chapter Review Exercises* may be used to identify any aspects of the material that deserve further attention.
- *Chapter Tests,* graded *A, B,* and *C* according to their level of difficulty, help students evaluate their own progress through the course.

- A *Sample Final Examination* at the end of the text helps students to determine their overall mastery of the material covered in *Intermediate Algebra.*
- An *Answer Key* for the *Self-Tests, Chapter Review Exercises, Chapter Tests,* and *Sample Final Examination* appears at the back of the book.

In addition to these items meant to assist the student, an *Instructor's Guide* is provided for the benefit of the teacher. The guide includes Suggested Course Outlines, a Diagnostic Pretest to check students' preparation for the course, Teaching Suggestions, five additional Chapter Tests for each chapter, and an extensive Test Bank of Final Exam Questions.

The authors owe a debt of gratitude to the many reviewers whose comments and suggestions aided in the development of *Intermediate Algebra.* Included among the reviewers were William F. Ward of Indian River Community College, David E. Conroy of Northern Virginia Community College, Maurice E. Nott of St. Petersburg Junior College, Paul P. Miller of the Community College of Baltimore, Joe C. Prater of the University of Southern Colorado, and Richard D. Semmler of Northern Virginia Community College. A special note of thanks is due the editorial staff of Scott, Foresman and Company, especially Pam Carlson for her encouragement, Dane Tyson for his guidance, and Betty Slack for her attention to detail and seeing the manuscript through production.

Anthony J. Pettofrezzo

Lee H. Armstrong

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Review of Basic Topics

1.1 SETS

A **set** is a collection of objects such as numbers, persons, places, or geometric figures. Objects which belong to a set are called **members**, or **elements**, of the set. Capital letters are often used to name a particular set. For example, we may write

A = the set of days of the week;
B = the set of vowels a, e, i, o, u.

1 The collection of numbers 1, 2, 3, 4, and 5 is a _____ of numbers. set

2 The collection of months of the year is a _____ . set

Consider the preceding sets A and B in Frames 3 through 7.

3 Wednesday is a member of set _____ . *A*

4 The letter e is a member of set _____ . *B*

5 Friday is a _____ of set *A*. member *or* element

6 The letter u is a _____ of set *B*. member *or* element

7 The letter t is _____ a member of set *B*. not

To represent a set, we list the elements and enclose them within braces. For example, if C is the set of numbers 1, 2, 3, 4, and 5, we write

$$C = \{1, 2, 3, 4, 5\}.$$

If an object a is an **element** of a set S, we write $a \in S$; if a is not an element of S, we write $a \notin S$.

The symbol $\in$, which is the Greek letter epsilon, is read "is an element of"; the symbol $\notin$ is read "is not an element of."

Since 1 is an element of the above set C, we may write $1 \in C$. Similarly, $2 \in C$, $3 \in C$, $4 \in C$, and $5 \in C$.

8 The collection of vowels a, e, i, o, and u may be denoted by using braces as _____ . {a, e, i, o, u}

9 The elements of the set {1, 3, 5} are _____ , _____ , and _____ . 1, 3, 5

10 An object is an _____ of a set if it is contained in the list within the braces. element

11 Represent the set of numbers 5, 10, 15, 20, and 25 by using braces. _____ . {5, 10, 15, 20, 25}

In Frames 12 through 16 let $S = \{1, a, 2, b, 3\}$.

12 True or false: $2 \in S$. _____ true

13 True or false: $0 \notin S$. _____ true

14 True or false: $4 \in S$. _____ false

15 Since 3 is an element of set S, we may write _____ . $3 \in S$

16 Since the letter d is not an element of set S, we may write _____ . $d \notin S$

Another method of defining a set involves the use of **set-builder notation**:

$\{x \mid$ _____$\}$,

which is read "the set of all elements x such that _____ ." The common property of each element x is described in the space _____ . For example,

$\{a, e, i, o, u\} = \{x \mid x \text{ is a vowel in the English alphabet}\}$;
$\{1, 2, 3, 4, 5\} = \{x \mid x \text{ is a counting number less than } 6\}$.

17 Consider the set

$W = \{x \mid x \text{ is a three-letter word}\}$.

The word "man" is a member of set W since "man" is a three-letter word. Is the word "house" a member of W? _____ no

18 Is the word "cow" a member of the set W in Frame 17? _____ yes

19 Represent the set

$\{x \mid x \text{ is a season of the year}\}$

by listing the members within braces.

_____ {winter, spring, summer, fall}

20 Represent the set

$\{x \mid x \text{ is a vowel in the word "mathematics"}\}$

by listing the members within braces. _____ {a, e, i}

21 Use set-builder notation to describe the set

$\{1, 2, 3, 4\}$.

_____ $\{x \mid x \text{ is a counting number less than } 5\}$

Consider the sets

$$A = \{2, 3, 1, 4\} \quad \text{and} \quad B = \{4, 3, 1, 2\}.$$

Every element of set A is also an element of set B, and every element of set B is also an element of set A. In such a case, sets A and B are **equal** and we write $A = B$; when A is not equal to B, we write $A \neq B$.

22 Is $\{a, b, c\} = \{c, a, b\}$? _____ yes

23 Consider $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Since $3 \in A$, but $3 \notin B$, A _____ B. $\neq$

24 Let $A = \{x | x$ is a country in North America that borders the United States$\}$, and let $B = \{$Canada, Mexico$\}$. Replace $\square$ by = or $\neq$ to make the statement $A \square B$ true. _____ =

Sets may have a limited or finite number of elements. Such sets are called **finite sets.** For example, the set

$$\{a, b, c, d, e\}$$

has five elements and is a finite set; the set $\{1, 2, 3\}$ has three elements and is a finite set.

