John J. Costello / Spenser O. Gowdy / Agnes M. Rash

FINITE MATHEMATICS WITH APPLICATIONS

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Harcourt Brace Jovanovich, Inc.
New York / San Diego / Chicago / San Francisco / Atlanta
London / Sydney / Toronto

Figures drawn by Arthur Ritter, Inc.

ISBN: 0-15-527400-7

Library of Congress Catalog Card Number: 80-85133

Printed in the United States of America

FINITE MATHEMATICS
WITH APPLICATIONS

DEDICATED TO

Helen, Kathy, and Bob

with love

PREFACE

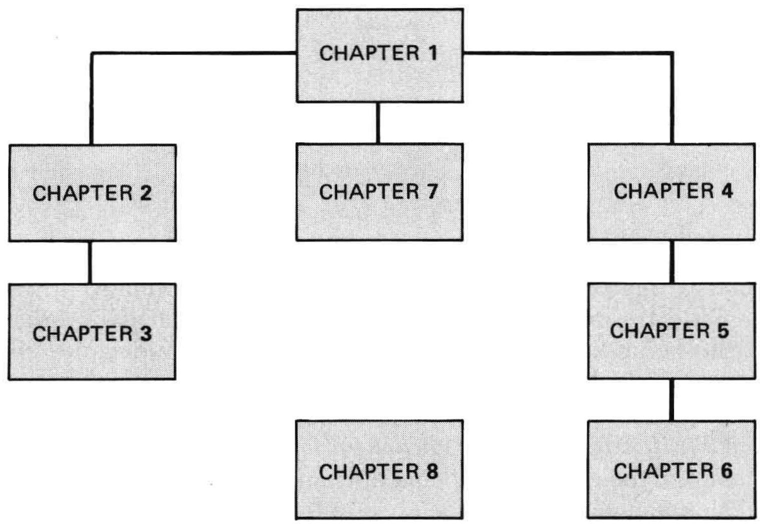
One of the attractive features of the natural sciences (biology, chemistry, physics) is that practitioners can formulate principles mathematically, and from these principles they can make predictions about the behavior of a system. Within the last few decades similar techniques have been applied to the social and managerial sciences with significant results. In view of these developments, we have written *Finite Mathematics with Applications* to acquaint students with some of the quantitative concepts and methods that have proved useful to management and social scientists. Throughout, two important objectives have guided our presentation:

1. To introduce students to mathematical techniques and to mathematical models in business and the social sciences.
2. To present these techniques and models within the framework of a text that is interesting and, above all, readable.

In order to achieve these objectives, we have organized each section around the mathematical modeling of a problem. First, a motivating problem is stated. Next, the mathematical techniques necessary for the solution of this type of problem are presented. The original problem is then solved and, usually, additional examples of the techniques and principles are presented. Finally, the section is summarized, and the student is given numerous exercises and problems to work on. In the text, marginal headings are used to indicate each step in the solution process and to show where key terms are defined in the text. Each chapter concludes with a summary and set of review exercises.

The style of the textbook is conversational and easy to read; we have attempted to make the material as interesting as possible by including many applications from diverse fields of study. Clearly displayed, step-by-step outlines of complicated procedures are another valuable feature of *Finite Mathematics.* For instance, outlines are provided for the Gauss-Jordan method of solving a system of linear equations, the simplex method, and approximating binomial probabilities with normal probabilities.

The book can be used for a one-semester or two-quarter course in finite mathematics, although there is ample material for a full-year course. Chapter dependence is indicated in the diagram below.



Appendix A contains a review of certain topics from algebra, which may be covered completely, or as needed, or not at all, depending on the background of the students in the course.

Chapter 1, "Modeling," introduces the student to the subject of mathematical modeling in Section 1. The remainder of Chapter 1 presents linear and quadratic functions, using the modeling techniques described in Section 1. We recommend that the instructor discuss Chapter 1, even if the students are familiar with the concepts described, because the remainder of the text is *styled* after this chapter.

Chapter 8, "Computers and Flowcharting," contains flowcharts and BASIC programs for some of the techniques and formulas developed in the text. Also, programs in BASIC for the Gauss-Jordan and simplex method are given in Appendix C. Thus the instructor or students can enter the program into a computer and use it to solve problems that would be unmanageable by hand calculations. This chapter covers only a minimal introduction to programming and may be included or omitted at the discretion of the instructor.

Certain sections containing advanced or supplemental topics are marked *Optional* both in the table of contents and in the text headings. Omitting any of these optional sections does not affect the continuity of the basic material.

Some problems in this text may involve lengthy calculations, and thus require the use of a hand-held calculator or computer for solution. The student is alerted to such problems by the presence of a **c** in front of the problem number.

We would like to express our thanks to Dr. William Kuhn, RCA Corporation, for his constructive suggestions on the simplex method; the administrators of Saint Joseph's University for their support and encouragement; the editorial and production staff at Harcourt Brace Jovanovich for their careful work in the development of this book.

