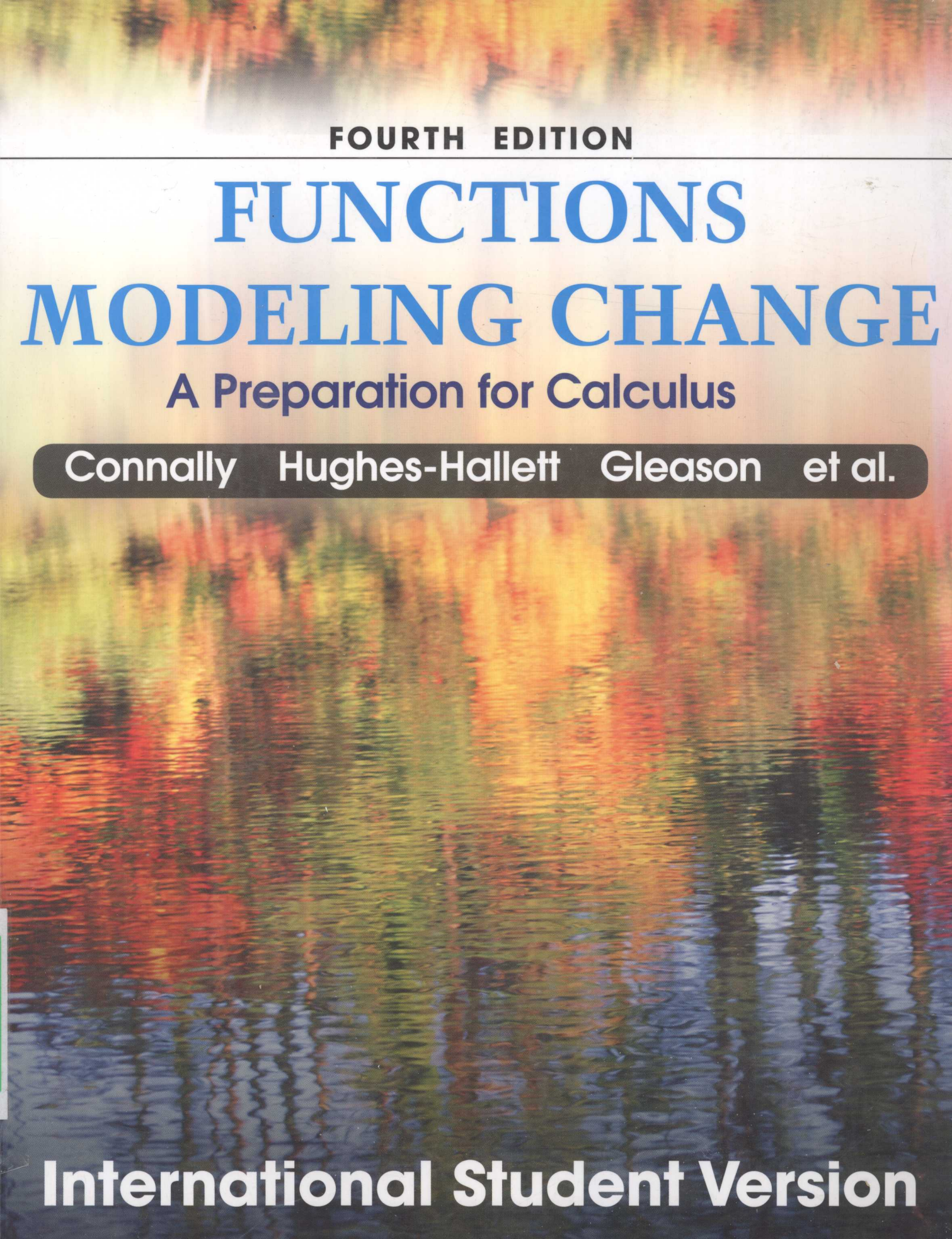
FOURTH EDITION

FUNCTIONS MODELING CHANGE

A Preparation for Calculus

Connally Hughes-Hallett Gleason et al.



International Student Version

FUNCTIONS MODELING CHANGE:
A Preparation for Calculus

Fourth Edition: International Student Version

Dedicated to Ben, Jonah, and Isabel

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Fourth Edition: International Student Version

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PREFACE

Mathematics has the extraordinary power to reduce complicated problems to simple rules and procedures. Therein lies the danger in teaching mathematics: it is possible to teach the subject as nothing but the rules and procedures—thereby losing sight of both the mathematics and its practical value. The fourth edition of *Functions Modeling Change: A Preparation for Calculus* continues our effort to refocus the teaching of mathematics on concepts as well as procedures.

