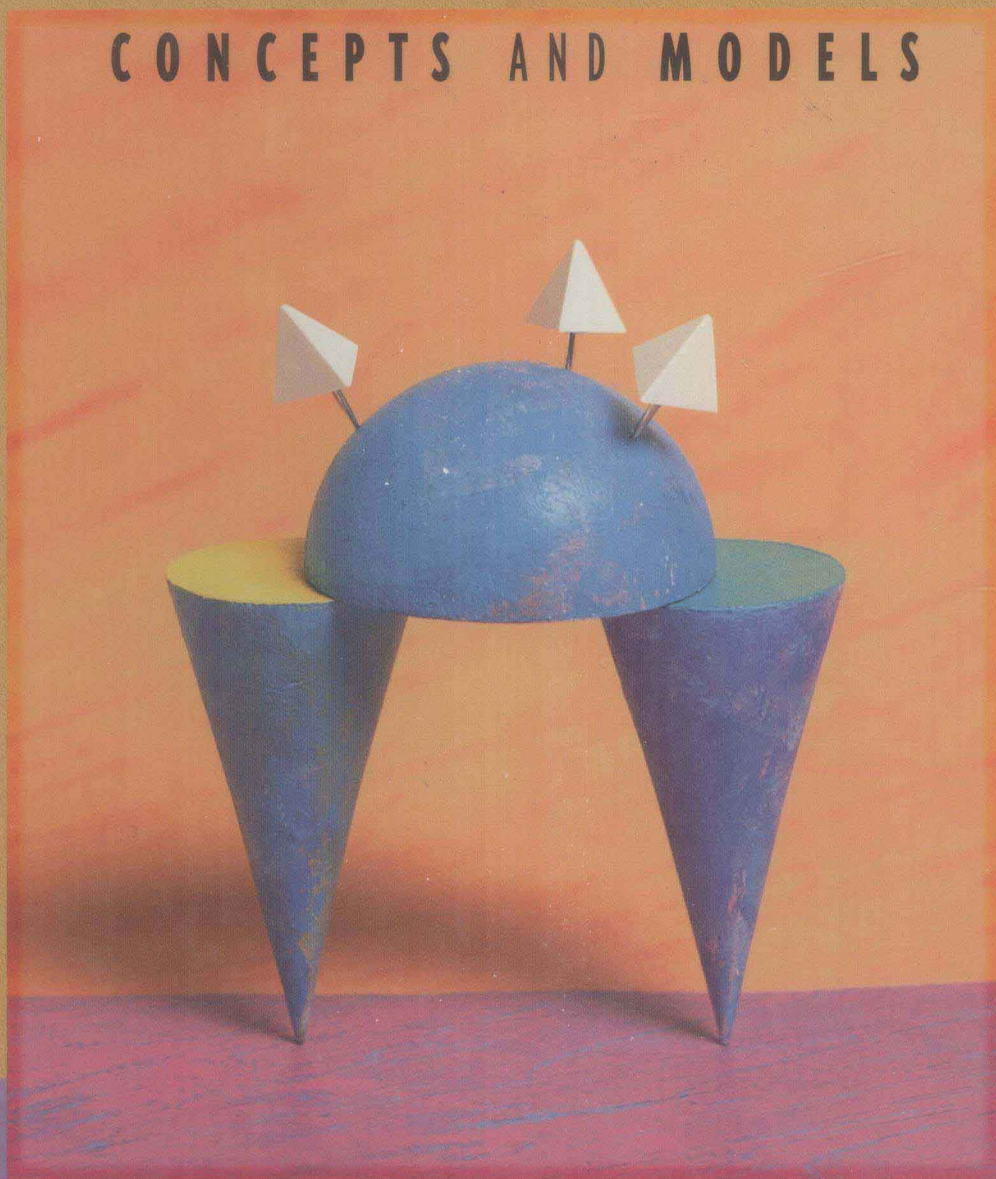


College Algebra

THIRD EDITION

CONCEPTS AND MODELS



LARSON ▽ HOSTETLER ▽ HODGKINS

College Algebra

Concepts and Models

Third Edition

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We have included examples and exercises that use real-life data as well as technology output from a variety of software. This would not have been possible without the help of many people and organizations. Our wholehearted thanks goes to all for their time and effort.

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A Word from the Authors

Welcome to *College Algebra: Concepts and Models*, Third Edition. In this revision, we continue to focus on developing students' conceptual understanding of college algebra while offering opportunities for them to hone their problem-solving skills. In preparing the Third Edition, we carefully considered all text elements for revision, as well as adding new examples, exercises, and applications.

In response to suggestions from college algebra instructors, we have revised the coverage of some topics in the Third Edition, especially in Chapters P, 3, 4, and 8. To allow greater flexibility at the beginning of the course, we streamlined the algebra review (Chapter P in the Second Edition) and moved it to Appendix A. In Chapter 3, *Transformations of Functions* (Section 3.3) and *The Algebra of Functions* (Section 3.4) are now separate sections. This allows us to cover transformations in greater detail and to expand the discussion of composite functions. We moved several sections from old Chapter 3 to Chapter 4: *Quadratic Functions and Models* (Section 4.1), *Polynomial Functions of Higher Degree* (Section 4.2), and *Rational Functions* (Section 4.7). In addition, we combined old Sections 4.2 and 4.3 as *Real Zeros of Polynomial Functions* (Section 4.4) and addressed the Intermediate Value Theorem in that context. Finally, we omitted the discussion of Descartes's Rule of Signs and the Bounds for Real Zeros from this edition. In Chapter 8, we added a new section, *Mathematical Induction* (8.7), for instructors who want to prove the Binomial Theorem.

We have found that many college algebra students grasp theoretical concepts more easily when they work with them in the context of a real-life situation. Throughout the Third Edition, students now have many more opportunities to collect, analyze, and model real data. We updated all real-data application examples or replaced them with new applications that use current data. In addition, all of the Chapter Opener/Chapter Project applications—many of which ask the student to manipulate real data—are new.

There are many more opportunities in the Third Edition for students to use a graphing utility in the problem-solving process—to visualize and explore theoretical concepts, to analyze real data, and to verify alternative solution methods. However, to accommodate a variety of teaching and learning styles, the use of graphing technology is always optional. At the suggestion of our users, we revised the Technology feature to use only generic references, omitting keystrokes and references to specific calculator models. At the beginning of the Third Edition, we have included “A Brief Introduction to Graphing Utilities.” This section addresses the basic functions of graphing calculators that are used in college algebra, including the equation editor and the table, zoom, and trace features.

