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# Advanced Microeconomic Theory

(Third Edition)

杰弗里·A·杰里 (Geoffrey A. Jehle) 菲利普·J·瑞尼 (Philip J. Reny)

四 中国人民大学出版社

PEARSON

高等学校经济类双语教学推荐教材



经济学经典教材・核心课系列

# 高级微观经济理论

Advanced Microeconomic Theo

菲利普·J·瑞尼 (Philip J. Reny)

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第二,英文原版教材特色。本系列教材依据国内实际教学需要以及广泛适应性,部分对原版教材进行了全文影印,部分在保持原版教材体系结构和内容特色的基础上进行了适当删减。

第三,内容紧扣学科前沿。本系列教材在原著选择上紧扣国外教学前沿,基本上都是国外最流行教材的最新版本。

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本系列教材既适合高等院校经济类专业的本科教学使用,也适合从事经济类工作和研究的广大从业者的阅读和学习。我们在选书、改编过程中虽然全面听取了专家、学者和教师的意见,努力做到满足广大读者的需求,但由于各教材的作者所处的政治、经济和文化背景不同,书中内容仍可能有不妥之处,我们真诚希望广大读者提出宝贵意见和建议,以便我们在以后的版本中不断改进和完善。

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# PREFACE

In preparing this third edition of our text, we wanted to provide long-time readers with new and updated material in a familiar format, while offering first-time readers an accessible, self-contained treatment of the essential core of modern microeconomic theory.

To those ends, every chapter has been revised and updated. The more significant changes include a new introduction to general equilibrium with contingent commodities in Chapter 5, along with a simplified proof of Arrow's theorem and a new, careful development of the Gibbard-Satterthwaite theorem in Chapter 6. Chapter 7 includes many refinements and extensions, especially in our presentation on Bayesian games. The biggest change – one we hope readers find interesting and useful – is an extensive, integrated presentation in Chapter 9 of many of the central results of mechanism design in the quasi-linear utility, private-values environment.

We continue to believe that working through exercises is the surest way to master the material in this text. New exercises have been added to virtually every chapter, and others have been updated and revised. Many of the new exercises guide readers in developing for themselves extensions, refinements or alternative approaches to important material covered in the text. Hints and answers for selected exercises are provided at the end of the book, along with lists of theorems and definitions appearing in the text. We will continue to maintain a readers' forum on the web, where readers can exchange solutions to exercises in the text. It can be reached at <a href="http://alfred.vassar.edu">http://alfred.vassar.edu</a>.

The two full chapters of the Mathematical Appendix still provide students with a lengthy and largely self-contained development of the set theory, real analysis, topology, calculus, and modern optimisation theory which are indispensable in modern microeconomics. Readers of this edition will now find a fuller, self-contained development of Lagrangian and Kuhn-Tucker methods, along with new material on the Theorem of the Maximum and two separation theorems. The exposition is formal but presumes nothing more than a good grounding in single-variable calculus and simple linear algebra as a starting point. We suggest that even students who are very well-prepared in mathematics browse both chapters of the appendix early on. That way, if and when some review or reference is needed, the reader will have a sense of how that material is organised.

Before we begin to develop the theory itself, we ought to say a word to new readers about the role mathematics will play in this text. Often, you will notice we make certain assumptions purely for the sake of mathemat-

PREFACE

ical expediency. The justification for proceeding this way is simple, and it is the same in every other branch of science. These abstractions from 'reality' allow us to bring to bear powerful mathematical methods that, by the rigour of the logical discipline they impose, help extend our insights into areas beyond the reach of our intuition and experience. In the physical world, there is 'no such thing' as a frictionless plane or a perfect vacuum. In economics, as in physics, allowing ourselves to accept assumptions like these frees us to focus on more important aspects of the problem and thereby helps to establish benchmarks in theory against which to gauge experience and observation in the real world. This does not mean that you must wholeheartedly embrace every 'unrealistic' or purely formal aspect of the theory. Far from it. It is always worthwhile to cast a critical eye on these matters as they arise and to ask yourself what is gained, and what is sacrificed, by the abstraction at hand. Thought and insight on these points are the stuff of which advances in theory and knowledge are made. From here on, however, we will take the theory as it is and seek to understand it on its own terms, leaving much of its critical appraisal to your moments away from this book.

