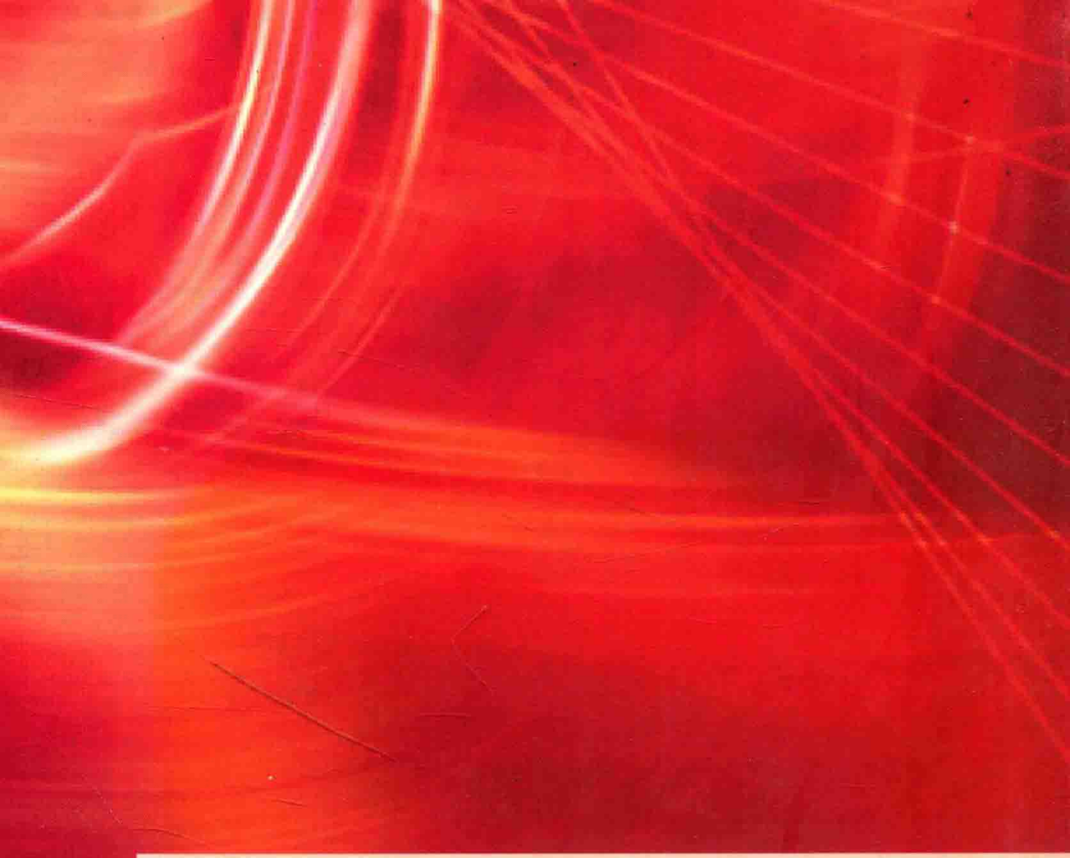

Ting-Chung Poon • Taegeun Kim

ENGINEERING OPTICS WITH MATLAB®

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Preface

This book serves two purposes: The first is to introduce the readers to some traditional topics such as the matrix formalism of geometrical optics, wave propagation and diffraction, and some fundamental background on Fourier optics. The second is to introduce the essentials of acousto-optics and electro-optics, and to provide the students with experience in modeling the theory and applications using MATLAB®, a commonly used software tool. This book is based on the authors' own in-class lectures as well as research in the area.

The key features of the book are as follows. Treatment of each topic begins from the first principles. For example, geometrical optics starts from Fermat's principle, while acousto-optics and electro-optics start from Maxwell equations. MATLAB examples are presented throughout the book, including programs for such important topics as diffraction of Gaussian beams, split-step beam propagation method for beam propagation in inhomogeneous as well as Kerr media, and numerical calculation of up to 10-coupled differential equations in acousto-optics. Finally, we cover acousto-optics with emphasis on modern applications such as spatial filtering and heterodyning.