To encourage mastery and understanding, we have included opportunities for student self-assessment at strategic points throughout each chapter. New to this edition, Learning Objectives at the beginning of each section provide students with a conceptual outline for reference. In the Chapter Summary, these Objectives are linked to the Review Exercises for students who require additional guided practice. Finally, Mid-Chapter Quizzes, Chapter Tests, and Cumulative Tests allow students to test understanding and learning retention at regular intervals.

We hope you will enjoy using the Third Edition in your college algebra class. We think its straightforward, readable style, engaging applications, and study tools will appeal to your students and contribute to their success in this course.



Ron Larson



Robert P. Hostetler

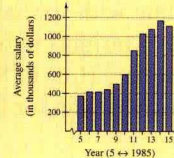


Anne Hodgkins

Features

1 Equations and Inequalities

- 1.1 Linear Equations
- 1.2 Mathematical Modeling
- 1.3 Quadratic Equations
- 1.4 The Quadratic Formula
- 1.5 Other Types of Equations
- 1.6 Linear Inequalities
- 1.7 Other Types of Inequalities



Today there are 30 teams playing in the National and American Leagues. Attendance for the 1998 season topped 70.5 million. Salaries for professional baseball players have steadily increased. The average salaries for players from 1985 to 1995 are listed in the table.

Year	1985	1986	1987	1988	1989	1990	1991	1992	1993	1994	1995
Average salary (in thousands)	\$371	\$413	\$412	\$439	\$497	\$598	\$851	\$1029	\$1076	\$1168	\$1111

The chapter project related to this information is on page 80.

1

Definition and Theorem

Definitions, theorems, rules, formulas, guidelines, and properties are highlighted for emphasis and easy reference.

Historical Note

Notes featuring mathematicians and their work, integrated throughout the text, place algebra in historical context.

Chapter Opener

Each chapter begins with a real-life application with current data. At the end of the chapter, students revisit this application as a *Chapter Project* after they have been exposed to the algebra techniques required for solving the problem. A list of section titles outlines the topics covered in the chapter.

Section Opener

Each section opens with a list of learning *Objectives*, the algebra skills presented in the section. This conceptual outline functions as a useful tool for reference and review for the students and class planning for the instructor. In the *Chapter Summary*, these objectives are linked to *Review Exercises* for additional student practice.

4.6 The Fundamental Theorem of Algebra

Objectives

- Use the Fundamental Theorem of Algebra and the Linear Factorization Theorem to write a polynomial as the product of linear factors.
- Find a polynomial with integer coefficients whose zeros are given.
- Factor a polynomial over the real and complex numbers.
- Find all real and complex zeros of a polynomial function.



The Granger Collection

Jean La Rond d'Alembert
(1717–1783)

Jean La Rond d'Alembert worked independently of Carl Gauss trying to prove the Fundamental Theorem of Algebra. His efforts were such that in France, the Fundamental Theorem of Algebra is frequently known as the theorem of d'Alembert.

The Fundamental Theorem of Algebra

You have been using the fact that an n th-degree polynomial can have at most n real zeros. In the complex number system, this statement can be improved. That is, in the complex number system, every n th-degree polynomial function has precisely n zeros. This important result is derived from the **Fundamental Theorem of Algebra**, first proved by the famous German mathematician Carl Friedrich Gauss (1777–1855).

▶ The Fundamental Theorem of Algebra

If $f(x)$ is a polynomial of degree n , where $n > 0$, then f has at least one zero in the complex number system.

Using the Fundamental Theorem of Algebra and the equivalence of zeros and factors, you obtain the following theorem.

▶ Linear Factorization Theorem

If $f(x)$ is a polynomial of degree n

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $n > 0$, then f has precisely n linear factors

$$f(x) = a_n(x - c_1)(x - c_2) \cdots (x - c_n)$$

where c_1, c_2, \dots, c_n are complex numbers and a_n is the leading coefficient of $f(x)$.

Note that neither the Fundamental Theorem of Algebra nor the Linear Factorization Theorem tells you *how* to find the zeros or factors of a polynomial. Such theorems are called **existence theorems**. To find the zeros of a polynomial function, you still rely on the techniques developed in the earlier parts of the text.

Remember that the n zeros of a polynomial function can be real or complex, and they may be repeated. Example 1 illustrates several cases.

To solve an equation involving an absolute value, remember that the expression inside the absolute value signs can be positive or negative. This results in two separate equations, each of which must be solved. For instance, the equation

$$|x - 2| = 3$$

results in the two equations

$$x - 2 = 3 \quad \text{and} \quad -(x - 2) = 3$$

which implies that the equation has two solutions: 5 and -1.

Example 7 An Equation Involving Absolute Value

Solve $|x^2 - 3x| = -4x + 6$.

Solution

Because the variable expression inside the absolute value signs can be positive or negative, you must solve the following two equations.

First Equation

$x^2 - 3x = -4x + 6$	Use positive expression.
$x^2 + x - 6 = 0$	Standard form
$(x + 3)(x - 2) = 0$	Factor.
$x + 3 = 0$ \Rightarrow $x = -3$	Set 1st factor equal to 0.
$x - 2 = 0$ \Rightarrow $x = 2$	Set 2nd factor equal to 0.

Second Equation

$-(x^2 - 3x) = -4x + 6$	Use negative expression.
$x^2 - 7x + 6 = 0$	Standard form
$(x - 1)(x - 6) = 0$	Factor.
$x - 1 = 0$ \Rightarrow $x = 1$	Set 1st factor equal to 0.
$x - 6 = 0$ \Rightarrow $x = 6$	Set 2nd factor equal to 0.

Check


$ (-3)^2 - 3(-3) = -4(-3) + 6$	-3 checks. ✓
$ (2)^2 - 3(2) = -4(2) + 6$	2 does not check. ✗
$ (1)^2 - 3(1) = -4(1) + 6$	1 checks. ✓
$ (6)^2 - 3(6) = -4(6) + 6$	6 does not check. ✗

The solutions are -3 and 1.

Study Tip

When solving an equation with absolute value, such as in Example 7, write the two equations to be solved. Then solve each equation independently. Be sure to check your solution.

Example

A wide variety of *Examples*, many with detailed, step-by-step solutions, enhance the usefulness of the Third Edition as a learning and study tool. Each is labeled for easy reference, and many are accompanied by side comments that clarify the solution method used. The icon  identifies *Examples* that reflect real life data.

Study Tip

Throughout the text, *Study Tips* at point of use offer students helpful suggestions for their study of algebra, including avoiding common errors, dealing with special cases, and developing further insights into theoretical concepts. In addition, “How to Study Algebra” (page xxi) outlines a plan for improving student study skills.

