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# SUPERCONDUCTIVITY

GNSM-CNR and CONSORZIO INFM

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SUPERCONDUCTIVITY — PROCEEDINGS OF THE XXIV ITALIAN NATIONAL SCHOOL ON CONDENSED MATTER PHYSICS

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# Preface

This volume is based on the lecture notes of the XXIV Italian National School on Condensed Matter organized by the Gruppo Nazionale di Struttura della Materia (GNSM) of the Consiglio Nazionale delle Ricerche (CNR) and by the Consorzio Interuniversitario Nazionale per la Fisica della Materia (INFM). The course was devoted to Low Temperature Superconductors and was held in Bra, Italy from 18th to 28th September 1989.

The aim of the School was mainly to give an up-to-date presentation of the "old" superconductivity in a form suitable to a rather inhomogeneous audience composed of graduate and post-graduate students with a background either in Physics, or Chemistry or Materials Engineering. In the rush of new theoretical models and experimental problems connected with the high-$T_c$ superconductors, the need was felt to assess what was the actual content of the "classical" theory of superconductivity. The aim was twofold: on one hand, as that topic is only marginally covered in the regular University courses in Italy, many young reseachers were approaching the field with insufficient information; on the other hand, there was the risk (not limited to young researchers) of considering some phenomenology as completely unusual, only because of a superficial knowledge of the BCS and derived models. As a typical example of such a misleading attitude one can mention the often heard claim that the smallness or absence of the isotope effect ruled out any phonon contribution to the pairing.

As for the style of the presentation, the authors have rearranged and expanded their notes in order to present them in a volume, trying carefully to preserve the original didactic bias. An effort was required to put all the participants at the same level of knowledge about the basic tools, particularly with respect to the theoretical methods for many-body problems. This explains the somehow detailed discussion of some basic formalism, which could otherwise, for a more homogeneous audience, have been taken for granted. However, we hope that this will make the book useful to a broader range of users. Not all the lectures given at the school can be found in this book, since some talks on particular topics, such as the electron-phonon interaction, have been given following the well-known literature on these subjects.

The order of the contributions have been changed with respect to that of the talks, grouping arguments with similar content. The presentation starts from a phenomenological approach, which is developed at increasing levels of sophistication up to the Landau-Ginzburg theory in the first four contibutions. Chapter 5 is, in a sense, the bridge to the three subsequent ones. Chapters 6 to 8 deal with specific microscopic interactions and model Hamiltonians, mainly by using the Green function technique, which is presented in some detail and applied in various forms.

The following Chapters 9 to 14 discuss some important aspects of the phenomenology of the tunnel effect. Both the single particle tunnelling and the Cooper pair tunnelling, i.e. the Josephson effect, are analysed. Even though the emphasis is on the experimental point of view, the necessary theoretical support is also summarized. Beside these arguments, the SQUIDs, viewed as the main application of the Josephson effect, and some particular topics on Josephson junction arrays are analyzed.

Finally, Chapters 15 to 17 are devoted to non-standard low-$T_c$ superconductors such as the heavy fermion systems, superconducting superlattice and magnetic superconductors. The last Chapter introduces the BSCCO, as a material representative of the high-$T_c$ superconductors. During the course, one lecture was given also on YBCO, but it was impossible to include the related text in this book.

In choosing the contributions to the school, it was unavoidable to exclude some topics which would deserve as much attention as the ones included. The present book is not an exception: indeed, nothing is said about NMR or optical properties, just to mention two very important fields of activity. However, we hope that the material presented here will provide the reader with a satisfactory basic knowledge, in such a way that he will be able to move confidently to other available publications about any specific field of interest.

*S. Pace*
*M. Acquarone*

*Salerno, Parma, 1991*

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# REMARKS ON PHENOMENOLOGY AND PHENOMENOLOGICAL THEORETICAL ANALYSIS

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## 1. Introduction

In this work we have restricted our analysis to low-temperature metallic superconductors even though, for the most of key points regarding the superconducting state, high-temperature ceramic superconductors appear to be qualitatively identical to the usual ancient superconductors. On this subject a large number of phenomenological reviews exist in the literature [1,2,3,4], so we believe that it is better to summarize the main properties which have been the basis for the understanding of the superconducting state from a quantum mechanical point of view.

Their well known main features are:

1) the resistive transition,

2) the perfect diamagnetism,

3) the existence of critical magnetic fields and critical currents,

4) the magnetic flux quantization,

5) the Josephson effect,

6) the existence of interference phenomena in superconducting loops linked

by weak links,

7) the isotopic effect,

8) the behaviour of the specific heat,

9) the behaviour of tunnel devices.