Finally, we wish to acknowledge the many readers and colleagues who have provided helpful comments and pointed out errors in previous editions. Your keen eyes and good judgements have helped us make this third edition better and more complete than it otherwise would be. While we cannot thank all of you personally, we must thank Eddie Dekel, Roger Myerson, Derek Neal, Motty Perry, Arthur Robson, Steve Williams, and Jörgen Weibull for their thoughtful comments.

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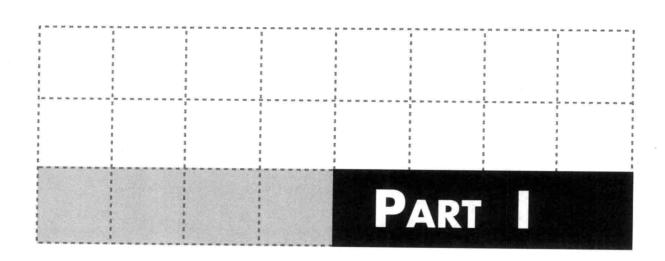
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**ECONOMIC AGENTS** 

# CHAPTER 1

# CONSUMER THEORY

In the first two chapters of this volume, we will explore the essential features of modern consumer theory – a bedrock foundation on which so many theoretical structures in economics are built. Some time later in your study of economics, you will begin to notice just how central this theory is to the economist's way of thinking. Time and time again you will hear the echoes of consumer theory in virtually every branch of the discipline – how it is conceived, how it is constructed, and how it is applied.

# 1.1 Primitive Notions

There are four building blocks in any model of consumer choice. They are the consumption set, the feasible set, the preference relation, and the behavioural assumption. Each is conceptually distinct from the others, though it is quite common sometimes to lose sight of that fact. This basic structure is extremely general, and so, very flexible. By specifying the form each of these takes in a given problem, many different situations involving choice can be formally described and analysed. Although we will tend to concentrate here on specific formalisations that have come to dominate economists' view of an individual consumer's behaviour, it is well to keep in mind that 'consumer theory' *per se* is in fact a very rich and flexible *theory of choice*.

The notion of a **consumption set** is straightforward. We let the consumption set, X, represent the set of all alternatives, or complete consumption plans, that the consumer can conceive – whether some of them will be achievable in practice or not. What we intend to capture here is the universe of alternative choices over which the consumer's mind is capable of wandering, unfettered by consideration of the realities of his present situation. The consumption set is sometimes also called the **choice set**.

Let each commodity be measured in some infinitely divisible units. Let  $x_i \in \mathbb{R}$  represent the number of units of good i. We assume that only non-negative units of each good are meaningful and that it is always possible to conceive of having no units of any particular commodity. Further, we assume there is a finite, fixed, but arbitrary number n of different goods. We let  $\mathbf{x} = (x_1, \dots, x_n)$  be a vector containing different quantities of each of the n commodities and call  $\mathbf{x}$  a **consumption bundle** or a **consumption plan**. A consumption bundle  $\mathbf{x} \in X$  is thus represented by a point  $\mathbf{x} \in \mathbb{R}_+^n$ . Usually, we'll simplify things and just think of the consumption set as the *entire* non-negative orthant,  $X = \mathbb{R}_+^n$ . In this case, it is easy to see that each of the following basic requirements is satisfied.

# ASSUMPTION 1.1 Properties of the Consumption Set, X

The minimal requirements on the consumption set are

- 2. X is closed.
- 3. X is convex.
- 4.  $0 \in X$ .

The notion of a **feasible set** is likewise very straightforward. We let B represent all those alternative consumption plans that are both conceivable and, more important, realistically obtainable given the consumer's circumstances. What we intend to capture here are precisely those alternatives that are *achievable* given the economic realities the consumer faces. The feasible set B then is that subset of the consumption set X that remains after we have accounted for any constraints on the consumer's access to commodities due to the practical, institutional, or economic realities of the world. How we specify those realities in a given situation will determine the precise configuration and additional properties that B must have. For now, we will simply say that  $B \subset X$ .

A preference relation typically specifies the limits, if any, on the consumer's ability to perceive in situations involving choice the form of consistency or inconsistency in the consumer's choices, and information about the consumer's tastes for the different objects of choice. The preference relation plays a crucial role in any theory of choice. Its special form in the theory of consumer behaviour is sufficiently subtle to warrant special examination in the next section.

Finally, the model is 'closed' by specifying some **behavioural assumption**. This expresses the guiding principle the consumer uses to make final choices and so identifies the ultimate objectives in choice. It is supposed that *the consumer seeks to identify and select an available alternative that is most preferred in the light of his personal tastes.* 

# 1.2 PREFERENCES AND UTILITY

In this section, we examine the consumer's preference relation and explore its connection to modern usage of the term 'utility'. Before we begin, however, a brief word on the evolution of economists' thinking will help to place what follows in its proper context.

