NONLINEAR DYNAMICS

INTEGRABILITY AND CHAOS

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Narosa

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Integrability and Chaos

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NAROSA PUBLISHING HOUSE

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2F–2G Shivam Chambers, 53 Syed Amir Ali Avenue, Calcutta 700 019
3 Henrietta Street, Covent Garden, London WC2E 8LU, UK

ISBN 81-7319-326-6

Published by N.K. Mehra for Narosa Publishing House, 6 Community Centre, Panchsheel Park, New Delhi 110 017 and printed at Rajkamal Electric Press, Delhi 110 033 (India).

NONLINEAR DYNAMICS: Integrability and Chaos

Preface

This book is the proceedings of the International Conference on *Nonlinear Dynamics: Integrability and Chaos* held at Bharathidasan University, Tiruchirapalli, India during 12 - 16 February 1998. The aim of the conference was to put together experts and active researchers from India and abroad working in Nonlinear Dynamics in the field of soliton theory and integrability with those in the field of classical and quantum chaos. The conference also marked the 50th birthday of Professor M. Lakshmanan who has made significant contributions to the areas of both integrability including soliton theory and chaos in the field of Nonlinear Dynamics during the past two decades. The conference was inaugurated by Professor Martin Kruskal, the discoverer of the concept of soliton. The conference, first of this kind being organized in India, covered a range of frontier topics and problems of current research in the general perspective of Nonlinear Dynamics, treating integrability and chaos as intertwined notions. This proceedings is a publication in honour of Professor Lakshmanan's active career in nonlinear dynamics research and his varied contributions to the field, by his colleagues, friends, students and well wishers. This is also a mark of celebration of his inspiring personality, his extraordinary ability to motivate young researchers to pursue active research even under trying conditions and his untiring efforts to build a strong group and an internationally recognised centre for Nonlinear Dynamics in the remote town of Tiruchirapalli in South India. Glancing over his research publications numbering around 200 (original research articles, reviews and books) might give the impression that here is an extraordinary person who can not be tied down to one single scientific topic but looks all the time for a unified understanding of the subject. His deep-rooted care for his students and colleagues have made him a personal friend to many and a scientific father- figure to his students.

Hailing from an agriculture family in a tiny village (Kurva Goundan Palayam) in Coimbatore District of Tamil Nadu, India, Professor Lakshmanan had his school education in a nearby rural public school and completed his undergraduate and post-graduate studies in Physics at Pollachi and Madras(Chennai) respectively. Professor Lakshmanan worked for his Ph.D. (1974) at the University of Madras under the supervision of the well known Theoretical Physicist Professor P.M.Mathews. After spending a couple of years at the University of Tübingen, Germany as an Alexander Von Humboldt Foundation Fellow and at the Eindhoven University of Technology, Holland as a Post-doctoral Fellow, he became one of the first few faculty members of the Autonomous Post-Graduate Centre of the University of Madras at Tiruchirapalli (the present Bharathidasan University) in 1978. He rose to a full Professor in 1984 and is heading the Centre for Nonlinear Dynamics since 1992 and the Department of Physics since 1994.

Professor Lakshmanan's several significant contributions to the field of soliton and integrability as well as chaos have had deep impact in Nonlinear Dynamics (ND) and in fact have been instrumental to the development of new lines of research in the respective topics. His significant contributions to the field of ND have been well recognised both inside and outside India and he has been bestowed with several top and prestigious scientific awards including the highest scientific award of Shanti Swarup Bhatnagar Prize (1989) in India. His discovery (Phys. Lett. **61A**, 53 (1977)) of magnetic soliton in isotropic classical continuum ferromagnetic spin system had initiated a new way to identify integrable spin models and his space curve mapping procedure thereof (M. Lakshmanan, Th.W. Ruijgrok and C. J. Thompson, Physica **84A**, 577 (1976)) has been revisited by several researchers in recent times. His two important papers on symmetries (M. Lakshmanan and P. Kaliappan, J. Math. Phys. **24**, 795 (1983)) and on singularity structures (M. Lakshmanan and R. Sahadevan, Phys. Rep. **224**, 1 (1993)) of nonlinear evolution equations of finite and infinite dimensions have played very crucial roles. His recent exciting work (R. Radhakrishnan, M. Lakshmanan and J. Hietarinta, Phys. Rev. **E56**, 2213 (1997)) on optical soliton in birefringent fibres has shown the possibility of shape changing collisions of solitons, a possibility which has led to the theoretical suggestion of constructing logic gates by implementing computation in a bulk nonlinear medium without interconnecting discrete components. The introduction of the simplest dissipative nonautonomous chaotic circuit (Murali-Lakshmanan-Chua Circuit) (IEEE Transcations on Circuits and Systems **I41**, 462 (1994)) has created new avenues of research in nonlinear electronic circuits. In the following, we bring out the salient features of some of his significant discoveries as we understand them.