Technology

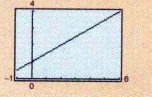
Students are encouraged to use a graphing utility as a problem-solving tool. The text offers many opportunities to visualize theoretical concepts, to discover alternative approaches, and to verify the results of other solution methods using technology. However, students are not required to have access to a graphing utility to use this text effectively. As appropriate, the text addresses both the benefits of using technology and its possible misuse or misinterpretation.

Graphics

The Third Edition has over 1500 figures to help students grasp mathematical concepts through visualization.

TECHNOLOGY

Your graphing utility may have a built-in statistical program to calculate the equation of the best-fitting line for linear data. This statistical method of fitting a line to a collection of points is called *linear regression*. A discussion of linear regression is beyond the scope of this text, but the program in most graphing utilities is easy to use and allows you to analyze linear data that may not be convenient to graph by hand. The graph below shows the best-fitting line for the data in Example 7.



Scatter Plots

Another type of linear modeling is a graphical approach that is commonly used in statistics. To find a mathematical model that approximates a set of actual data points, plot the points on a rectangular coordinate system. This collection of points is called a **scatter plot**. Once the points have been plotted, try to find the line that most closely represents the plotted points. (In this section, we will rely on a visual technique for fitting a line to a set of points. If you take a course in statistics, you will encounter *regression analysis* formulas that can fit a line to a set of points.)

Example 7 Fitting a Line to a Set of Points

The scatter plot in Figure 2.53(a) shows 35 different points in the plane. Find the equation of a line that approximately fits these points.

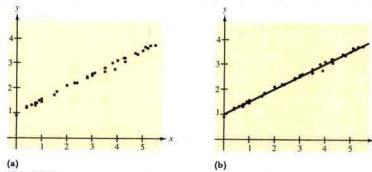


Figure 2.53

Solution

From Figure 2.53(a), you can see that there is no line that *exactly* fits the given points. The points, however, do appear to resemble a linear pattern. Figure 2.53(b) shows a line that appears to best describe the given points. (Notice that about as many points lie above the line as below it.) From this figure, you can see that the “best-fitting line” has a *y*-intercept at about (0, 1) and has a slope of about $\frac{1}{2}$. So, the equation of the line is

$$y = \frac{1}{2}x + 1.$$

Study Tip

Use a scatter plot to first determine whether a line is a reasonable model for a given set of data.

If you had been given the coordinates of the 35 points, you could have checked the accuracy of this model by constructing a table that compared the actual *y*-values with the *y*-values given by the model.

DISCOVERY

Graph each of the following functions with a graphing utility. Determine whether the function is *odd*, *even*, or *neither*.

$f(x) = x^2 - x^4$
 $g(x) = 2x^3 + 1$
 $h(x) = x^5 - 2x^3 + x$
 $k(x) = x^5 - 2x^4 + x - 2$
 $j(x) = 2 - x^6 - x^8$
 $p(x) = x^9 + 3x^5 - x^3 + x$

What do you notice about the equations of functions that are odd? What do you notice about the equations of functions that are even? Can you describe a way to identify a function as odd or even by inspecting the equation? Can you describe a way to identify a function as neither odd nor even by inspecting the equation?

Even and Odd Functions

In Section 2.2, you studied different types of symmetry of a graph. In the terminology of functions, a function is said to be **even** if its graph is symmetric with respect to the y -axis and to be **odd** if its graph is symmetric with respect to the origin. The symmetry tests in Section 2.2 yield the following tests for even and odd functions.

Test for Even and Odd Functions

A function given by $y = f(x)$ is **even** if, for each x in the domain of f ,

$f(-x) = f(x)$.

A function given by $y = f(x)$ is **odd** if, for each x in the domain of f ,

$f(-x) = -f(x)$.

Example 7. Even and Odd Functions

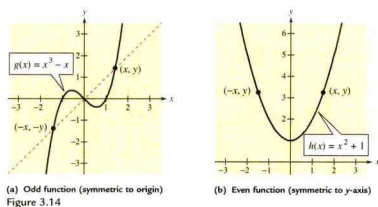
a. The function $g(x) = x^3 - x$ is odd because

$g(-x) = (-x)^3 - (-x) = -x^3 + x = -(x^3 - x) = -g(x)$.

b. The function $h(x) = x^2 + 1$ is even because

$h(-x) = (-x)^2 + 1 = x^2 + 1 = h(x)$.

The graphs of the two functions are shown in Figure 3.14.



Discussing the Concept

Each text section ends with *Discussing the Concept*. Designed to encourage students to think, reason, and write about algebra, these exercises help synthesize the concepts and methods presented in the section. Depending on the nature of the course, these problems can be assigned as individual student work or collaborative projects, or they may form the basis for class discussion.

Discovery

Discovery activities offer opportunities for the exploration of selected mathematical concepts. Students are encouraged to use techniques such as visualization and modeling to develop their intuitive understanding of theoretical topics. These optional activities can be omitted at the instructor's discretion without affecting the flow of the material.

Example 8. Prize Money at the Indianapolis 500

The total prize money p (in millions of dollars) awarded at the Indianapolis 500 race from 1990 to 1998 is given in the following table. Construct a scatter plot that represents the data and find a linear model that approximates the data. (Source: Indianapolis Motor Speedway Hall of Fame)

Year	1990	1991	1992	1993	1994	1995	1996	1997	1998
p	\$6.63	\$7.01	\$7.53	\$7.68	\$7.86	\$8.06	\$8.11	\$8.62	\$8.72

Solution

Let $t = 0$ represent 1990. The scatter plot for the points is shown in Figure 2.54. From the scatter plot, draw a line that approximates the data. Then, to find the equation of the line, approximate two points on the line: $(1, 7)$ and $(5, 8)$. The slope of this line is

$m = \frac{p_2 - p_1}{t_2 - t_1} = \frac{8 - 7}{5 - 1} = 0.25$.

Using the point-slope form, you can determine that the equation of the line is

$p = 0.25t + 6.75$.

To check this model, compare the actual p -values with the p -values given by the model (these values are labeled p^* in the table below).

t	0	1	2	3	4	5	6	7	8
p	\$6.63	\$7.01	\$7.53	\$7.68	\$7.86	\$8.06	\$8.11	\$8.62	\$8.72
p^*	\$6.75	\$7.00	\$7.25	\$7.50	\$7.75	\$8.00	\$8.25	\$8.50	\$8.75

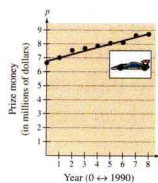


Figure 2.54

Discussing the Concept. Gathering and Analyzing Data

Measure the height (h) and forearm (f) of each person in the class with a tape measure. Gather the data in the form (h, f) and plot the points on a set of coordinate axes. Do the points appear to follow a linear model? Find an equation for a line that approximately represents the points.