25 The set of letters in the English alphabet

{a, b, c, d, e, f, g, h, i, j, k, l, m,
n, o, p, q, r, s, t, u, v, w, x, y, z}

has twenty-six elements and is an example of a _____ _____ . finite set

26 The set of months of the year is a finite set since it has a finite number of elements. This set has _____ elements. twelve

27 Although the number of people in the world is large, the set of people in the world is a _____ _____ . finite set

28 The set $\{1, 2, 3, 4\}$ is a finite set. The set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ is a finite set. The set of the first one thousand counting numbers is a finite set. Is the set of all counting numbers 1, 2, 3, 4, and so on a finite set? _____ no

Some sets, such as the set of all counting numbers, may have an unlimited number of elements. Such sets are called **infinite sets.**

29 Is the set of counting numbers greater than 100 an infinite set? _____

yes

30 The set of all counting numbers may be represented by

$\{1, 2, 3, 4, \ldots\}$.

Three dots are used in this manner to list the elements of an infinite set whenever it is understood what the missing elements are. The set $\{1, 2, 3, 4\}$ is a _____ _____, but the set $\{1, 2, 3, 4, \ldots\}$ is an _____ _____.

finite set, infinite set

31 True or false: $\{1, 3, 5, 7\}$ is a finite set. _____

true

32 True or false: $\{1, 3, 5, 7, \ldots\}$ is an infinite set. _____

true

Every element of the set $\{a, c\}$ is an element of the set $\{a, b, c, d\}$. In such a case we say that the first set is a *subset* of the second set.

Set A is a **subset** of set B if and only if every element of A is also an element of B. When A is a subset of B, we write $A \subseteq B$; when A is not a subset of B, we write $A \not\subseteq B$.

In Frames 33 through 35 consider the sets

$A = \{1, 3, 4\}$ *and* $B = \{1, 2, 3, 4, 5\}$.

33 Note that $1 \in B$, $3 \in B$, and $4 \in B$. Since every element of A is also an element of B, set A is a _____ of set B.

subset

34 Since A is a subset of B, we may write _____.

$A \subseteq B$

35 Since $2 \in B$ but $2 \notin A$, not every element of set B is in set A. Therefore, set B is _____ a subset of A.

not

36 True or false: The finite set $\{1, 2, 3, 4\}$ is a subset of the infinite set $\{1, 2, 3, 4, \ldots\}$. _____

true

37 True or false: $A \subseteq A$ for every set A (every set is a subset of itself).
_____ true

38 True or false: The set of dogs is a subset of the set of animals.
_____ true

39 True or false: The set of all oak trees is a subset of the set of all trees.
_____ true

A finite set may have no elements. For example, the following are finite sets with no elements.

The set of counting numbers between 2 and 3
The set of triangles with five sides

A set that contains no elements is called the **null set** or **empty set.** The null set is denoted either by { } or by the special symbol $\emptyset$.

40 The set of counting numbers less than 1 is the _____ _____. null set *or* empty set

41 True or false: The set of people born on the sun is the null set.
_____ true

42 The null set is a set with _____ elements. zero

43 The null set is denoted either by _____ or by the special symbol _____. { }, $\emptyset$

44 Since no state is larger than Alaska, the set of states in the United States larger than Alaska is the _____ _____. null set *or* empty set

45 The null set is a subset of every set and every set is a subset of itself. The subsets of {3} are { } and {3}. The subsets of {7} are _____ and _____. { }, {7}

46 The subsets of $\{k\}$ are _____ and _____. { }, $\{k\}$

47 The subsets of $\{a, b\}$ are { }, $\{a\}$, $\{b\}$, and $\{a, b\}$. The subsets of {5, 8} are _____, _____, _____, and _____. { }, {5}, {8}, {5, 8}

1.2 ALGEBRA OF SETS

The set of all objects under study is called the **universal set** and is represented by the symbol U.

The universal set serves as a reference set for a particular discussion, and every set considered must be a subset of U.

48 If the universal set is the set of days of the week, then

_____ = {Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday}. U

49 Let the set of letters of the English alphabet be the universal set U.
May we consider the set of vowels in our discussion? _____ yes
May we consider the set of consonants in our discussion? _____ yes

50 Describe the set V of vowels: V = _____ . {a, e, i, o, u}

51 Describe the set C of consonants: C = _____ {b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z}

In Frames *49* through *51* every element of the set U of letters is either in set V or in set C. Furthermore, no letter is both a vowel and a consonant. Each of the sets V and C is the *complement* of the other.

The **complement** of a set A is the set of elements in the universal set U that are not in set A. The complement of set A is denoted by $\overline{A}$.

52 If $U = \{1, 2, 3, 4, 5\}$ and $A = \{2, 3\}$, then the elements in U that are not in A are _____ , _____ , and _____ . Therefore, $\overline{A}$ = _____ . 1, 4, 5, {1, 4, 5}

53 If $U = \{a, b, c, d, e\}$, the complement of $\{a, d, e\}$ is _____ . $\{b, c\}$

54 If U is the set of counting numbers and A is the set of odd counting numbers, then $\overline{A}$ is the set of _____ counting numbers. even