Fourth Edition: Focus

The focus of this edition remains the same as in previous editions: a balance of understanding and skills. We stress conceptual understanding and multiple ways of representing mathematical ideas. Our goal is to provide students with a clear understanding of functions as a solid foundation for subsequent courses in mathematics and other disciplines. When we designed this curriculum under an NSF grant, we started with a clean slate. We focused on the key concepts, emphasizing depth of understanding.

Skills are developed in the context of problems and reinforced in a variety of settings, thereby encouraging retention. This balance of skills and understanding enables students to realize the power of mathematics in modeling.

Fourth Edition: Flexibility

Precalculus courses are taken by a wide range of students and are taught in a wide variety of styles. As instructors ourselves, we know that the balance we choose depends on the students we have: sometimes a focus on conceptual understanding is best; sometimes more skill-building is needed.

To enable instructors to select the balance appropriate for their students, we have increased the options available in the fourth edition. Instructors will find that by selecting sections and exercises that reflect their goals, they can tailor the materials to their students and their institution.

Origin of Text: The Calculus Consortium for Higher Education

This book is the work of faculty at a diverse consortium of institutions, and was originally generously supported by the National Science Foundation. It represents the first consensus among such a diverse group of faculty to have shaped a mainstream precalculus text. Bringing together the results of research and experience with the views of many users, this text is designed to be used in a wide range of institutions.

Guiding Principles: Varied Problems and the Rule of Four

Since students usually learn most when they are active, the exercises in a text are of central importance. In addition, we have found that multiple representations encourage students to reflect on the meaning of the material. Consequently, we have been guided by the following principles.

- Problems should be varied and some should be challenging. Many of our problems cannot be done by following a template in the text.
- The Rule of Four: each concept and function should be represented symbolically, numerically, graphically, and verbally. This principle, originally introduced by the consortium, promotes multiple representations.
- Students and instructors should have a quick way to pinpoint misunderstandings before moving on. To this end, we include Check Your Understanding true-false problems at the end of each chapter.

- The components of a precalculus curriculum should be tied together by clearly defined themes. Functions as models of change is our central theme, and algebra is integrated where appropriate.
- Topics should be fewer in number than is customary so that they can be treated in greater depth. The core syllabus of precalculus should include only those topics that are essential to the study of calculus.
- Problems involving real data should be included to prepare students to use mathematics in other fields.
- To use mathematics effectively, students should develop skill in both symbolic manipulation and the use of technology. The exact proportions of each may vary widely, depending on the preparation of the student and the wishes of the instructor.
- Materials for precalculus should allow for a broad range of teaching styles. They should be flexible enough to use in large lecture halls, small classes, or in group or lab settings.

Changes in the Fourth Edition

The fourth edition retains the hallmarks of earlier editions and reflects the many helpful suggestions from users in the following changes.

- **Algebraic Skills Refreshers** are introduced in two formats, both integrated with the text:
  - **Skills Refresher exercises** are included at the start of many exercise sets, identified by **S1, S2**, etc.
  - **Skills Refresher sections** are included at the end of some chapters.
- A new **Chapter 3** on **Quadratic Functions** brings together and expands the material on quadratics in the former Sections 2.6 and 5.5.
- **Section 4.4** on **Compound Interest** and **Section 4.5** on **the Number** e have been reorganized to give instructors more flexibility in deciding how to introduce e.
- **Limit Notation** has been moved to the end of **Section 5.3** to provide additional flexibility in how, or whether, limit notation is introduced.
- A new **Section 6.5** on **Combining Transformations** investigates the effect of varying the order in which transformations are applied to functions and their graphs.
- The material on **trigonometry** in the former Chapters 6 and 7 has been expanded and reorganized into three chapters, allowing instructors to tailor their approach to the needs of their students.
  - **Chapter 7 defines the trigonometric functions** in circles and triangles.
  - **Chapter 8 studies the behavior of the trigonometric functions** and their graphs, and introduces trigonometric identities, polar coordinates, and complex numbers.
  - **Chapter 9 investigates trigonometric identities** and their applications to modeling.
- **Data and problems** have been updated and revised as appropriate. Many new problems have been added.
- **ConcepTests** for precalculus are available for instructors looking for innovative ways to promote active learning in the classroom. Further information is provided under Supplementary Materials on page viii.

What Student Background is Expected?

Students using this book should have successfully completed a course in intermediate algebra or high school algebra II. The book is thought-provoking for well-prepared students while still accessible to students with weaker backgrounds. Providing numerical and graphical approaches as well as algebraic gives students various ways to master the material. This encourages students to persist, thereby lowering failure rates.

