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PROCEEDINGS OF THE THIRD INTERNATIONAL SCHOOL

ADVANCED COURSES OF MATHEMATICAL ANALYSIS III

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And, though the warrior's sun has set,
Its light shall linger round us yet,
Bright, radiant, blest.

(J. Manrique; H. W. Longfellow)

To Antonio Aizpuru

In Memoriam

PREFACE

The III International Course of Mathematical Analysis in Andalusia, held in La Rábida (Huelva), 3–7 September 2007, continued the tradition of previous courses in Cádiz (2002) and Granada (2004). Five years ago, representatives of several Andalusian universities made a concerted effort organizing a course to provide an extensive overview of the research in different areas of Mathematical Analysis. The friendly cooperation of many Andalusian research groups in these areas and the initiative of Antonio Aizpuru and Fernando León made possible the organization of the first course in Cádiz. A new and wider cooperation of the Andalusian research groups in Real Analysis, Complex Variable and Functional Analysis and, mainly, the encouragement and hard work of María Victoria Velasco were the cornerstone to support the second course in Granada. During the Gala Dinner in this course (held in a beautiful house which was used as a summer palace by the latter Arab–Andalusian Kings), a group of professors from the universities of Sevilla and Huelva agreed to organize the third course. With the support of the Spanish National Government, the universities of Seville and Huelva, sponsored by several private and official institutions and hosted by the International University of Andalusia (Sede Iberoamericana de La Rábida) where we could invite some leading researchers in this area to give three seminars and eleven plenary lectures. The course brought more than 70 participants from different countries and provided an ideal forum for learning and exchanging of ideas. The high scientific quality of the lectures and seminars offered in this course made us think about the interest of a book collecting these talks. We asked the speakers for a written version of their lectures. The lecturers kindly facilitated us and we could agree with World Scientific Publishing Co. the publication of these proceedings which can be of high interest to graduate students and researchers in several areas of Mathematical Analysis.

The present book includes the contributions corresponding to the seminars by Marco Abate, Eleonor Harboure and Edward Odell and to the plenary talks by Óscar Blasco, Joaquim Bruna, Bernardo Cascales, Francisco

L. Hernández, Lawrence Narici, Héctor Salas, Bertram Schreiber, Antonio Villar and Wiesław Żelazko. Seminars and talks lectured by these prestigious researchers attracted the interest of a big number of graduate students and researchers who attended this conference. The excellent work of the lecturers and the scientific contributions from them and all participants made possible the success of this course.

The talks of M. Abate concerns the theory of local discrete dynamical systems in complex dimension 1, describing what is known on the topological and dynamical structure of the stable set, and on topological, holomorphic and formal conjugacy classes.

O. Blasco presents DeLeeuw-type transference theorems for bilinear multipliers. They allow us to obtain the boundedness of the periodic and discrete versions of bilinear multipliers (even for their maximal versions) and to get new applications of these results in Ergodic Theory.

J. Bruna studies functions which generate the Lebesgue space by translations. He shows that the discrete translation parameter sets $\Lambda \subset \mathbb{R}$ for which some $\varphi \in L_1(\mathbb{R})$ exists such that the translates $\varphi(x-\lambda)$, $\lambda \in \Lambda$, span $L_1(\mathbb{R})$ are exactly the uniqueness sets for certain quasianalytic classes, and gives explicit constructions of such generators φ.

B. Cascales' lecture shows that several classical results about compactness in functional analysis can all be derived from some suitable inequalities involving distances to spaces of continuous or Baire one functions. In particular, he gives quantitative versions of Grothendieck's characterization of weak compactness in spaces $C(K)$, and Eberlein–Grothendieck and Krein–Smulyan theorems.

E. Harboure poses several situations in analysis where some kind of smooth functions play a fundamental role. In connection with the study of Laplace equation, she analyzes the behavior of the fractional integral operator on L_p-spaces and presents a brief description of Besov spaces and their connection with a problem of non-linear approximation of a function by its wavelet expansion.

The domination problem for positive operators between Banach function spaces consists in given two positive operators $0 \leq R \leq T$ between two Banach lattices E and F and assuming that T belongs to a certain operator class, should R belong to the same class? F. Hernández surveys recent results on the behavior of related operator classes like strictly singular (or *Kato*) operators, strictly co-singular (or *Pelczynski*) operators as well as their local versions.

L. Narici explains to us many facts concerning the Hanh–Banach Theorem and the significant role played by the Austrian mathematician Eduard Helly in the development of this theorem.