The book can be used for a general text book for Optics/Optical Engineering classes as well as acousto-optics and electro-optics classes for advanced students. It is our hope that this book will stimulate the readers' general interest in optics as well as provide them with an essential background in acousto-optics and electro-optics. The book is geared towards a senior/first-year graduate level audience in engineering and physics. This is suitable for a two-semester course. The book may also be useful for scientists and engineers who wish to learn about the basics of beam propagation in inhomogeneous media, acousto-optics and electro-optics.

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Contents

Chapter 1

Geometrical Optics

When we consider optics, the first thing that comes to our minds is probably light. Light has a dual nature: light is particles (called photons) and light is waves. When a particle moves, it processes momentum, p. And when a wave propagates, it oscillates with a wavelength, λ. Indeed, the momentum and the wavelength is given by the *de Broglie relation*

$$\lambda = \frac{h}{p} \ ,$$

where $h \approx 6.62 \times 10^{-34}$ Joule-second is Planck's constant. Hence from the relation, we can state that every particle is a wave as well.

Each particle or photon is specified precisely by the frequency ν and has an energy E given by

$$E = h\nu.$$

If the particle is traveling in free space or in vacuum, $\nu = c/\lambda$, where c is a constant approximately given by $3 \times 10^8 \ m/s$. The speed of light in a transparent linear, homogeneous and isotropic material, which we term v, is again a constant but less than c. This constant is a physical characteristic or signature of the material. The ratio c/v is called the *refractive index*, n, of the material.

In *geometrical optics*, we treat light as particles and the trajectory of these particles follows along paths that we call *rays*. We can describe an optical system consisting of elements such as mirrors and lenses by tracing the rays through the system.

Geometrical optics is a special case of *wave* or *physical* optics, which will be mainly our focus through the rest of this Chapter. Indeed, by taking the limit in which the wavelength of light approaches zero in wave optics, we recover geometrical optics. In this limit, diffraction and the wave nature of light is absent.

1.1 Fermat's Principle

Geometrical optics starts from *Fermat's Principle.* In fact, Fermat's Principle is a concise statement that contains all the physical laws, such as the *law of reflection* and *the law of refraction*, in geometrical optics. Fermat's principle states that the path of a light ray follows is an extremum in comparison with the nearby paths. The extremum may be a minimum, a maximin, or stationary with respect to variations in the ray path. However, it is usually a minimum.

We now give a mathematical description of Fermat's principle. Let $n(x, y, z)$ represent a position-dependent refractive index along a path C between end points A and B, as shown in Fig. 1.1. We define the *optical path length* (OPL) as

$$OPL = \int_C n(x, y, z)\, ds, \tag{1.1-1}$$

where ds represents an infinitesimal arc length. According to Fermat's principle, out the many paths that connect the two end points A and B, the light ray would follow that path for which the OPL between the two points is an extremum, i.e.,

$$\delta(OPL) = \delta \int_C n(x, y, z)\, ds = 0 \tag{1.1-2}$$

in which δ represents a small variation. In other words, a ray of light will travel along a medium in such a way that the total OPL assumes an extremum. As an extremum means that the rate of change is zero, Eq. (1.1-2) explicitly means that

$$\frac{\partial}{\partial x}\int n\, ds + \frac{\partial}{\partial y}\int n\, ds + \frac{\partial}{\partial z}\int n\, ds = 0. \tag{1.1-3}$$

Now since the ray propagates with the velocity $v = c/n$ along the path,

$$nds = \frac{c}{v}ds = cdt, \tag{1.1-4}$$

where dt is the differential time needed to travel the distance ds along the path. We substitute Eq. (1.1-4) into Eq. (1.1-2) to get

$$\delta \int_C n\,ds = c\,\delta \int_C dt = 0. \tag{1.1-5}$$

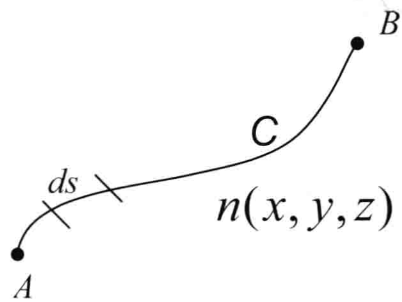

Fig. 1.1 A ray of light traversing a path C between end points A and B.