Our Experiences

Previous editions of this book were used by hundreds of schools around the country. In this diverse group of schools, the first three editions were successfully used with many different types of students in semester and quarter systems, in large lectures and small classes, as well as in full-year courses in secondary schools. They were used in computer labs, small groups, and traditional settings, and with a number of different technologies.

Content

The central theme of this course is functions as models of change. We emphasize that functions can be grouped into families and that functions can be used as models for real-world behavior. Because linear, quadratic, exponential, power, and periodic functions are more frequently used to model physical phenomena, they are introduced before polynomial and rational functions. Once introduced, a family of functions is compared and contrasted with other families of functions.

A large number of the examples and problems that students see in this precalculus course are given in the context of real-world problems. Indeed, we hope that students will be able to create mathematical models that help them understand the world in which they live. The inclusion of non-routine problems is intended to establish the idea that such problems are not only part of mathematics, but in some sense are the point of mathematics.

The book does not require any specific software or technology. Instructors have used the material with graphing calculators and graphing software. Any technology with the ability to graph functions will suffice.

Chapter 1: Linear Functions and Change

This chapter introduces the concept of a function and graphical, tabular, symbolic, and verbal representations of functions, discussing the advantages and disadvantages of each representation. It introduces rates of change and uses them to characterize linear functions. A section on fitting a linear function to data is included.

The **Skills Refresher** section for Chapter 1 reviews linear equations and the coordinate plane.

Chapter 2: Functions

This chapter studies function notation in more detail, after its introduction in Chapter 1. It introduces domain, range, and the concepts of composite and inverse functions and investigates the idea of concavity using rates of change. A section on piecewise functions is included.

Chapter 3: Quadratic Functions

This chapter introduces the standard, factored, and vertex forms of a quadratic function and explores their relationship to graphs. The family of quadratic functions provides an opportunity to see the effect of parameters on functional behavior.

The **Skills Refresher** section for Chapter 3 reviews factoring, completing the square, and quadratic equations.

Chapter 4: Exponential Functions

This chapter introduces the family of exponential functions and the number e. It compares exponential and linear functions, solves exponential equations graphically, and gives applications to compound interest.

The **Skills Refresher** section for Chapter 4 reviews the properties of exponents.

Chapter 5: Logarithmic Functions

This chapter introduces logarithmic functions with base 10 and base e, both in order to solve exponential equations and as inverses of exponential functions. After discussing manipulations with logarithms, the chapter focuses on modeling with exponential functions and logarithms. Logarithmic scales and a section on linearizing data conclude the chapter.

The **Skills Refresher** section for Chapter 5 reviews the properties of logarithms.

Chapter 6: Transformations of Functions and Their Graphs

This chapter investigates transformations—shifting, reflecting, and stretching. The last section investigates the effect of changing the order of transformations and suggests a standard way of writing transformations.

Chapter 7: Trigonometry in Circles and Triangles

This chapter, which opens with modeling periodic phenomena, introduces the trigonometric functions of an angle measured in degrees: sine, cosine, tangent, and, briefly, secant, cosecant, and cotangent. Definitions of these functions use the unit circle and are related to definitions using right triangles. The graphs of sine, cosine, and tangent are introduced. The treatment of triangles includes both right and non-right triangles and the inverse trigonometric functions for angles in a right triangle.

The **Skills Refresher** section for Chapter 7 reviews the special angles in 30°-60°-90°and 45°-45°-90°triangles.

Chapter 8: The Trigonometric Functions

This chapter opens with the definition of radians and then studies sinusoidal behavior. It introduces basic identities and revisits inverse trigonometric functions. It also introduces polar coordinates and complex numbers, including Euler's and de Moivre's formulas.

Chapter 9: Trigonometric Identities and Their Applications

The first two sections of this chapter provide a thorough treatment of identities, including double-angle identities and identities involving the sum and difference of angles. The third section shows the uses of trigonometry in mathematical modeling.

Chapter 10: Compositions, Inverses, and Combinations of Functions

This chapter discusses combinations of functions. It investigates composite and inverse functions, which were introduced in Chapter 2, in more detail.

Chapter 11: Polynomial and Rational Functions

This chapter discusses power functions, polynomials, and rational functions. The chapter concludes by comparing several families of functions, including polynomial and exponential functions, and by fitting functions to data.

The **Skills Refresher** section for Chapter 11 reviews algebraic fractions.

Chapter 12: Vectors

This chapter contains material on vectors and operations involving vectors. An introduction to matrices is included in the last section.