The contribution by E. Odell discusses the Banach space structure of $L_p[0,1]$, mostly in the reflexive setting, $1 < p < \infty$. This classical Banach space has been a prime case study for abstraction to a more general study of Banach space structure. E. Odell revises the most relevant properties of this space, concerning complemented and non-complemented subspaces, embeddings, normalized unconditional sequences, distortions of the norm, etc.

H. Salas revises properties of hypercyclic operators and presents several problems, some of them in the new classes of dual hypercyclic operators and frequently hypercyclic operators.

In B. Schreiber's lecture, the Operator Algebra Basic Theory is outlined and some applications are described. Many of these applications lead easily to open problems worthy of investigation, both in the area of the application and in the development of the basic theory.

A. Villar's lecture deals with the mathematical tools that permits to understand the logics of competitive markets. It refers to the solvability of a finite system of equations with non-negativity restrictions. Some changes in the environment are also considered: non-competitive behavior, non-convex feasible sets, non-finite sets of markets, a continuum of agents, etc.

W. Żelazco discusses some recent results and some open problems concerning unital F-algebras (i.e., a topological algebra which is an F-space). The following questions are considered:

(1) When are all maximal ideals closed?
(2) When are all ideals closed?
(3) When does a dense principal ideal exist?

There were many other pluses to the course; the tourist trip across Huelva Ría on a typical boat with dinner on board, the memorable trip to the Riotinto Mine Park and to the Marvels's Cave in Sierra de Aracena Natural Park, not to mention the BBQ and Gala Dinner held in the garden of La Rábida Residence followed by an amazing "Cheen–Cheen-Poom" dancing.

No conference can succeed without a lot of generous support and we would like to express our gratitude to the other organizers, specially to Cándido Piñeiro, Ramón Rodríguez and Enrique Serrano, who were in charge of almost all necessary organization duties, the Sede Iberoameri-

cana of the University International of Andalusia and its Director Luis C. Contreras, the Local Committee, the many official and private sponsors, listed at the end of this preface, and, of course, the participants themselves without whom there could have been no conference.

Sevilla and Huelva, Spring 2008

The editors

J. M. Delgado and T. Domínguez

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CONTENTS

AN INTRODUCTION TO DISCRETE HOLOMORPHIC LOCAL DYNAMICS IN ONE COMPLEX VARIABLE

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1. Introduction

In this survey, by *one–dimensional discrete holomorphic local dynamical system*, we mean a holomorphic function $f\colon U \to \mathbb{C}$ such that $f(0) = 0$, where $U \subseteq \mathbb{C}$ is an open neighbourhood of 0; we shall also assume that $f \not\equiv \mathrm{id}_U$. We shall denote by End $(\mathbb{C}, 0)$ the set of one–dimensional discrete holomorphic local dynamical systems.

Remark 1.1. Since in this survey we shall only be concerned with the one–dimensional discrete case, we shall often drop the adjectives "one–dimensional" and "discrete" and we shall call an element of End $(\mathbb{C}, 0)$ simply a holomorphic local dynamical system. We shall not discuss at all continuous holomorphic local dynamical systems (e.g., holomorphic ODEs or foliations); however, replacing $\mathbb{C}$ by a complex manifold M and 0 by a point $p \in M$, we recover the general definition of discrete holomorphic local dynamical system in M at p.

Remark 1.2. Since we are mainly concerned with the behaviour of f nearby 0, we shall sometimes replace f by its restriction to some suitable open neighbourhood of 0. It is possible to formalize this fact by using germs of maps and germs of sets at the origin, but for our purposes, it will be enough to use a somewhat less formal approach.

To talk about the dynamics of an $f \in$ End $(\mathbb{C}, 0)$, we need to introduce the iterates of f. If f is defined on the set U then the second iterate $f^2 = f \circ f$ is defined on $U \cap f^{-1}(U)$, which is still an open neighbourhood of the origin. More generally, the k-th iterate $f^k = f \circ f^{k-1}$ is only defined on $U \cap f^{-1}(U) \cap \cdots \cap f^{-(k-1)}(U)$. Thus, it is natural to introduce the *stable set* K_f of f by setting

$$K_f = \bigcap_{k=0}^{\infty} f^{-k}(U).$$

Clearly, $0 \in K_f$ and so, the stable set is never empty (but it can happen that $K_f = \{0\}$; see the next section for an example). The stable set of f is the set of all points $z \in U$ such that the *orbit* $\{f^k(z) : k \in \mathbb{N}\}$ is well–defined. If $z \in U \setminus K_f$, we shall say that z (or its orbit) *escapes* from U.