As mentioned before, the extremum is usually a minimum, we can, therefore, restate Fermat's principle as a *principle of least time*. In a *homogeneous medium*, i.e., in a medium with a constant refractive index, the ray path is a straight line as the shortest OPL between the two end points is along a straight line which assumes the shortest time for the ray to travel.

1.2 Reflection and Refraction

When a ray of light is incident on the interface separating two different optical media characterized by n_1 and n_2, as shown in Fig. 1.2, it is well known that part of the light is reflected back into the first medium, while the rest of the light is refracted as it enters the second medium. The directions taken by these rays are described by the laws of reflection and refraction, which can be derived from Fermat's principle.

In what follows, we demonstrate the use of the principle of least time to derive the law of refraction. Consider a reflecting surface as shown in Fig. 1.3. Light from point A is reflected from the reflecting surface to point B, forming the angle of incidence ϕ_i and the angle of reflection ϕ_r, measured from the normal to the surface. The time required for the ray of light to travel the path AO + OB is given by $t = (\text{AO} + \text{OB})/v$, where v is the velocity of light in the medium containing the points AOB. The medium is considered isotropic and homogeneous. From the geometry, we find

$$t(z) = \frac{1}{v}\left([h_1^2 + (d-z)^2]^{1/2} + [h_2^2 + z^2]^{1/2}\right). \quad (1.2\text{-}1)$$

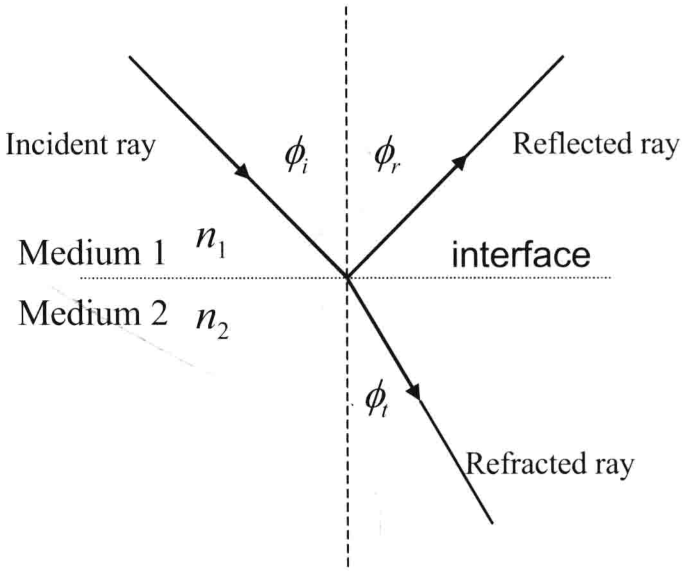

Fig. 1.2 Reflected and refracted rays for light incident at the interface of two media.

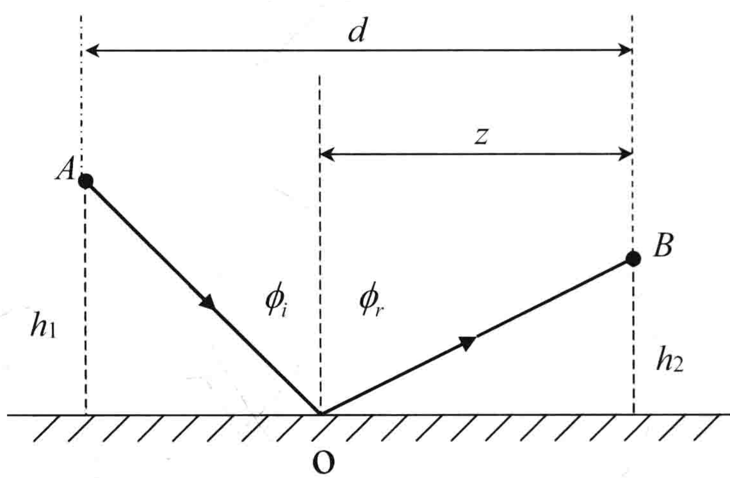

Fig. 1.3 Incident (AO) and reflected (OB) rays.