Warm Up

The following warm-up exercises involve skills that were covered in earlier sections. You will use these skills in the exercise set for this section.

1. Find $f(3)$ for $f(x) = x^2 - 4x + 15$.
 2. Find $f(-x)$ for $f(x) = 2x/(x - 3)$.
- In Exercises 3 and 4, solve the equation.
3. $-x^2 + 10x = 0$
 4. $3x^2 + 2x - 8 = 0$
- In Exercises 5–10, sketch the graph of the function.
5. $f(x) = -2$
 6. $f(x) = -x$
 7. $f(x) = x + 5$
 8. $f(x) = 2 - x$
 9. $f(x) = 3x - 4$
 10. $f(x) = 9x + 10$

3.3 Exercises

LAB I

In Exercises 1–8, use the graph of $f(x) = x^2$ to sketch the graph of the function by hand. Verify with a graphing utility.

1. $g(x) = x^2 + 3$
2. $g(x) = x^2 - 2$
3. $g(x) = (x + 3)^2$
4. $g(x) = (x - 4)^2$
5. $g(x) = (x - 2)^2 + 2$
6. $g(x) = (x + 1)^2 - 3$
7. $g(x) = -x^2 + 1$
8. $g(x) = -(x - 2)^2$

In Exercises 9–14, use the graph of $f(x) = |x|$ to sketch the graph of the function by hand. Verify with a graphing utility.

9. $g(x) = |x| + 2$
10. $g(x) = |x - 1|$
11. $g(x) = -|x| + 3$
12. $g(x) = |x + 2| - 3$
13. $g(x) = 4 - |x - 2|$
14. $g(x) = |x - 2| + 2$

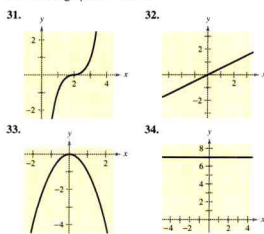
In Exercises 15–22, use the graph of $f(x) = \sqrt{x}$ to sketch the graph of the function by hand. Verify with a graphing utility.


15. $y = \sqrt{x - 2}$
16. $y = \sqrt{x + 3}$
17. $y = \sqrt{x - 3} + 1$
18. $y = \sqrt{x + 5} - 2$
19. $y = 2 - \sqrt{x - 4}$
20. $y = \sqrt{2x}$
21. $y = \sqrt{-x} + 1$
22. $y = \sqrt{2x} - 5$

In Exercises 23–30, use the graph of $f(x) = \sqrt[3]{x}$ to sketch the graph of the function by hand. Verify with a graphing utility.

23. $y = \sqrt[3]{x} - 1$
24. $y = \sqrt[3]{x + 1}$
25. $y = 2 - \sqrt[3]{x + 1}$
26. $y = -\sqrt[3]{x - 1} - 4$
27. $y = \sqrt[3]{x + 1} - 1$
28. $y = \frac{1}{2}\sqrt[3]{x}$
29. $y = \frac{1}{3}\sqrt[3]{x} - 3$
30. $y = 2\sqrt[3]{x - 2} + 1$

In Exercises 31–36, identify the common function and the transformation shown in the graph. Write the equation for the graphed function.



There are over 5000 exercises in the Third Edition, including multi-part, exploratory, modeling, data analysis, reasoning, writing, estimation, geometry, and technology-required problems. A rich variety of relevant applications—many of which utilize real data—represent a range of disciplines and clearly demonstrate the applicability of the mathematics to real-world situations. In addition, many section exercises have been labeled in the Instructor’s Annotated Edition with the icon  to indicate that they are appropriate for group work as well.

Warm Up

Starting with Section 1.1, each exercise set is preceded by ten warm-up exercises. These *Warm Ups* provide students with the opportunity to review and practice previously learned skills necessary to master the new skills presented in the section. Answers to all *Warm Ups* are given in the back of the text.

Exercises

The Third Edition contains a variety of computational, conceptual, and applied problems to accommodate a range of teaching and learning styles. Designed to build competence, skill, and understanding, each exercise set is graded in difficulty to allow students to gain confidence as they progress. Answers to odd-numbered exercises are given in the back of the text, while detailed solutions are available in the *Study and Solutions Guide*.

59. **Hockey Players’ Weights** From 1971 to 1990, the average weight of a hockey player in the National Hockey League increased by about 11 pounds. A model that approximates the average weight of a hockey player is
- $$W = 188.36 + 0.202t + 0.0042t^2, \quad -9 \leq t \leq 10$$
- where W is the weight in pounds and t represents the playing season, with $t = 0$ corresponding to the 1980–81 playing season. (Source: National Hockey League)

- (a) Use a graphing utility to graph the model.
- (b) Estimate the playing season in which the average weight was 190 pounds.
- (c) In the 1997–98 playing season, the average weight was 200.3 pounds. Does this follow the model? Explain.

60. **Total Investment Capital** From 1990 to 1997, the total investment capital for Northwest Natural Gas Company increased by about 280 million dollars. A model that approximates the total investment capital for this corporation is
- $$C = 461.3 + 59.74t - 10.53t^2 + 1.097t^3, \quad 0 \leq t \leq 7$$

where C is the total investment capital (in millions of dollars) and $t = 0$ represents 1990. (Source: Northwest Natural Gas Company)

- (a) Use a graphing utility to graph the model.
 - (b) Estimate the year (between 1990 and 1997) in which the total investment capital was the least.
61. **Advertising Expenses** The total revenue for a soft-drink company is related to its advertising expense by the function

$$R = \frac{1}{50,000}(-x^3 + 600x^2), \quad 0 \leq x \leq 400$$

where R is the total revenue in millions of dollars and x is the amount spent on advertising (in tens of thousands of dollars). Use the graph of this function to estimate the point on the graph at which the function is increasing most rapidly. This point is called the *point of diminishing returns* because any expenses above this amount will yield less return per dollar invested in advertising.