Then the first natural question in local holomorphic dynamics is

Question 1.1. What is the topological structure of K_f?

For instance, when does K_f have non-empty interior? As we shall see in Section 5, holomorphic local dynamical systems such that 0 belongs to the interior of the stable set enjoy special properties.

Remark 1.3. Both the definition of stable set and Question 1.1 (as well as several other definitions or questions we shall meet later on) are topological in character; we might state them for local dynamical systems which are only continuous. As we shall see, however, the *answers* will strongly depend on the holomorphicity of the dynamical system.

Clearly, the stable set K_f is *completely f-invariant*, that is, $f^{-1}(K_f) = K_f$ (this implies, in particular, that $f(K_f) \subseteq K_f$). Therefore, the pair (K_f, f) is a discrete dynamical system in the usual sense and so, the second natural question in local holomorphic dynamics is

Question 1.2. What is the dynamical structure of (K_f, f)?

For instance, what is the asymptotic behaviour of the orbits? Do they converge to the origin, or have they a chaotic behaviour? Is there a dense orbit? Do there exist proper *f-invariant* subsets, that is, sets $L \subset K_f$ such that $f(L) \subseteq L$? If they do exist, what is the dynamics on them?

To answer all these questions, the most efficient way is to replace f by a "dynamically equivalent" but simpler (e.g., linear) map g. In our context, "dynamically equivalent" means "locally conjugated"; and we have at least three kinds of conjugacy to consider.

Let $f_1 : U_1 \to \mathbb{C}$ and $f_2 : U_2 \to \mathbb{C}$ be two holomorphic local dynamical system. We shall say that f_1 and f_2 are *holomorphically* (respectively, *topologically*) *locally conjugated* if there are open neighbourhoods $W_1 \subseteq U_1$ and

$W_2 \subseteq U_2$ of the origin and a biholomorphism (respectively, a homeomorphism) $\varphi\colon W_1 \to W_2$ with $\varphi(0) = 0$ such that

$$f_1 = \varphi^{-1} \circ f_2 \circ \varphi \quad \text{on} \quad \varphi^{-1}\left(W_2 \cap f_2^{-1}(W_2)\right) = W_1 \cap f_1^{-1}(W_1).$$

In particular, we have

$$f_1^k = \varphi^{-1} \circ f_2^k \circ \varphi \quad \text{on} \qquad \varphi^{-1}\left(W_2 \cap \cdots \cap f_2^{-(k-1)}(W_2)\right) = W_1 \cap \cdots \cap f_1^{-(k-1)}(W_1),$$

for every $k \in \mathbb{N}$ and thus, $K_{f_2|_{W_2}} = \varphi(K_{f_1|_{W_1}})$. So the local dynamics of f_1 is to all purposes equivalent to the local dynamics of f_2.

Whenever we have an equivalence relation in a class of objects, there are classification problems. So the third natural question in local holomorphic dynamics is

Question 1.3. Find a (possibly small) class $\mathcal{F}$ of holomorphic local dynamical systems such that every holomorphic local dynamical system $f \in \operatorname{End}(\mathbb{C}, 0)$ is holomorphically (respectively, topologically) locally conjugated to a (possibly) unique element of $\mathcal{F}$, called the *holomorphic* (respectively, *topological*) *normal form* of f.

Unfortunately, the holomorphic classification is often too complicated to be practical; the family $\mathcal{F}$ of normal forms might be uncountable. A possible replacement is looking for invariants instead of normal forms:

Question 1.4. Find a way to associate a (possibly small) class of (possibly computable) objects, called *invariants*, to any holomorphic local dynamical system f so that two holomorphically conjugated local dynamical systems have the same invariants. The class of invariants is furthermore said *complete* if two holomorphic local dynamical systems are holomorphically conjugated if and only if they have the same invariants.

As remarked before, up to now all the questions we asked make sense for topological local dynamical systems; the next one instead makes sense only for holomorphic local dynamical systems.