According to the least time principle, light will find a path that extremizes $t(z)$ with respect to variations in z. We thus set $dt(z)/dz = 0$ to get

$$\frac{d-z}{[h_1^2+(d-z)^2]^{1/2}} = \frac{z}{[h_2^2+z^2]^{1/2}} \tag{1.2-2}$$

or

$$\sin\phi_i = \sin\phi_r \tag{1.2-3}$$

so that

$$\phi_i = \phi_r, \tag{1.2-4}$$

which is the law of reflection. We can readily check that the second derivative of $t(z)$ is positive so that the result obtained corresponds to the least time principle. In addition, Fermat's principle also demands that the incident ray, the reflected ray and the normal all be in the same plane, called the *plane of incidence*.

Similarly, we can use the least time principle to derived the law of refraction

$$n_1\sin\phi_i = n_2\sin\phi_t \,, \tag{1.2-5}$$

which is commonly known as *Snell's law of refraction*. In Eq. (1.2-5), ϕ_i is the angle of incidence for the incident ray and ϕ_t is the angle of transmission (or angle of refraction) for the refracted ray. Both angles are measured from the normal to the surface. Again, as in reflection, the incident ray, the refracted ray, and the normal all lie in the same plane of incidence. Snell's law shows that when a light ray passes obliquely from a medium of smaller refractive index n_1 into one that has a larger refractive index n_2, or an optically denser medium, it is bent toward the normal. Conversely, if the ray of light travels into a medium with a lower refractive index, it is bent away from the normal. For the latter case, it is possible to visualize a situation where the refracted ray is bent away from the normal by exactly 90^o. Under this situation, the angle of incidence is called the *critical angle* ϕ_c, given by

$$\sin\phi_c = n_2/n_1. \tag{1.2-6}$$

When the incident angle is greater than the critical angle, the ray originating in medium 1 is totally reflected back into medium 1. This phenomenon is called *total internal reflection*. The optical fiber uses this principle of total reflection to guide light, and the mirage on a hot summer day is a phenomenon due to the same principle.

1.3 Ray Propagation in an Inhomogeneous Medium: Ray Equation

In the last Section, we have discussed refraction between two media with different refractive indices, possessing a discrete inhomogeniety in the simplest case. For a general inhomogeneous medium, i.e., $n(x, y, z)$, it is instructive to have an equation that can describe the trajectory of a ray. Such an equation is known as the *ray equation*. The ray equation is analogous to the equations of motion for particles and for rigid bodies in classical mechanics. The equations of motion can be derived from *Newtonian mechanics* based on Netwon's laws. Alternatively, the equations of motion can be derived directly from *Hamilton's principle of least action*. Indeed Fermat's principle in optics and Hamilton's principle of least action in classical mechanics are analogous. In what follows, we describe Hamilton's principle so as to formulate the so called *Lagrange's equations* in mechanics. We then re-formulate Lagrange's equations for optics to derive the ray equation.

Hamilton's principle states that the trajectory of a particle between times t_1 and t_2 is such that the variation of the line integral for fixed t_1 and t_2 is zero, i.e.,

$$\delta \int_{t_1}^{t_2} L(q_k, \dot{q_k}, t)\, dt = 0, \qquad (1.3\text{-}1)$$

where $L = T - V$ is known as the *Lagrangian function* with T being the kinetic energy and V the potential energy of the particle. The q_k's are called *generalized coordinates* with $k = 1, 2, 3, ...n$. Also, $\dot{q_k} = dq_k/dt$.