In earlier periods, the so-called 'Law of Demand' was built on some extremely strong assumptions. In the classical theory of Edgeworth, Mill, and other proponents of the utilitarian school of philosophy, 'utility' was thought to be something of substance. 'Pleasure' and 'pain' were held to be well-defined entities that could be measured and compared between individuals. In addition, the 'Principle of Diminishing Marginal Utility' was accepted as a psychological 'law', and early statements of the Law of Demand depended on it. These are awfully strong assumptions about the inner workings of human beings.

The more recent history of consumer theory has been marked by a drive to render its foundations as general as possible. Economists have sought to pare away as many of the traditional assumptions, explicit or implicit, as they could and still retain a coherent theory with predictive power. Pareto (1896) can be credited with suspecting that the idea of a measurable 'utility' was inessential to the theory of demand. Slutsky (1915) undertook the first systematic examination of demand theory without the concept of a measurable substance called utility. Hicks (1939) demonstrated that the Principle of Diminishing Marginal Utility was neither necessary, nor sufficient, for the Law of Demand to hold. Finally, Debreu (1959) completed the reduction of standard consumer theory to those bare essentials we will consider here. Today's theory bears close and important relations to its earlier ancestors, but it is leaner, more precise, and more general.

## 1.2.1 PREFERENCE RELATIONS

Consumer preferences are characterised axiomatically. In this method of modelling as few meaningful and distinct assumptions as possible are set forth to characterise the struc-

ture and properties of preferences. The rest of the theory then builds logically from these axioms, and predictions of behaviour are developed through the process of deduction.

These **axioms of consumer choice** are intended to give formal mathematical expression to fundamental aspects of consumer behaviour and attitudes towards the objects of choice. Together, they formalise the view that the consumer *can* choose and that choices are *consistent* in a particular way.

Formally, we represent the consumer's preferences by a *binary relation*,  $\succeq$ , defined on the consumption set, X. If  $\mathbf{x}^1 \succeq \mathbf{x}^2$ , we say that ' $\mathbf{x}^1$  is at least as good as  $\mathbf{x}^2$ ', for this consumer.

That we use a binary relation to characterise preferences is significant and worth a moment's reflection. It conveys the important point that, from the beginning, our theory requires relatively little of the consumer it describes. We require only that consumers make binary comparisons, that is, that they only examine two consumption plans at a time and make a decision regarding those two. The following axioms set forth basic criteria with which those binary comparisons must conform.

**AXIOM 1:** Completeness. For all 
$$\mathbf{x}^1$$
 and  $\mathbf{x}^2$  in  $X$ , either  $\mathbf{x}^1 \gtrsim \mathbf{x}^2$  or  $\mathbf{x}^2 \gtrsim \mathbf{x}^1$ .

Axiom 1 formalises the notion that the consumer *can* make comparisons, that is, that he has the ability to discriminate and the necessary knowledge to evaluate alternatives. It says the consumer can examine *any* two distinct consumption plans  $\mathbf{x}^1$  and  $\mathbf{x}^2$  and decide whether  $\mathbf{x}^1$  is at least as good as  $\mathbf{x}^2$  or  $\mathbf{x}^2$  is at least as good as  $\mathbf{x}^1$ .

**AXIOM 2:** Transitivity. For any three elements  $\mathbf{x}^1$ ,  $\mathbf{x}^2$ , and  $\mathbf{x}^3$  in X, if  $\mathbf{x}^1 \succeq \mathbf{x}^2$  and  $\mathbf{x}^2 \succeq \mathbf{x}^3$ , then  $\mathbf{x}^1 \succeq \mathbf{x}^3$ .

Axiom 2 gives a very particular form to the requirement that the consumer's choices be *consistent*. Although we require only that the consumer be capable of comparing two alternatives at a time, the assumption of transitivity requires that those pairwise comparisons be linked together in a consistent way. At first brush, requiring that the evaluation of alternatives be transitive seems simple and only natural. Indeed, were they not transitive, our instincts would tell us that there was something peculiar about them. Nonetheless, this is a controversial axiom. Experiments have shown that in various situations, the choices of real human beings are not always transitive. Nonetheless, we will retain it in our description of the consumer, though not without some slight trepidation.