A holomorphic local dynamical system is clearly given by an element of $\mathbb{C}_0\{z\}$, the space of converging power series in z without constant terms. The space $\mathbb{C}_0\{z\}$ is a subspace of the space $\mathbb{C}_0[[z]]$ of formal power series without constant terms. An element $\Phi \in \mathbb{C}_0[[z]]$ has an inverse (with respect to composition) still belonging to $\mathbb{C}_0[[z]]$ if and only if its linear part is not zero, that is, if and only if it is not divisible by z^2. We shall then say that two holomorphic local dynamical systems $f_1, f_2 \in \mathbb{C}_0\{z\}$ are *formally conjugated* if there exists an invertible $\Phi \in \mathbb{C}_0[[z]]$ such that $f_1 = \Phi^{-1} \circ f_2 \circ \Phi$ in $\mathbb{C}_0[[z]]$.

It is clear that two holomorphically locally conjugated dynamical systems are both formally and topologically locally conjugated too. On the other hand, we shall see (in Remark 4.2) examples of holomorphic local dynamical systems that are topologically locally conjugated without being neither formally nor holomorphically locally conjugated and (in Remarks 4.2 and 5.3) examples of holomorphic local dynamical systems that are formally conjugated without being neither holomorphically nor topologically locally conjugated. So the last natural question in local holomorphic dynamics we shall deal with is

Question 1.5. Find normal forms and invariants with respect to the relation of formal conjugacy for holomorphic local dynamical systems.

In this survey we shall present some of the main results known on these questions. But before entering the main core of the paper, I would like to thank heartily François Berteloot, Salvatore Coen, Santiago Díaz–Madrigal, Vincent Guedj, Giorgio Patrizio, Mohamad Pouryayevali, Jasmin Raissy, Francesca Tovena and Alekos Vidras, without whom this survey would never has been written.

2. Hyperbolic dynamics

As remarked in the previous section, an one–dimensional discrete holomorphic local dynamical system is given by a converging power series f without constant term:

$$f(z) = a_1 z + a_2 z^2 + a_3 z^3 + \cdots \in \mathbb{C}_0\{z\}.$$

The number $a_1 = f'(0)$ is the *multiplier* of f. Since $a_1 z$ is the best linear approximation of f, it is sensible to expect that the local dynamics of f will be strongly influenced by the value of a_1. We then introduce the following definitions:

- if $|a_1| < 1$ we say that the fixed point 0 is *attracting*;
- if $a_1 = 0$ we say that the fixed point 0 is *superattracting*;
- if $|a_1| > 1$ we say that the fixed point 0 is *repelling*;
- if $|a_1| \neq 0, 1$ we say that the fixed point 0 is *hyperbolic*;
- if $a_1 \in S^1$ is a root of unity we say that the fixed point 0 is *parabolic* (or *rationally indifferent*);

- if $a_1 \in S^1$ is not a root of unity we say that the fixed point 0 is *elliptic* (or *irrationally indifferent*).

Remark 2.1. If $a_1 \neq 0$ then f is locally invertible, that is, there exists $f^{-1} \in \mathrm{End}\,(\mathbb{C}, 0)$ so that $f^{-1} \circ f = f \circ f^{-1} = \mathrm{id}$ where defined. In particular, if 0 is an attracting fixed point for $f \in \mathrm{End}\,(\mathbb{C}, 0)$ with non-zero multiplier then it is a repelling fixed point for the inverse function f^{-1}.

As we shall see in a minute, the dynamics of one–dimensional holomorphic local dynamical systems with a hyperbolic fixed point is pretty elementary; so we start with this case.

Assume first that 0 is attracting (but not superattracting) for the holomorphic local dynamical system $f \in \mathrm{End}\,(\mathbb{C}, 0)$. Then we can write $f(z) = a_1 z + O(z^2)$, with $0 < |a_1| < 1$; hence, we can find a large constant $M > 0$, a small constant $\varepsilon > 0$ and $0 < \delta < 1$ such that if $|z| < \varepsilon$ then

$$|f(z)| \leq (|a_1| + M\varepsilon)|z| \leq \delta |z|. \tag{1}$$

In particular, if Δ_ε is the disk of center 0 and radius ε, we have $f(\Delta_\varepsilon) \subset \Delta_\varepsilon$ for $\varepsilon > 0$ small enough and the stable set of $f|_{\Delta_\varepsilon}$ is Δ_ε itself (in particular, it contains the origin in its interior). Furthermore, since Δ_ε is f-invariant, we can apply (1) to $f(z)$; arguing by induction we get

$$|f^k(z)| \leq \delta^k |z| \to 0 \tag{2}$$

as $k \to +\infty$ and thus, every orbit starting in Δ_ε is attracted by the origin, which is the reason of the name "attracting" for such a fixed point.