Before a brief description of each point, three remarks are necessary:

a) very often the actual experimental behaviour is quite different from the expected one, which is actually the result of an idealization process;

b) in spite of the idealization process, only points 1-6 are verified in each low temperature superconducting material, so that points 7-9 do not appear necessary for the traditional superconducting state, neither does their absence necessarily imply a new superconducting state;

c) if points 1-6 led to a particular microscopical state, it seems to us highly unlikely that the new ceramic materials, which substantially show the same qualitative behaviour, can correspond to a very different microscopical state.

Coming back to a short description of each point, the existence of a resistive transition means the existence of a well defined temperature, named "critical temperature" ($T_c$), below which the direct current (DC) metallic resistance becomes zero. In DC conditions this fact implies the complete absence of power dissipation. On the contrary, frequency-dependent dissipative effects are present in AC conditions.

Anyway, more than perfect conductivity, the existence of perfect diamagnetic behaviour is the main characteristics of the superconducting state; it appears in two different ways:

a) as an almost perfect shielding effect, so that, after a cooling down below the transition temperature in the absence of an external magnetic field H (Z.F.C. conditions), the application of a "low" field leaves the magnetic induction $B = 0$ inside the material,

b) as the Meissner effect, that is the expulsion of low magnetic fields obtained by cooling down the material below the transition temperature. As a matter of fact, 22 years (from 1911 to 1933) were necessary for the discovery of such an effect since it actually only appears in almost perfect materials. In the presence of impurities the magnetic flux is trapped inside the material and only partial expulsion occurs.

The existence of critical fields means that perfect diamagnetic behaviour persists up to an upper threshold, above which the magnetic field penetrates the sample. For ideal materials two classes of behaviour can be distinguished:

a) in type-one superconductors, above a critical field $H_c$ , the materials revert to the normal state;

b) in type-two superconductors, above a lower critical field $H_{c1}$, (lower than $H_c$), a continuous penetration of the magnetic field starts; as the field H increases, the magnetic induction increases more and more, however the perfect conductivity state persists up to an upper threshold $H_{c2}$ corresponding to the complete penetration of the magnetic field. In ideal samples (soft superconductors) this behaviour is completely reversible. Actually in the presence of localized impurities (hard superconductors) a more complex irreversible behaviour appears, as described in more details by A. Testa in a different paper in this book. Critical fields are monotonically decreasing functions of the temperature, almost constant for $T < T_c/2$ and going continuously to zero as T approaches $T_c$.

Analogously in direct current measurements too, the absence of dissipative effects persists up to a current threshold, named critical current $I_c$ above which power dissipation appears; as the current still increases the details of the reestablishement of the normal state depend on the superconductor type.

The presence of the Meissner effect in ideal samples guarantees the uniqueness of the state for a fixed external field; however this result is valid only for simply connected samples. Indeed, the magnetic state of a superconducting ring is a function of its history: when the ring is cooled down in an external magnetic field, the magnetic induction becomes zero only in the body of the ring and it is different from zero inside the vacuum core of the ring. On the contrary, if the same field is applied after Z.F.C. , the shielding effect leads to zero magnetic induction both in the body and in the core. Moreover a subsequent removal of the field leads in the first case (F.C. condition) to magnetic flux trapping inside the hole, while in the second one (Z.F.C. condition) the field removal leads to the complete absence of the field. The flux trapping effect is strictly related to the perfect conductivity since by Maxwell equations $d\Phi(\mathbf{B})/dt = \mathbf{E}$ and by Ohm law $\mathbf{E} = \rho\mathbf{j}$, so that in the superconducting state the perfect conductivity implies that the flux linked to a ring must be a constant. Nevertheless it generated a serious problem since in classical thermodynamics the state of any thermodynamic system must be completely determined by the values of thermodynamic variables. Luckily quantum mechanics removes the problem. Indeed not only the flux is constant but it is quantized:

$$\Phi(B) = n\Phi_o \tag{1}$$

where $n$ is an integer and $\Phi_o$ is the flux quantum: $\Phi_o = hc/2e$. In this way an infinite number of metastable states exists and the ground state corresponds to the thermodynamic equilibrium.

The presence of a flux quantization phenomenon is the first evidence of the existence of a macroscopical quantum state of particles with charge $2e$.