Theories of intrinsically nonlinear phenomena have developed tremendously in the years since the concepts of soliton and chaos in nonlinear dynamical systems were discovered. The contributions in this book represent a snapshot of the current status of the above nonlinear theories in different areas of Physics and Mathematics and also the personal interests of Professor Lakshmanan. The specific topics covered in this volume include integrability, symmetries and Painlevé singularity structures, Hirota's bilinearisation, soliton perturbation, integrability in discrete and periodic systems, multidimensional solitons, low dimensional chaos including quantum chaos, spatio-temporal chaos and pattern formation, applications of soliton and chaos in magnetic systems, fluids, electronic circuits, biological systems, etc. We have grouped the contributions into six chapters:

1. Nonlinear Dynamics : General
2. Soliton Systems and Integrability
3. Discrete Nonlinear Systems: Integrability and Chaos
4. Classical and Quantum Chaos
5. Spatio-Temporal Chaos and Pattern Formation
6. Nonlinear Dynamics : Applications

It is with great pleasure that we express our gratitude to those who have worked hard to make this tribute both enjoyable and meaningful. The financial support received from the Department of Science and Technology, Government of India, the National Board for Higher Mathematics, Bharathidasan University, Tiruchirapalli and the Indian National Science Academy, New Delhi is gratefully acknowledged. Finally, we thank V. Veerakumar and P. Muruganandam who helped in setting the manuscript on the computer.

M.Daniel
K.M.Tamizhmani
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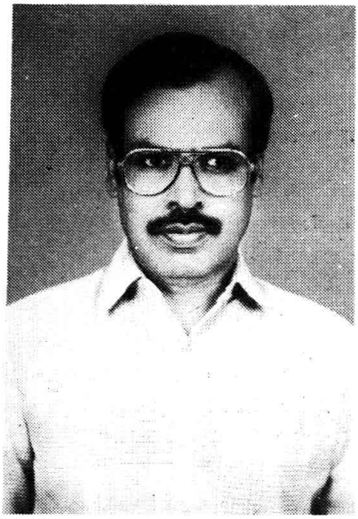
A Short Resume of Professor M. Lakshmanan's Scientific Contributions



Professor M. Lakshmanan's scientific career consists of several phases, each one marked by his uncanny ability to identify challenging problems and approach them with ingenious ability. Several major contributions marked the formative years (1970-74) of his research career as a doctoral student. The very first publication of Professor Lakshmanan (along with his research supervisor Prof. P.M. Mathews), on the apparent visual forms of relativistically moving objects wherein it was pointed out that such fast moving objects can have apparent speeds that may exceed the velocity of light (Nuovo Cimento **12B**, 168 (1972)), has drawn a laudatory write up by W. H. McCrea in the "News and Views" column of Nature (**241**, 423 (1973)). His entry into the field of Nonlinear Dynamics, working in isolation and without contact with the mainstream development of the field, was through a study of the nonlinear excitations and wave propagation in the ϕ^4 field model (Ann. Phys. (N.Y.) **79**, 171 (1973) ; J. Phys. **7A**, 889 (1974); **8A**, 788 (1975)) and the study of the semiclassical dynamics of anharmonic oscillators (Lett. Nuov. Cim. **7**, 689 (1973); **8**, 743 (1973)). In this period he also identified a unique nonlinear oscillator (Quart. Appl. Maths. **32**, 215 (1974)) exhibiting amplitude-dependent sinusoidal oscillations, whose generalizations and quantization (Nuov. Cim **26A**, 299 (1975); J. Phys. **8A**, 1658 (1975)) lead to the study of exactly solvable quantum nonlinear chiral models equivalent to an oscillator moving on a three sphere, which P.W. Higgs (J. Phys. **12A**, 309 (1979)) recognized as the starting point for the study of dynamical symmetries in spherical geometry.