Chapter 13: Sequences and Series

This chapter introduces arithmetic and geometric sequences and series and their applications.

Chapter 14: Parametric Equations and Conic Sections

The concluding chapter looks at parametric equations, implicit functions, hyperbolic functions, and conic sections: circles, ellipses, and hyperbolas. The chapter includes a section on the geometrical properties of the conic sections and their applications to orbits.

Supplementary Materials

The following supplementary materials are available for the Fourth Edition:

- **The Instructor's Manual** contains teaching tips, lesson plans, syllabi, and worksheets. It has been expanded and revised to include worksheets, identification of technology-oriented problems, and new syllabi.

- **The Instructor's Solution Manual** has complete solutions to all problems.
- **The Computerized Test Bank**, available in both PC and Macintosh formats, allows instructors to create, customize, and print a test containing any combinations of questions from a large bank of questions. Instructors can also customize the questions or create their own.
- **Classroom Activities** are posted at the book companion website. These activities were developed to facilitate in-class group work as well as to introduce new concepts and to practice skills. In addition to the blank copies for each activity that can be handed out to the students, a copy of the activity with fully worked out solutions is also available.
- **The Book Companion Site** contains all instructor supplements.
- **WileyPLUS** is a powerful and highly integrated suite of online teaching and learning resources providing course management options to instructors and students. Instructors can automate the process of assigning, delivering, and grading algorithmically generated homework exercises, and giving hints and solutions, while providing students with immediate feedback. In addition, WileyPLUS provides student tutorials, an instructor gradebook, integrated links to the electronic version of the text, and all of the text supplemental materials. For more information, visit www.wiley.com/college/wileyplus or contact your local Wiley representative for more details.
- **The Faculty Resource Network** is a peer network of academic faculty dedicated to the effective use of technology in the classroom. This group can help you apply innovative classroom techniques, implement specific software packages, and tailor the technology experience to the specific needs of each individual class. Ask your Wiley representative for more details.

ConcepTests

ConcepTests, modeled on the pioneering work of Harvard physicist Eric Mazur, are questions designed to promote active learning during class, particularly (but not exclusively) in large lectures. Our evaluation data show students taught with ConcepTests outperformed students taught by traditional lecture methods 73% versus 17% on conceptual questions, and 63% versus 54% on computational problems. ConcepTests arranged by section are available in PowerPoint and Classroom Response System-ready formats from your Wiley representative.

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To Students: How to Learn from this Book

- This book may be different from other math textbooks that you have used, so it may be helpful to know about some of the differences in advance. At every stage, this book emphasizes the *meaning* (in practical, graphical or numerical terms) of the symbols you are using. There is much less emphasis on "plug-and-chug" and using formulas, and much more emphasis on the interpretation of these formulas than you may expect. You will often be asked to explain your ideas in words or to explain an answer using graphs.
- The book contains the main ideas of precalculus in plain English. Success in using this book will depend on reading, questioning, and thinking hard about the ideas presented. It will be helpful to read the text in detail, not just the worked examples.
- There are few examples in the text that are exactly like the homework problems, so homework problems can't be done by searching for similar–looking "worked out" examples. Success with the homework will come by grappling with the ideas of precalculus.
- Many of the problems in the book are open-ended. This means that there is more than one correct approach and more than one correct solution. Sometimes, solving a problem relies on common-sense ideas that are not stated in the problem explicitly but which you know from everyday life.
- This book assumes that you have access to a calculator or computer that can graph functions and find (approximate) roots of equations. There are many situations where you may not be able to find an exact solution to a problem, but can use a calculator or computer to get a reasonable approximation. An answer obtained this way can be as useful as an exact one. However, the problem does not always state that a calculator is required, so use your own judgment.
- This book attempts to give equal weight to four methods for describing functions: graphical (a picture), numerical (a table of values), algebraic (a formula) and verbal (words). Sometimes it's easier to translate a problem given in one form into another. For example, you might replace the graph of a parabola with its equation, or plot a table of values to see its behavior. It is important to be flexible about your approach: if one way of looking at a problem doesn't work, try another.
- Students using this book have found discussing these problems in small groups helpful. There are a great many problems that are not cut-and-dried; it can help to attack them with the other perspectives your colleagues can provide. If group work is not feasible, see if your instructor can organize a discussion session in which additional problems can be worked on.
- You are probably wondering what you'll get from the book. The answer is, if you put in a solid effort, you will get a real understanding of functions as well as a real sense of how mathematics is used in the age of technology.

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Chapter One

LINEAR FUNCTIONS AND CHANGE

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