As shown in more details in two other papers in this book by G. Costabile and C. Cosmelli, the quantum macroscopical behaviour is also the basis of the Josephson effect and of quantum interference phenomena which clearly appear in superconducting rings closed by Josephson junctions. Indeed, in the first case the superconducting current flows through a junction weakly coupling two superconducting electrodes in the absence of applied voltage. This current is a sinusoidal function of a macroscopical phase which characterizes the two electrodes. In the second case the maximum superconducting current, which can cross through two junctions in parallel in a superconducting ring, shows a sinusoidal dependence on the flux inside the ring. Such an effect is particularly surprising from a classical point of view, since it is also present in the complete absence of any field inside the superconducting material.

Coming back to properties 6), 7), and 8), such features are not always present even in classical superconductors, so that their presence or absence cannot play any basic role for the definition of the microscopical nature of the superconducting state.

Anyway the isotopic effect, that is the dependence of the critical temperature on the isotopic mass M of the superconducting element ($T_c \propto M^{-1/2}$), opened the way to the understanding of role of the electron-phonon interaction in low $T_c$ superconductors. However metals or metallic alloys with a higher critical temperature ($T_c \cong 10K$) show a lower dependence or even do not show any appreciable dependence on the isotopic mass. Indeed, as reported in more details by A. Saggese and S. Pace in another paper in this book, the Bardeen-Cooper-Schrieffer (BCS) microscopical theory shows that the electron-phonon interaction gives rise to the $M^{-1/2}$ dependence only in the "weak coupling" limit.

As far as the specific heat is concerned, at the transition point, in the absence of an external magnetic field, a jump of the specific heat appears with the transition from a linear metallic behaviour to an exponential one. These features indicate respectively the presence of a second-order phase transition, and an energy band completely empty of states, a gap, at the Fermi surface. However, in this case too, the presence of impurities is sufficient to drastically change the height of the jump. Moreover the exponential behaviour below $T_c$ is not present in some superconducting material like in the presence of particular magnetic impurities concentrations.

In an analogous way in most of the cases the current-voltage characteristics of tunnel junctions show the presence of an energy gap at the Fermi surface. As shown in more details by S. Pace and A. Saggese in another paper in this book, tunnel measurements show the presence of a gap not only in the dispersion relation but also in the density of states (DOS) (the difference between the two cases is analysed by the contribution of M. Acquarone). However, as stated before, in particular conditions, states can be present in the forbidden band, leading in some cases to the complete absence of any gap both in the dispersion relation and in the DOS. This fact appears, for instance, in the presence of well defined magnetic impurities concentrations or in metal-superconducting sandwiches where by the proximity effect superconductivity is induced in the normal metal. In these cases, the survival of the perfect conductivity, of the perfect diamagnetism and of macroscopical quantum phenomena demonstrate that the presence of a gap is not really necessary for the superconducting state.

Coming back to our main purpose, the implication of the described experimental features can be analysed.

Considering the first two fundamental properties, i.e. perfect conductivity and diamagnetism, at the beginning it was not clear that they are related to the same phenomenon. As in any other material, the magnetic behaviour was related to microscopical atomic currents. On the contrary by the Maxwell equation $curl\,\mathbf{h} = \mathbf{j}$,* it is clear that both the shielding and the trapping effect of a ring can arise only due to the presence of persistent macroscopic currents in the sample. In this way perfect conductivity and perfect diamagnetism are easily explained by a superfluid model: the current is driven by a superfluid electron gas.

Obviously the presence of a finite current density can generate only a gradient of the magnetic field, in such a way that the field can penetrate near the surface on the superconducting sample at a depth of the order of a characteristic length $\lambda$ depending on the material.

In any case the presence of dissipative effects in AC conditions proves the contemporary presence of "normal" electrons. For these reasons, in close analogy to the superfluid $He^4$ theory, the two-fluid model has been developed, starting with the equations:

$$n = n_s + n_n \tag{2a}$$

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* Some care has to be taken about the symbols used. Usually **h** is the microscopical magnetic field: $\mathbf{h} = curl\,\mathbf{A}$, where **A** is the vector potential, **H** is the field generated by the macroscopical external currents, while **B** is the mean value of **h**. In our case a difficulty arises, since in a superconductor the diamagnetic currents, which usually are microscopical, are almost macroscopical, with spatial variations in ranges of the order of hundreds of angstroms, and the real microscopical currents do not play any role. For these reasons we prefer to indicate by **h** the magnetic field generated by both external and "macroscopical" internal currents, while **H** is generated only by external currents. All the equations will be written in Gaussian units.