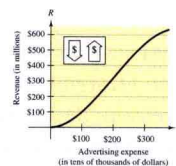
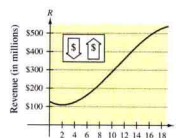


Figure for 61

62. **Advertising Expenses** The total revenue for a hotel corporation is related to its advertising expense by the function
- $$R = -0.148x^3 + 4.889x^2 - 17.778x + 125.185, \quad 0 \leq x \leq 20$$

where R is the total revenue in millions of dollars and x is the amount spent on advertising (in millions of dollars). Use the accompanying graph of this function to estimate the point on the graph at which the function is increasing most rapidly. This point is called the *point of diminishing returns* because any expenses above this amount will yield less return per dollar invested in advertising.



63. **Think About It** Use a graphing utility to graph the following functions: $f(x) = x^2$, $g(x) = x^4$, $h(x) = x^6$. Do the three functions have a common shape? Are their graphs identical? Why or why not?

55. Ice Cream During the mid-1990s, the number of gallons of regular ice cream produced in the United States fell slightly, then rose again according to the model

$$G(x) = 11.25x^2 - 111.25x + 1158.75, \quad 4 \leq x \leq 7$$

where G is the number of gallons of regular ice cream (in millions) and t represents the calendar year, with $t = 4$ corresponding to 1994. Use a graphing utility to determine the year, from 1994 to 1997, when the number of gallons produced was at its lowest. (Source: U.S. Department of Agriculture)

56. Employment Rate The annual average employment rate for single women between 16 and 19 years of age from 1990 to 1994 can be approximated by the model

$$P(x) = 0.564t^2 - 2.39t + 51.89, \quad 0 \leq t \leq 4$$

where P is the percent of single women who were employed and t represents the calendar year, with $t = 0$ corresponding to 1990. Use a graphing utility to determine the year in which the employment rate was at its lowest. (Source: U.S. Bureau of Labor Statistics)

Math Matters • Circles and Pi

Pi is a special number that represents a relationship that is found in *all circles*. The relationship is this: The ratio of the circumference of any circle to the diameter of the circle always produces the same number—the number denoted by the Greek symbol π .

Calculation of the decimal representation of the number π has consumed the time of many mathematicians (and computers). Because the number π is irrational, its decimal representation does not repeat or terminate. Here is a decimal representation of π that is accurate to 27 decimal places.

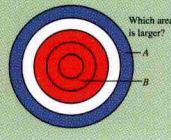
$\pi = 3.141592653589793238462643383$

In October 1995, a team of mathematicians in British Columbia discovered a formula for calculating the n th digit of π without calculating the preceding digits. Run on a computer, the algorithm for the n th digit requires that numbers be written in base 16 (hexadecimal representation) instead of base 10 (decimal representation).

The number π is used in most formulas that deal with circles. For instance, the area of a circle is given by

$$A = \pi r^2$$

where r is the radius of the circle. Use this formula to determine which of the colored regions inside the circle below has the greater area. Is the area of the red circle larger than that of the blue ring, or do both regions have the same area? (The answer is given in the back of the book.)



Supplementary Lab Assignments

Additional guided discovery activities and applications for selected text topics are available in the supplement *Problem Solving, Modeling, and Data Analysis Labs*. These labs are referenced in the section exercise sets.

Math Matters

In each chapter, a *Math Matters* feature engages student interest with an historical note or mathematical problem. Answers to those that pose a question are given in the back of the text.

Review Exercises

Review Exercises at the end of each chapter offer students an additional opportunity for practice and review. Answers to odd-numbered review exercises are given in the back of the text.

REVIEW EXERCISES

In Exercises 1–4, decide whether the equation represents y as a function of x .

1. $3x - 4y = 12$
2. $y^2 = x^2 - 9$
3. $y = \sqrt{x+3}$
4. $x^2 + y^2 - 6x + 8y = 0$

In Exercises 5 and 6, decide whether the set represents a function from A to B .

$A = \{1, 2, 3\}$ $B = \{-3, -4, -7\}$

5. $\{(1, -3), (2, -7), (3, -3)\}$
6. $\{(1, -4), (2, -3), (3, -9)\}$

In Exercises 7–10, evaluate the function and simplify the results.

7. $f(x) = 3x - 5$
(a) $f(1)$ (b) $f(-2)$ (c) $f(m)$ (d) $f(x+1)$
8. $f(x) = \sqrt{x+9} - 3$
(a) $f(7)$ (b) $f(0)$ (c) $f(-5)$ (d) $f(x+2)$
9. $f(x) = |x| + 5$
(a) $f(0)$ (b) $f(-3)$ (c) $f(\frac{1}{2})$ (d) $f(-x^2)$
10. $f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ x + 3, & x > 2 \end{cases}$
(a) $f(0)$ (b) $f(2)$ (c) $f(3)$ (d) $f(-5)$

In Exercises 11 and 12, find all real values of x such that $f(x) = 0$.

11. $f(x) = \frac{2x+7}{3}$
12. $f(x) = x^3 - 4x$

In Exercises 13–18, find the domain of the function.

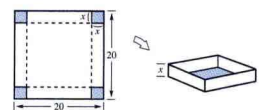
13. $f(x) = 3x^2 + 8x + 4$
14. $g(t) = \frac{5}{t^2 - 9}$
15. $h(x) = \sqrt{x+9}$
16. $f(t) = \sqrt[3]{t-5}$
17. $g(t) = \frac{\sqrt{t-3}}{t-5}$
18. $h(x) = \sqrt[3]{9-x^2}$

19. Reasoning A student has difficulty understanding why the domains of

$h(x) = \frac{x^2-4}{x}$ and $k(x) = \frac{x}{x^2-4}$ are different. How would you explain their respective domains algebraically? How could you use a graphing utility to explain their domains?

20. Reasoning A student has difficulty understanding why the domains of $h(x) = \sqrt{x+1}$ and $k(x) = \sqrt[3]{x+1}$ are different. How would you explain their respective domains algebraically? How could you use a graphing utility to explain their domains?

21. Volume of a Box An open box is to be made from a square piece of material 20 inches on a side by cutting equal squares from the corners and turning up the sides (see figure).
(a) Write the volume of the box as a function of x .
(b) What is the domain of this function?
(c) Use a graphing utility to sketch the graph of this function.



22. Balance in an Account A person deposits \$5000 in an account that pays 6.25% interest compounded quarterly.

- (a) Write the balance of the account in terms of the time t that the principal is left in the account.
- (b) What is the domain of this function?

23. Vertical Motion The velocity of a ball thrown vertically upward from ground level is given by $v(t) = -32t + 48$, where t is the time in seconds and v is the velocity in feet per second.
(a) Find the velocity when $t = 1$.
(b) Find the time when the ball reaches its maximum height. [Hint: Find the time when $v(t) = 0$.]
(c) Find the velocity when $t = 2$.