These two axioms together imply that the consumer can completely *rank* any finite number of elements in the consumption set, *X*, from best to worst, possibly with some ties. (Try to prove this.) We summarise the view that preferences enable the consumer to construct such a ranking by saying that those preferences can be represented by a *preference relation*.

## DEFINITION 1.1 Preference Relation

The binary relation  $\succeq$  on the consumption set X is called a preference relation if it satisfies Axioms 1 and 2.

There are two additional relations that we will use in our discussion of consumer preferences. Each is determined by the preference relation,  $\gtrsim$ , and they formalise the notions of *strict preference* and *indifference*.

## DEFINITION 1.2 Strict Preference Relation

The binary relation  $\succ$  on the consumption set X is defined as follows:

$$\label{eq:control_equation} \boldsymbol{x}^1 \succ \boldsymbol{x}^2 \qquad \text{if and only if} \qquad \boldsymbol{x}^1 \succsim \boldsymbol{x}^2 \quad \text{and} \quad \boldsymbol{x}^2 \not\succsim \boldsymbol{x}^1.$$

The relation  $\succ$  is called the strict preference relation induced by  $\succsim$ , or simply the strict preference relation when  $\succsim$  is clear. The phrase  $\mathbf{x}^1 \succ \mathbf{x}^2$  is read, ' $\mathbf{x}^1$  is strictly preferred to  $\mathbf{x}^2$ '.

## **DEFINITION 1.3** Indifference Relation

The binary relation  $\sim$  on the consumption set X is defined as follows:

$$\mathbf{x}^1 \sim \mathbf{x}^2$$
 if and only if  $\mathbf{x}^1 \succsim \mathbf{x}^2$  and  $\mathbf{x}^2 \succsim \mathbf{x}^1$ .

The relation  $\sim$  is called the indifference relation induced by  $\succeq$ , or simply the indifference relation when  $\succeq$  is clear. The phrase  $\mathbf{x}^1 \sim \mathbf{x}^2$  is read, ' $\mathbf{x}^1$  is indifferent to  $\mathbf{x}^2$ '.

Building on the underlying definition of the preference relation, both the strict preference relation and the indifference relation capture the usual sense in which the terms 'strict preference' and 'indifference' are used in ordinary language. Because each is derived from the preference relation, each can be expected to share some of its properties. Some, yes, but not all. In general, both are transitive and neither is complete.

Using these two supplementary relations, we can establish something very concrete about the consumer's ranking of any two alternatives. For any pair  $\mathbf{x}^1$  and  $\mathbf{x}^2$ , exactly one of three mutually exclusive possibilities holds:  $\mathbf{x}^1 \succ \mathbf{x}^2$ , or  $\mathbf{x}^2 \succ \mathbf{x}^1$ , or  $\mathbf{x}^1 \sim \mathbf{x}^2$ .

To this point, we have simply managed to formalise the requirement that preferences reflect an ability to make choices and display a certain kind of consistency. Let us consider how we might describe graphically a set of preferences satisfying just those first few axioms. To that end, and also because of their usefulness later on, we will use the preference relation to define some related sets. These sets focus on a single alternative in the consumption set and examine the ranking of all other alternatives relative to it.

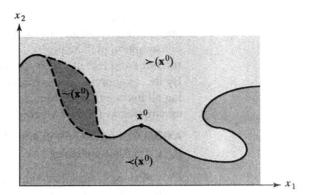
# **DEFINITION 1.4** Sets in X Derived from the Preference Relation

Let  $\mathbf{x}^0$  be any point in the consumption set, X. Relative to any such point, we can define the following subsets of X:

- 1.  $\succeq (\mathbf{x}^0) \equiv {\mathbf{x} \mid \mathbf{x} \in X, \mathbf{x} \succeq \mathbf{x}^0}$ , called the 'at least as good as' set.
- 2.  $\leq (\mathbf{x}^0) \equiv \{\mathbf{x} \mid \mathbf{x} \in X, \mathbf{x}^0 \succeq \mathbf{x}\}$ , called the 'no better than' set.
- 3.  $\langle (\mathbf{x}^0) \equiv \{\mathbf{x} \mid \mathbf{x} \in X, \mathbf{x}^0 \succ \mathbf{x}\}$ , called the 'worse than' set.
- 4.  $\succ (\mathbf{x}^0) \equiv {\mathbf{x} \mid \mathbf{x} \in X, \mathbf{x} \succ \mathbf{x}^0}$ , called the 'preferred to' set.
- 5.  $\sim (\mathbf{x}^0) \equiv {\mathbf{x} \mid \mathbf{x} \in X, \mathbf{x} \sim \mathbf{x}^0}$ , called the 'indifference' set.