If instead 0 is a repelling fixed point, a similar argument (or the observation that 0 is attracting for f^{-1}) shows that for $\varepsilon > 0$ small enough the stable set of $f|_{\Delta_\varepsilon}$ reduces to the origin only: all (non-trivial) orbits escape.

It is also not difficult to find holomorphic and topological normal forms in this case, as shown in the following result, which has marked the beginning of the theory of holomorphic dynamical systems:

Theorem 2.1 (Kœnigs, 1884 [19]). *Let $f \in \mathrm{End}\,(\mathbb{C}, 0)$ be an one–dimensional discrete holomorphic local dynamical system with a hyperbolic fixed point at the origin and let $a_1 \in \mathbb{C}^* \setminus S^1$ be its multiplier. Then:*

(i) f is holomorphically (and hence, formally) locally conjugated to its linear part $g(z) = a_1 z$. The conjugation φ is uniquely determined by the condition $\varphi'(0) = 1$.

(ii) Two such holomorphic local dynamical systems are holomorphically conjugated if and only if they have the same multiplier.
(iii) f is topologically locally conjugated to the map $g_<(z) = z/2$ if $|a_1| < 1$ and to the map $g_>(z) = 2z$ if $|a_1| > 1$.

Proof. Let us assume $0 < |a_1| < 1$; if $|a_1| > 1$, it will suffice to apply the same argument to f^{-1}.

(i) Choose $0 < \delta < 1$ such that $\delta^2 < |a_1| < \delta$. Writing $f(z) = a_1 z + z^2 r(z)$ for a suitable holomorphic germ r, we can find $\varepsilon > 0$ such that $|a_1| + M\varepsilon < \delta$, where $M = \max_{z\in\bar{\Delta}_\varepsilon} |r(z)|$. So we have

$$|f(z) - a_1 z| \le M|z|^2 \tag{3}$$

that implies (1) and hence, we get (2) for all $z \in \bar{\Delta}_\varepsilon$ and $k \in \mathbb{N}$.

Put $\varphi_k = f^k/a_1^k$; we claim that the sequence $\{\varphi_k\}$ converges to a holomorphic map $\varphi\colon \Delta_\varepsilon \to \mathbb{C}$. Indeed (3) and (2) yield

$$\begin{aligned}|\varphi_{k+1}(z) - \varphi_k(z)| &= \frac{1}{|a_1|^{k+1}} \left|f\left(f^k(z)\right) - a_1 f^k(z)\right| \\ &\le \frac{M}{|a_1|^{k+1}} |f^k(z)|^2 \le \frac{M}{|a_1|} \left(\frac{\delta^2}{|a_1|}\right)^k |z|^2\end{aligned}$$

for all $z \in \bar{\Delta}_\varepsilon$ and so, the telescopic series $\sum_k (\varphi_{k+1} - \varphi_k)$ is uniformly convergent in Δ_ε to $\varphi - \varphi_0$.

Since $\varphi_k'(0) = 1$ for all $k \in \mathbb{N}$, by Weierstrass' theorem we have $\varphi'(0) = 1$ and so, up to possibly shrink ε, we can assume that φ is a biholomorphism with its image. Moreover, we have

$$\varphi\left(f(z)\right) = \lim_{k\to+\infty} \frac{f^k\left(f(z)\right)}{a_1^k} = a_1 \lim_{k\to+\infty} \frac{f^{k+1}(z)}{a_1^{k+1}} = a_1\varphi(z),$$

that is, $f = \varphi^{-1} \circ g \circ \varphi$, as claimed.

If ψ is another local holomorphic function such that $\psi'(0) = 1$ and $\psi^{-1} \circ g \circ \psi = f$, it follows that $\psi \circ \varphi^{-1}(\lambda z) = \lambda \psi \circ \varphi^{-1}(z)$; comparing the power series expansion of both sides we find $\psi \circ \varphi^{-1} \equiv \mathrm{id}$, that is, $\psi \equiv \varphi$, as claimed.

(ii) Since $f_1 = \varphi^{-1} \circ f_2 \circ \varphi$ implies $f_1'(0) = f_2'(0)$, the multiplier is invariant under holomorphic local conjugation and so, two one–dimensional discrete holomorphic local dynamical systems with a hyperbolic fixed point are holomorphically locally conjugated if and only if they have the same multiplier.

(iii) It suffices to build a topological conjugacy between g and $g_<$ on Δ_ε. First choose a homeomorphism χ between the annulus $\{|a_1|\varepsilon \le |z| \le \varepsilon\}$