Chapter Project

Salaries in Professional Baseball

Year	Average Salary (in thousands)
1985	\$371
1986	\$413
1987	\$412
1988	\$439
1989	\$497
1990	\$598
1991	\$851
1992	\$1029
1993	\$1076
1994	\$1168
1995	\$1111

The record-breaking attendance (70.5 million people) for major league baseball was not the only record that was broken in the 1998 season. Mark McGwire (St. Louis) hit a record 70 home runs and drew a record 152 walks. Bret Boone (Cincinnati) became the first player to lead a league in fielding percentage for four consecutive years. Pitcher Tom Gordon (Boston) had a record 42 consecutive saves. Cal Ripkin, Jr. (Baltimore) holds the record for consecutive games played at 2632. As players reach and exceed previous records, they expect to be compensated accordingly. The average salary for a major league baseball player has tripled in the last 11 years.

The table at the left gives the average salaries (in thousands of dollars) for professional baseball players from 1985 to 1995. The average salary can be modeled by the equation

$$S = 3.86t^2 + 16.0t + 140, \quad 5 \leq t \leq 15$$

where $t = 5$ represents 1985. Use this information to investigate the following questions.

- Compare the Data** Make a table that compares the actual average salary for 1985 to 1995 with the average found using the model.
- Compare Projected Average Salaries with Actual Average Salaries** Use the model to predict the average salary in 1996 and in 1997. Use a library or other reference source to find the average salary. How well did the model predict the average salary?
- Predict Future Average Salaries** According to this model, when will the average salary reach \$2,000,000?
 - Answer the question numerically by creating a table of values.
 - Answer the question algebraically by solving $2000 = 3.86t^2 + 16.0t + 140$.
- Find a Model** Enter the table at the left in a graphing utility. Enter 5 for 1985, enter 6 for 1986, and so on. Use the quadratic fit program to find a quadratic model for the data. Do you get the same model as the one above?
- Research** Use a reference source to find data that can be closely modeled with a quadratic model. Compare the model with the actual data numerically and graphically.

Chapter Project

Chapter Projects are extended applications designed to enhance students' understanding of mathematical concepts. Real data is previewed at the beginning of the chapter and then analyzed in detail in the *Project* at the end of the chapter. Here the student is guided through a set of multi-part exercises using modeling, graphing, and critical thinking skills to analyze the data.

Chapter Summary

The *Chapter Summary* is another vehicle for student self-assessment. For the student who needs additional practice and review, each section learning *Objective* is keyed to the appropriate *Review Exercises*.

CHAPTER SUMMARY

After studying this chapter, you should have acquired the following skills. These skills are keyed to the Review Exercises that begin on page 82. Skills to odd-numbered Review Exercises are given in the back of the book.

- | | | |
|------------|---|---------------------------------------|
| 1.1 | Classify an equation as an identity or a conditional equation. | Review Exercises 1, 2 |
| | Determine whether a given value is a solution. | Review Exercises 3, 4 |
| | Solve a linear equation in one variable. | Review Exercises 5–1 |
| 1.2 | Use mathematical models to solve word problems. | Review Exercises 13, 23 |
| | Model and solve percent and mixture problems. | Review Exercises 14–16, 21, 22, 24–26 |
| | Use common formulas to solve geometry and simple interest problems. | Review Exercises 17–20 |
| 1.3 | Solve a quadratic equation by factoring. | Review Exercises 27–30 |
| | Solve a quadratic equation by extracting square roots. | Review Exercises 31–34 |
| | Analyze a quadratic equation. | Review Exercises 35, 36 |
| | Construct and use a quadratic model to solve an application problem. | Review Exercises 37–40 |
| 1.4 | Use the discriminant to determine the number of real solutions to a quadratic equation. | Review Exercises 41, 42 |
| | Solve a quadratic equation using the Quadratic Formula. | Review Exercises 43–51 |
| | Use the Quadratic Formula to solve an application problem. | Review Exercises 52, 53 |
| 1.5 | Solve a polynomial equation by factoring. | Review Exercises 54–57 |
| | Rewrite and solve an equation involving radicals or rational exponents. | Review Exercises 58–61 |
| | Rewrite and solve an equation with fractions or absolute value. | Review Exercises 62–65 |
| | Construct and use a nonquadratic model to solve an application problem. | Review Exercises 66–68 |
| 1.6 | Solve and graph a linear inequality. | Review Exercises 69–74 |
| | Construct and use a linear inequality to solve an application problem. | Review Exercises 75, 76 |
| 1.7 | Solve and graph a polynomial inequality. | Review Exercises 77–79, 83, 84 |
| | Solve and graph a rational inequality. | Review Exercises 80–82, 85, 86 |
| | Determine the domain of an expression involving a square root. | Review Exercises 87–90 |
| | Construct and solve a polynomial inequality to solve an application problem. | Review Exercises 91–96 |

Mid-Chapter Quiz

Take this quiz as you would take a quiz in class. After you are done, check your work against the answers given in the back of the book.

In Exercises 1–4, find the value of $f(x) = 2x^2 - x + 1$.

1. $f(3)$ 2. $f(-1)$ 3. $f(0)$ 4. $f(a)$

In Exercises 5 and 6, find all real values of x such that $f(x) = 0$.

5. $f(x) = x^3 - 4x$ 6. $f(x) = 3 - \frac{4}{x}$

7. A company produces a product for which the variable cost is \$11.40 per unit and the fixed costs are \$85,000. The product sells for \$16.89. Let x be the number of units produced. Write the total profit P as a function of x .

In Exercises 8–10, find the domain of the function.

8. $f(x) = \frac{3}{x^2 - x}$ 9. $h(x) = \sqrt{5 - x}$ 10. $g(x) = \frac{\sqrt{x-1}}{x+2}$

In Exercises 11–14, use a graphing utility to graph the function. Then estimate the open intervals on which the function is increasing or decreasing.

11. $f(x) = -x^2 + 5x + 3$ 12. $f(x) = 2x^3 - 5x^2$
 13. $f(x) = x^2 + 2$ 14. $f(x) = \sqrt{x+1}$

In Exercises 15 and 16, compare the graphs of $f(x) = \sqrt[3]{x}$ and g .

15. $g(x) = \sqrt[3]{x-2}$ 16. $g(x) = -\sqrt[3]{x+1}$

17. Write the equation for the transformation of $f(x) = x^3$ shown in the figure.

In Exercises 18 and 19, write the equation for the indicated transformation. Verify your result with a graphing utility.