Generalized coordinates are any collection of independent coordinates q_k (not connected by any equations of constraint) that are sufficient to specify uniquely the motion. The number n of generalized coordinates is the number of *degrees of freedom*. For example, a simple pendulum has one degree of freedom, i.e., $q_k = q_1 = \phi$, where ϕ is the angle the pendulum makes with the vertical. Now if the simple pendulum is complicated such that the string holding the bob is elastic. There will be two generalized coordinates, $q_k = q_1 = \phi$, and $q_k = q_2 = x$, where x is the length of the string. As another example, let us consider a particle constrained to move along the surface of a sphere with radius R. The coordinates (x, y, z) do not constitute an independent set as they are connected by the equation of constraint $x^2 + y^2 + z^2 = R^2$. The particle has only two degrees of freedom and two independent coordinates are needed to specify its position on the sphere uniquely. These coordinates could be taken as latitude and longitude or we could choose angles θ and ϕ from spherical coordinates as our generalized coordinates.

Now, if the force field $\boldsymbol{F}$ is conservative, i.e.,$\nabla \times \boldsymbol{F} = 0$, the total energy $E = T + V$ is a constant during the motion, and Hamilton's principle leads to the following equations of motion of the particle called *Lagrange's equations*:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \frac{\partial L}{\partial q_k}. \tag{1.3-2}$$

As a simple example illustrating the use of Lagrange's equations, let us consider a particle with mass m having kinetic energy $T = \frac{1}{2}m|\dot{\boldsymbol{r}}|^2$ under potential energy $V(x, y, z)$, where

$$\boldsymbol{r}(x, y, x) = x(t)\boldsymbol{a_x} + y(t)\boldsymbol{a_y} + z(t)\boldsymbol{a_z}$$

is the position vector with $\boldsymbol{a_x}$, $\boldsymbol{a_y}$, and $\boldsymbol{a_z}$ being the unit vector along the x, y, and z direction, respectively. According to Newton's second law,

$$\boldsymbol{F} = m\ddot{\boldsymbol{r}}, \tag{1.3-3}$$

where $\ddot{\boldsymbol{r}}$ is the second derivative of $\boldsymbol{r}$ with respect to t. As usual the force is given by the negative gradient of the potential, i.e., $\boldsymbol{F} = -\nabla V$. Hence, we have the vector equation of motion for the particle

$$m\ddot{\boldsymbol{r}} = -\nabla V \tag{1.3-4}$$

according to Newtonian mechanics. Now from the Lagrange's equations, we identify

$$L = T - V = \frac{1}{2}m|\dot{\boldsymbol{r}}|^2 - V.$$

Considering $q_1 = x$, we have

$$\frac{d}{dt}(\frac{\partial L}{\partial \dot{x}}) = m\ddot{x} \quad \text{and} \quad \frac{\partial L}{\partial x} = -\frac{\partial V}{\partial x}. \tag{1.3-5}$$

Now, from Eq. (1.3-2) and using the above results, we have

$$m\ddot{x} = -\frac{\partial V}{\partial x}, \tag{1.3-6}$$

and similarly for the y and z components as $q_2 = y$ and $q_3=z$. Therefore, we come up with Eq. (1.3-4), which is directly from Newtonian mechanics. Hence, we see that Newton's equations can be derived from Lagrange's equations and in fact, the two sets of equations are equally fundamental. However, the Lagrangian formalism has certain advantages over the conventional Newtonian laws in that the physics problem has been transformed into a purely mathematical problem. We just need to find T and V for the system and the rest is just mathematical consideration through the use of Lagrange's equations. In addition, there is no need to consider any vector equations as in Newtonian mechanics as Lagrange's equations are scalar quantities. As it turns out, Lagrange's equations are much better adapted for treating complex systems such as in the areas of quantum mechanics and general relativity.

After having some understanding of Hamilton's principle, and the use of Lagrange's equations to obtain the equations of motion of a particle, we now formulate Lagrange's equations in optics. Again, the particles of concern in optics are photons. Starting from Fermat's principle as given by Eq. (1.1-2),

$$\delta \int_C n(x, y, z)\, ds = 0\,. \tag{1.3-7}$$

We write the arc length ds along the path of the ray as

$$ds^2 = dx^2 + dy^2 + dz^2 \tag{1.3-8}$$

with reference to Fig. 1.4, where for brevity, we have only shown the 2-