In the second phase, during the period 1975-80, wherein he got into direct contact with fellow scientists in Europe, he moved onto the field of soliton theory proper, exploring many a topics. During this time, he made the seminal contribution that the continuum Heisenberg ferromagnetic spin system is a completely integrable soliton system (Phys. Lett. **61A**, 53 (1977)), following the earlier collaborative work with Th.W. Ruijgrok and C.J. Thompson (Physica **84A**, 577 (1976)) wherein the geometry of the system was utilized. Further generalization of the geometric approach to cover AKNS class of soliton systems (Phys. Lett. **64A**, 354 (1978); J. Math. Phys. **20**, 1667 (1979)) and inhomogeneous systems with R.K. Bullough (Phys. Lett. **80A**, 287 (1980)) followed, leading to important advances.

Next, towards the end of 1970s and in early 1980s Professor Lakshmanan started forming a Nonlinear Dynamics group of young research students and collaborating with them. With P. Kaliappan (1979-82) he started a many faceted investigation on the connection between Lie symmetries, similarity reductions and Painlevé equations associated with nonlinear evolution equations (J. Phys. **A12**, L249 (1979); J. Math. Phys. **24**, 795 (1983)), identifying soliton-type similarity solutions to axisymmetric Einstein-Maxwell equations (J. Math. Phys. **22**, 2447 (1981)), then extending these studies to the Lie-Bäcklund symmetries with K. M. Tamizhmani (J. Math. Phys. **23**, 456 (1982); J. Phys **A16**, 3773 (1983); J. Math. Phys. **26**, 1189 (1985)) including perturbations around the solutions of the nonlinear evolution equations.

Parallely, Professor Lakshmanan continued his earlier studies on Heisenberg ferromagnetic spin systems, in different directions. With M. Daniel, he considered the spherically symmetric chain and formulated the problem of higher dimensional systems in terms of stereographic variables (Physica **107A**, 533 (1981)) and perturbed Heisenberg systems (Physica **120A**, 125 (1983)); with S. Ganesan on the integrability aspects of generalized inhomogeneous spin chains (Physica **132A**, 117 (1985)); with K. Porsezian on the effect of discreteness (Phys. Lett. **133A**, 483 (1988)) and integrability of spherically symmetric spin chains (Phys. Lett. **146A**, 305 (1990); J. Math. Phys. **32**, 2923 (1991)) and with Tamizhmani and R. Sahadevan on the integrability of coupled nonlinear Schrödinger equations occurring in optical soliton theory (J. Phys. **A19**, 1783 (1986)). This was also the time when Professor Lakshmanan along with R. Sahadevan started looking at the symmetry and singularity properties of finite dimensional nonlinear systems and based on these studies have made significant contributions in identifying integrable cases of coupled nonlinear oscillators (Phys. Rev. **A31**, 861 (1985); **A33**, 3563 (1986); J. Phys. **A19**, L949 (1986); Phys. Reports, **224**, 1 (1993)). This was continued further to understand the connection between generalized symmetries, integrability and separability with M. Senthil Velan (J. Phys. **A25**, 1259 (1992); J. Math. Phys. **33**, 4068 (1992)).