18. The graph of $f(x) = x^2$ shifted three units to the right and two units downward.

19. The graph of $f(x) = \sqrt{x}$ shifted two units to the left and reflected about the x -axis.

20. The marketing department of a company estimates that the demand for a product is given by $p = 80 - 0.001x$, where p is the price per unit and x is the number of units. The cost of producing x units is given by $C = 300,000 + 25x$ and the profit for producing x units is given by $P = xp - C$. Write the equation for the profit function, sketch the graph, and estimate the number of units that would produce a maximum profit.

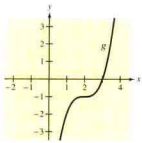


Figure for 17

Chapter Test

Take this test as you would take a test in class. After you are done, check your work against the answers given in the back of the book.

In Exercises 1–4, find the domain of the function.

1. $f(x) = 2x^2 - 3x + 8$ 2. $g(x) = \sqrt{x-7}$
 3. $g(t) = \frac{\sqrt{t-2}}{t-7}$ 4. $h(x) = \frac{3}{x^2-4}$

In Exercises 5–7, decide whether the statement is true or false. Explain.

5. The equation $2x - 3y = 5$ identifies y as a function of x .
 6. The equation $y = \pm\sqrt{x^2 - 16}$ identifies y as a function of x .
 7. If $A = \{3, 4, 5\}$ and $B = \{-1, -2, -3\}$, the set $\{(3, -9), (4, -2), (5, -3)\}$ represents a function from A to B .

In Exercises 8 and 9, (a) find the range and domain of the function, (b) determine the intervals over which the function is increasing, decreasing, or constant, and (c) determine whether the function is even or odd.

8. $f(x) = x^2 + 2$ (See figure.) 9. $g(x) = \sqrt{x^2 - 4}$ (See figure.)

10. Evaluate the function and simplify the results.

$$f(x) = \begin{cases} x+1, & x < 0 \\ x^2+2, & x \geq 0 \end{cases}$$

(a) $g(-1)$ (b) $g(0)$ (c) $g(3)$

In Exercises 11 and 12, sketch the graph of the function.

11. $g(x) = \begin{cases} x+1, & x < 0 \\ \frac{1}{x}, & x > 0 \end{cases}$ 12. $h(x) = (x-3)^2 + 4$

13. Find $f^{-1}(x)$ for the function $f(x) = x^3 - 5$.

In Exercises 14–19, use $f(x) = x^2 + 2$ and $g(x) = 2x - 1$ to find the function.

14. $(f+g)(x)$ 15. $(f-g)(x)$ 16. $(fg)(x)$
 17. $(\frac{f}{g})(x)$ 18. $(f \circ g)(x)$ 19. $(g \circ f)(x)$

20. A manufacturer determines that the variable costs for a new product are \$3.17 per unit and the fixed costs are \$75,000. The product is to be sold for \$5.25. Let x be the number of units sold.

- (a) Write the total cost C as a function of the number of units sold.
 (b) Write the total revenue R as a function of the number of units sold.
 (c) Write the total profit P as a function of the number of units sold.

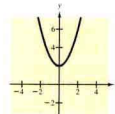


Figure for 8

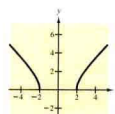


Figure for 9

Cumulative Test: Chapters 1–2

Take this test as you would take a test in class. After you are done, check your work against the answers given in the back of the book.

In Exercises 1–6, solve the equation.

1. $4y^2 - 64 = 0$ 2. $\frac{x}{3} + \frac{x}{4} = 1$
 3. $x^4 - 17x^2 + 16 = 0$ 4. $|2x - 3| = 5$
 5. $\sqrt{y+3} + y = 3$ 6. $2x^2 + x = 5$

7. You deposit \$5000 in an account that pays 8.5%, compounded monthly. Find the balance after 10 years.

In Exercises 8–10, solve the inequality.

8. $|\frac{2x-5}{4}| \leq 3$ 9. $(x+4)^2 \leq 4$ 10. $\frac{3+2x}{4-x} > 2$

11. Find the intercepts of the graph of $y = x\sqrt{x+4}$.
 12. Describe the symmetry of the graph of $y = -\sqrt{4-x^2}$.
 13. Write the equation of the circle in standard form and sketch its graph.
 $x^2 + y^2 - 6x + 4y - 3 = 0$
 14. Find the constant C such that $(-2, 3)$ is a solution point of $y = x^3 + C$.
 15. Find an equation of the line passing through $(3, -2)$ and $(-1, 5)$.
 16. Find an equation of the line with slope $\frac{1}{3}$ passing through $(2, -1)$.
 17. Find an equation of the line with zero slope passing through $(-1, -3)$.
 18. You have accepted a sales job that pays a base salary of \$1000 a month plus a 7% commission on monthly sales x . Write a linear model that describes your total monthly salary y in terms of your base salary and commission.
 19. During the second and third quarters of the year, a business had sales of \$210,000 and \$230,000, respectively. If the sales growth follows a linear pattern, what will the sales be during the fourth quarter?
 20. The revenue and cost equations for a product are given by
 $R = x(100 - 0.001x)$ and $C = 20x + 30,000$
 where R and C are measured in dollars and x represents the number of units sold. How many units must be sold for the revenue to equal the cost?

Mid-Chapter Quiz

Each chapter contains a 20-question *Mid-Chapter Quiz*. Appearing midway through the chapter, these are designed as a self-assessment tool for the student. Answers are provided in the back of the text.

Chapter Test

Each chapter ends with a *Chapter Test*, providing students with another self-assessment opportunity. Answers are provided in the back of the text.

Cumulative Test

The *Cumulative Tests* that appear after Chapters 2, 5, and 8 give students several opportunities throughout the course to judge their mastery of previously covered material.

Supplements

College Algebra: Concepts and Models, Third Edition, by Larson, Hostetler, and Hodgkins, is accompanied by a comprehensive supplements package with ancillaries for students, for instructors, and for use as classroom resources.