A hypothetical set of preferences satisfying Axioms 1 and 2 has been sketched in Fig. 1.1 for  $X = \mathbb{R}^2_+$ . Any point in the consumption set, such as  $\mathbf{x}^0 = (x_1^0, x_2^0)$ , represents a consumption plan consisting of a certain amount  $x_1^0$  of commodity 1, together with a certain amount  $x_2^0$  of commodity 2. Under Axiom 1, the consumer is able to compare  $\mathbf{x}^0$  with any and every other plan in X and decide whether the other is at least as good as  $\mathbf{x}^0$  or whether  $\mathbf{x}^0$  is at least as good as the other. Given our definitions of the various sets relative to  $\mathbf{x}^0$ , Axioms 1 and 2 tell us that the consumer must place *every* point in X into

**Figure 1.1.** Hypothetical preferences satisfying Axioms 1 and 2.



one of three mutually exclusive categories relative to  $\mathbf{x}^0$ ; every other point is worse than  $\mathbf{x}^0$ , indifferent to  $\mathbf{x}^0$ , or preferred to  $\mathbf{x}^0$ . Thus, for any bundle  $\mathbf{x}^0$  the three sets  $\prec (\mathbf{x}^0)$ ,  $\sim (\mathbf{x}^0)$ , and  $\succ (\mathbf{x}^0)$  partition the consumption set.

The preferences in Fig. 1.1 may seem rather odd. They possess only the most limited structure, yet they are entirely consistent with and allowed for by the first two axioms alone. Nothing assumed so far prohibits any of the 'irregularities' depicted there, such as the 'thick' indifference zones, or the 'gaps' and 'curves' within the indifference set  $\sim (\mathbf{x}^0)$ . Such things can be ruled out only by imposing additional requirements on preferences.

We shall consider several new assumptions on preferences. One has very little behavioural significance and speaks almost exclusively to the purely mathematical aspects of representing preferences; the others speak directly to the issue of consumer tastes over objects in the consumption set.

The first is an axiom whose only effect is to impose a kind of topological regularity on preferences, and whose primary contribution will become clear a bit later.

From now on we explicitly set  $X = \mathbb{R}^n_+$ .

**AXIOM 3:** Continuity. For all  $\mathbf{x} \in \mathbb{R}^n_+$ , the 'at least as good as' set,  $\succeq (\mathbf{x})$ , and the 'no better than' set,  $\preceq (\mathbf{x})$ , are closed in  $\mathbb{R}^n_+$ .

Recall that a set is closed in a particular domain if its complement is open in that domain. Thus, to say that  $\succeq (\mathbf{x})$  is closed in  $\mathbb{R}^n_+$  is to say that its complement,  $\prec (\mathbf{x})$ , is open in  $\mathbb{R}^n_+$ .

The continuity axiom guarantees that sudden preference reversals do not occur. Indeed, the continuity axiom can be equivalently expressed by saying that if each element  $\mathbf{y}^n$  of a sequence of bundles is at least as good as (no better than)  $\mathbf{x}$ , and  $\mathbf{y}^n$  converges to  $\mathbf{y}$ , then  $\mathbf{y}$  is at least as good as (no better than)  $\mathbf{x}$ . Note that because  $\succeq (\mathbf{x})$  and  $\preceq (\mathbf{x})$  are closed, so, too, is  $\sim (\mathbf{x})$  because the latter is the intersection of the former two. Consequently, Axiom 3 rules out the open area in the indifference set depicted in the north-west of Fig. 1.1.

Additional assumptions on tastes lend the greater structure and regularity to preferences that you are probably familiar with from earlier economics classes. Assumptions of this sort must be selected for their appropriateness to the particular choice problem being analysed. We will consider in turn a few key assumptions on tastes that are ordinarily imposed in 'standard' consumer theory, and seek to understand the individual and collective contributions they make to the structure of preferences. Within each class of these assumptions, we will proceed from the less restrictive to the more restrictive. We will generally employ the more restrictive versions considered. Consequently, we let axioms with primed numbers indicate alternatives to the norm, which are conceptually similar but slightly less restrictive than their unprimed partners.

When representing preferences over ordinary consumption goods, we will want to express the fundamental view that 'wants' are essentially unlimited. In a very weak sense,