Professor Lakshmanan has all along been interested not only on the integrability aspects of nonlinear dynamical systems, but also on other aspects including bifurcations and chaos. In fact he has been stressing all the time that the subject should be studied in a unified way both for its integrability and chaos aspects together. However the total lack of computational facilities at Tiruchirapalli until late 1980s was a great impediment in Professor Lakshmanan's scheme of things. During 1984-85, Professor Lakshmanan had spent an year at Kyoto University in Japan and came into personal contact with several Japanese physicists. During this time he realized that it is imperative to study chaotic dynamical systems also in order to have a better perspective of nonlinear dynamics as a whole. With H. Hasegawa he started looking at the Kepler problem/hydrogen atom in external fields (J. Phys. **A17**, L889 (1984)), which then he continued further on returning to India with K. Ganesan in a series of papers on the classical, quasi-classical and quantum aspects of the chaotic dynamics of hydrogen atom in a generalized van der Waals potential (Phys. Rev. Lett. **62**, 232 (1989); Phys. Rev. **A42**, 3940 (1990); **A45**, 1548 (1992); **A48**, 964 (1993)). While in Japan, Professor Lakshmanan along with K. Nakamura had also made the seminal contribution to the topic of quantum chaos by formulating the'motion' equations for the eigenvalues and eigenfunctions of quantum chaotic systems and showed their equivalence with the dynamics of the completely integrable Calogero-Moser N-particle system with an internal complex vector space (Phys. Rev. Lett. **57**, 1661 (1986); Phys. Rev. **A46**, 6311 (1992)). At this time he had formulated again with K. Nakamura that the Landau-Lifshitz equation of ferromagnetism with Gilbert damping can be shown to be formally mappable on that of the no damping case but with a complex time (Phys. Rev. Lett. **53**, 2497 (1984)), which is yet another remarkable contribution to the topic of magnetic solitons.

During the next phase, starting around 1986, Professor Lakshmanan and several of his students have started giving equal importance in their study to chaotic dynamics. During a short span of about ten years several problems in the forefront have been taken up for investigation and impressive results obtained. These include analytical, numerical and experimental (through nonlinear electronic circuits) studies involving such topics as bifurcations, singularity structure, chaos, controlling, synchronization, secure communications, strange nonchaotic attractors and spatio-temporal patterns. New nonlinear electronic circuits were introduced and novel underlying phenomena were unravelled. With S. Rajasekar, existence of various kinds of bifurcations and new routes to chaos were established in the case of Bonhoeffer-van der Pol oscillator (J. Theor. Biology **133, 4**, 73 (1988); Physica **32D**, 146 (1988)) and FitzHugh-Nagumo equation (J. Theor. Biology **166**, 275 (1994)). Algorithms for controlling chaos were also investigated in detail with S. Rajasekar (Physica **D67**, 282 (1993)). With S. Parthasarathy, Professor Lakshmanan made a comprehensive analysis of singularity structure of chaotic nonlinear oscillators such as Duffing, driven pendulum and Morse oscillators (J. Phys. **A23**, L1223 (1990); Phys. Lett. **A157**, 365 (1991)) and with T. Bountis on the nonintegrability of Dufffing-van der Pol oscillator (J. Phys. **A26**, 6927 (1993)). He has also made a comprehensive analysis of the singularity structure aspects associated with the nonintegrability of Heisenberg spin system in a transverse magnetic field in collaboration with M. Daniel, M.D. Kruskal and K. Nakamura (J. Math. Phys. **33**, 771 (1992)).

A many faceted investigation on chaotic nonlinear electronic circuits was carried out with K. Murali, and later on in collaboration with Leon Chua, on the bifurcation, chaos, controlling and synchronization aspects of the driven Chua's circuit (Phys. Lett. **A151**, 412 (1990); Int. J. Bifurcation & Chaos **1**, 359 (1991); IEEE Transcations Circuits & Systems **I39**, 264 (1992); IJBC **3**, 1057 (1993)). Transmission of signals by chaos synchronization was demonstrated in different situations and secure communication aspects explored (Phys. Rev. **E48**, R1624 (1993); **E49**, 4882 (1994); IJBC **7**, 415 (1997); Phys. Rev. **E56**, 251 (1997); Phys. Lett. **A241**, 303 (1998)). The simplest dissipative nonautonomous chaotic circuit was proposed along with K. Murali and Leon Chua and its dynamical properties were explored (IEEE Transcations on Circuits & Systems **I41**, 462 (1994); IJBC**4**, 1511 (1994); **5**, 563 (1995)). The various developments concerned with the chaos studies are also reviewed in M. Lakshmanan and K. Murali, Phil. Trans. Roy. Soc. Lond **353**, 33 (1995); *Chaos in Nonlinear Oscillators*, World Scientific (1996).