Printed Resources

For the Student

Study and Solutions Guide by Dianna L. Zook, Purdue University and Indiana University at Fort Wayne, and Anne V. Hodgkins, Phoenix College
(0-395-97623-5)

- Section summaries of key concepts
- Detailed, step-by-step solutions to odd-numbered Section and Review Exercises and to all Mid-Chapter Quizzes, Chapter Tests, and Cumulative Tests

Graphing Technology Keystroke Guide
(0-395-97676-6)

- Keystroke instructions for *TI-80*, *TI-81*, *TI-82*, *TI-83*, *TI-83 Plus*, *TI-85*, *TI-86*, *Casio fx-7700GE*, *Casio fx-9700GE*, *Casio CFX-9800G*, *HP 38G*, and *Sharp EL-9200/9300*
- Examples with step-by-step solutions
- Extensive graphics screen output
- Technology tips

Problem Solving, Modeling, and Data Analysis Labs by Wendy Metzger, Palomar College
(0-395-97624-3)

- Guided discovery activities and applications with Answer Key
- Keyed to the text by topic

For the Instructor

Instructor's Annotated Edition
(0-395-97626-X)

- Complete student edition, including student answers section: answers to all odd-numbered exercises, and answers to all Mid-Chapter Quizzes, Chapter Tests, Cumulative Tests, and Math Matters
- Instructor's answers section: answers to all even-numbered exercises, and answers to all Discovery features, Technology features, and Discussing the Concept activities
- Annotations at point of use: teaching strategies, suggestions for implementing features, common student errors, and additional examples, exercises, and group activities

Complete Solutions Guide by Dianna L. Zook, Indiana University and Purdue University at Fort Wayne, and Anne V. Hodgkins, Phoenix College (0-395-97625-1)

- Detailed, step-by-step solutions to all Section and Review Exercises; Mid-Chapter Quiz, Chapter Test, and Cumulative Test questions; and Chapter Project Questions

Instructor's Resource Guide and Test Item File

(0-395-97622-7)

- Printed test bank with approximately 1700 test items coded by level of difficulty
- Test items include multiple-choice and open-ended format
- Technology-required test items coded for easy reference
- Three tests per chapter
- Two final exams
- Transparency masters
- Lab activities contributed by Phyllis Shaw, Paradise Valley Community College

Media Resources

Internet Site

- Student and Instructor resources
- Graphing calculator programs
- Career Interviews

For the Student

Tutor

(Windows: 0-395-97627-8; Macintosh: 0-395-97638-3)

- Interactive tutorial software keyed to the text by section
- Diagnostic feedback
- Additional practice
- Chapter warm-ups and self-tests
- Glossary

Videotapes by Dana Mosely

(0-395-97641-3)

- Comprehensive coverage keyed to the text by section
- Detailed explanations of important concepts
- Numerous examples and applications

For the Instructor

Computerized Test Bank

(Windows: 0-395-97639-1; Macintosh: 0-395-97640-5)

- Test-generating software
- Approximately 1700 test items
- Also available as a printed test item file

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If you have suggestions for improving the text, please feel free to write to us. Over the past two decades, we have received many useful comments from both instructors and students, and we value these very much.

Ron Larson
Robert P. Hostetler
Anne V. Hodgkins

A Brief Introduction to Graphing Utilities

Graphing utilities such as graphing calculators and computers with graphing software are very valuable tools for visualizing mathematical principles, verifying solutions to equations, exploring mathematical ideas, and developing mathematical models. While graphing utilities are extremely helpful in learning mathematics, their use does not mean that learning algebra is any less important. In fact, the combination of knowledge of mathematics and the use of graphing utilities allows you to explore mathematics more easily and to a greater depth. If you are using a graphing utility in this course, it is up to you to learn its capabilities and to practice using this tool to enhance your mathematical learning.

This introduction gives a brief description of two basic features of your graphing utility: the equation editor and the table feature. If you will be using your graphing utility more extensively in Chapter 1, you may want to review Section 2.3. You may also find it helpful to consult the user's guide for your graphing utility. Additionally, graphing utility keystroke guides are available for most models, and your college library may have a videotape on how to use your graphing utility.

The Equation Editor

Many graphing utilities are designed to act as “function graphers.” In Chapter 3, you will study functions and their graphs in detail. You may recall from previous courses that a function can be thought of as a rule that describes the relationship between two variables. These rules are frequently written in terms of x and y . For example, the equation $y = 3x + 5$ represents y as a function of x .

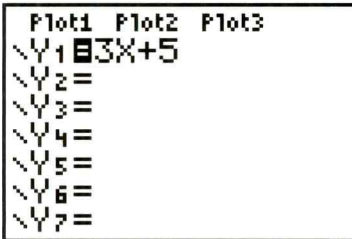


Figure 1

Many graphing utilities have an equation editor that requires an equation to be written in “ $y =$ ” form in order to be entered, as shown in Figure 1. (You should note that your equation editor screen may not look like the screen shown in Figure 1.) To determine exactly how to enter an equation into your graphing utility, consult your user's guide.

The Table Feature

Most graphing utilities have the capability to display a table of values with x and one or more y -values. These tables can be used to check solutions of an equation and to generate ordered pairs (x, y) to assist in graphing an equation.

To use the table feature, enter an equation in the equation editor in “ $y =$ ” form. The table may have a setup screen, which will allow you to select the starting x -value and the table step or x -increment. You may then have an option to automatically generate values for x and y or to build your own table using the ASK mode. In the ASK mode, you enter a value for x and the graphing utility will display the y -value.

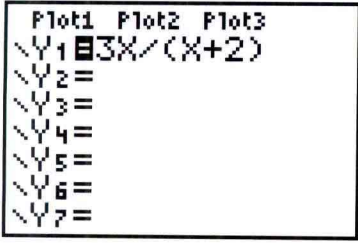


Figure 2

For example, enter the equation

$$y = \frac{3x}{x + 2}$$

in the equation editor as shown in Figure 2. In the table setup screen, set the table to start at $x = -4$ and set the table step to 1. When you view the table, notice that the first x -value is -4 and each value after it increases by one. Also notice that the Y_1 column gives the resulting y -value for each x -value, as shown in Figure 3. The table shows that the y -value when $x = -2$ is ERROR. This means that the variable x may not take on the value -2 in this equation. You can use the arrows on the keypad to scroll through the table.

X	Y ₁
-4	6
-3	9
-2	ERROR
-1	-3
0	0
1	1
2	1.5

X = -4

Figure 3

With the same equation in the equation editor, set the table to ASK mode. In this mode you do not need to set the starting x -value or the table step, since you are entering any value you choose for x . You may enter any real value for x —integers, fractions, decimals, irrational numbers, and so forth. If you enter $x = 1 + \sqrt{3}$, the graphing utility may rewrite the number as a decimal approximation, as shown in Figure 4. You can continue to build your own table by entering additional x -values in order to generate y -values.

X	Y ₁
2.7321	1.7321

X =

Figure 4

If you have several equations in the equation editor, the table may generate y -values for each equation.

Before beginning Chapter 1, be sure you are familiar with the use of the equation editor and the table feature. Remember that the screens and names of features and modes may not be the same as your graphing utility, so you should always consult your user's guide for help.