We would also like to thank the following reviewers whose constructive criticism was most valuable.

Professor Philip Cheifetz *Nassau Community College*

Professor Murray Eisenberg *University of Massachusetts, Amherst*

Professor John Gresser *Bowling Green University*

Professor Kenneth Heimes *Iowa State University*

Professor Roland Lamberson *Humboldt State University*

Professor Stanley Lukawecki *Clemson University*

Professor Kenneth Rager *Metropolitan State University*

Professor Edward Rozema *University of Tennessee*

Professor Erik Schreiner *Western Michigan University*

Professor Karen Schroeder *Bentley College*

Professor Patrick Weatley *California Polytechnic State University*

Professor David Wend *Montana State University*

Finally, we wish to thank the classes at Saint Joseph's University who used the book in its manuscript form and provided valuable suggestions; Mr. Gerald Perham, who typed our manuscript into the text editor of our computer; and Mrs. Edith Muldowney for typing the first draft of the manuscript.

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CHAPTER 1

MODELING

One of the most distinguishing features of the last half of the twentieth century is the speed with which change occurs. Although many factors contribute to this phenomenon, certainly one of the most important is the development of electronic media, by which ideas and information can be transmitted around the world in seconds. Forty years ago a well-established business could leisurely study the problems presented by a new competitor and be able to react successfully. Today this is not possible. In a very short time indeed even large corporations (such as the Penn Central Railroad or Pantry Pride) can be forced into bankruptcy by the pressures of competition or new technological developments. From the social point of view, poor management decisions by governments, for example, can cause severe hardship to a great many people. Management needs to solve its problems with more emphasis on mathematical methods, and less on trial and error. In the past there was time to experiment. For example, a toy manufacturer in 1920 could make a limited run of a new toy to see if its customers liked it. If they did, the manufacturer produced more of them. However, in 1980 mass production is used, and a trial run may consist of 10,000 copies of the toy. If the consumers do not buy the toy, the experiment has been too expensive. One example of a failure is the Edsel car. Good methods of predicting success are needed in place of trial runs.

Modeling 1.1

Few problems, whether they be in the physical sciences, business, or social sciences, come to us prepackaged in a manageable size or organized form. The study of a problem begins with observation and collection of data. An attempt to explain the observations or to make predictions based on the data is the role of mathematical modeling.

In a mathematical model, symbols represent the properties of a system or problem being studied. For instance, $y = 0.023x$ is a representation of the amount of income tax (y) paid on an income of x dollars in the state of Pennsylvania. Thus, if your income is \$10,000, your state income tax would be $0.023(10{,}000) = \$230$, which is the actual tax. The formula is a representation which allows you to compute the tax in advance of the payment. The symbols are used to make calculations for predicting the amount of income tax to be paid if you anticipate earning a particular amount or to calculate the tax when you are filing your return in April.

A mathematical model is a representation of a problem. In some cases, the model is simply an equation. In other cases, it may be a set of equations or inequalities. In a third system, it may be a set of axioms and theorems. The Euclidean geometry taught in high school is a model of the physical world around you.

model

A ***model*** is an approximation of a phenomenon in the real world which exhibits some of the observed relationships. However, for the sake of simplicity, not all relationships are modeled. The first step in formulating a model is to state the given problem as simply as possible. Next, the purpose of the model should be made clear, since this frequently determines the type of model to be used.

Sometimes a model is used to explain the observed data. For instance, in the construction of appliances, such as ovens and refrigerators, parts are riveted together. The equation $y = 0.06N + 0.04$ is used to obtain the number of minutes needed (y) to insert a number of rivets (N). Once the model is found to fit the information, it can be used in future predictions. Using this model, if 1000 rivets must be inserted, the worker needs $y = 0.06(1000) + 0.04 = 60.04$ minutes or slightly more than one hour to do the job. Other models are used to determine optimal use of space, time, or money. These models are designed to tell the user which of several choices is the best for obtaining the desired results without the need for trial and error.

In this textbook we begin each section with the statement of a problem. If the problem is too complex for immediate solution, it is simplified to abstract the essential features and minimize extraneous information. Once this has been done, we proceed to formulate a mathematical model of the problem.

The next step in the solution of the problem is to develop the mathematical techniques appropriate to the model. For instance, if the model consists of a set of equations to be solved, methods of solving the set of equations are presented. Having developed the techniques, we apply them to the particular problem at hand. The result is a mathematical solution to the problem.

Once we have a solution, we then apply it to the original problem and interpretations are made. The solution is checked in the statement of the problem for adequacy of representation. Occasionally, the solution is suitable for the simplified problem but inadequate for the original problem. In this case, a new model must be constructed. In the process of modeling a problem, frequently we develop techniques which can be applied to the solution of many similar problems. These related problems are discussed, and the original problem is solved. Figure 1.1 depicts the situation.

In this book several mathematical models are developed. Each type of model is designed to solve a particular type of problem. By the end of this course you will be able to solve each of the following problems and many other similar problems with these techniques.