In recent times Professor Lakshmanan has initiated further detailed studies with A. Venkatesan on several other chaotic nonlinear oscillators and identified transitions to chaos via strange nonchaotic attractors, identifying several new routes and mechanisms (Phys. Rev. **E 55**, 5134 (1997); **E56**, 6321 (1997); **E58**, 3008 (1998); Phys. Lett. A (1999) to appear). In recent times with P. Muruganandam, Professor Lakshmanan has also initiated a series of studies on spatio-temporal patterns, size dependent instabilities and chaos in arrays of nonlinear oscillators (Chaos **7**, 476 (1997); IJBC **9**, 805 (1999)) including nonlinear electronic circuits.

In a simultaneous program along with the above chaotic dynamical studies since 1986, Professor Lakshmanan and his group consisting essentially of M. Senthilvelan, R. Radha, R. Radhakrishnan and S. Vijayalakshmi along with the collaboration of R. Myrzakulov have been making concerted efforts on the integrability aspects of soliton systems both in (1+1) and (2+1) dimensions along with pioneering developments related to optical solitons. With M. Senthilvelan, symmetry aspects of several (2+1) dimensional evolution equations and associated infinite dimensional Kac-Moody symmetry algebras have been explored (J. Nonlinear Math. Phys **5**, 1 (1998)). Using Painlevé singularity structure analysis and Hirota bilinearization, the existence of localized dromion solutions in several (2+1) dimensional systems such as the generalized KdV, breaking soliton, (2+1) sine-Gordon, NLS class of equations have been demonstrated (J. Math. Phys. **35**, 4746 (1994); Phys. Lett. **A197**, 7 (1995); J. Phys. **A28**, 6997 (1995); **A29**, 1551 (1996); J. Math. Phys.**38**, 292 (1997)). An integrable (2+1) dimensional spin model was proposed and its relation to the (2+1) dimensional nonlinear Schrödinger equation and the nature of solutions were explored along with S. Vijayalakshmi and R. Myrzakulov (Phys. Lett. **A233**, 430 (1997); J. Math. Phys. **39**, 2122 (1998)). Further the connection between moving curves, surfaces and integrable (2+1) dimensional NLEEs was established (J. Math. Phys. **39**, 3765 (1998)).

Finally, in a series of important works with R. Radhakrishnan and in collaboration with M. Daniel, J. Hietarinta and A. Kundu, Professor Lakshmanan has been investigating the optical soliton propagation in birefringent fibers modelled by Manakov equations and its various generalizations (J. Phys. **A28**, 2683 (1995); **A28**, 7299 (1995); Phys. Rev. **E54**, 2949 (1996)). These studies ultimately resulted in the realization that solitons in the Manakov model and generalizations can undergo a kind of energy sharing/shape changing inelastic collisions in two mode fibers (Phys. Rev. **E56**, 2216 (1997); several papers to appear in Phys. Rev. E), leading to impressive implications in optical soliton communication and computation. Mention must also be made over a long term program Professor Lakshmanan has been carrying out with Professors Nanny and Per Olof Frömans of Uppsala on the application of higher order phase integral method to physical systems such as one and three dimensional quartic anharomonic oscillators (Phys. Rev. **D24**, 2586 (1981); Phys. Rev. **A49**, 3296 (1994)) and a comprehensive analysis of two centre Coulomb problem including the hydrogen molecular ion with N. Athavan (several manuscripts under publication).

To summarise, it has been rather hard for us to review in the best possible way the multifaceted and deep contributions of Professor Lakshmanan encompassing a multitude of areas in Theoretical Physics in which he has been working for almost three decades. This resumé should be taken only as a guideline as to what has been achieved by him rather than as a comprehensive write up on his remarkable career.

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