Calculus for College Students

second edition

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CALCULUS FOR COLLEGE STUDENTS

Second Edition

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PREFACE

This book is designed to meet the needs of a majority of students taking a two-semester (or three-quarter) course in calculus at a college or university. The text leans heavily on the intuitive approach, gives many illustrative examples, emphasizes physical applications wherever suitable, and has a large selection of graded exercises. Definitions and theorems are stated with care and proofs of simple theorems are given in full.

The changes in this second edition take into account the many suggestions we have received from teachers and students who used the first edition during the last several years.

In Chapter 1 we discuss inequalities with emphasis on the use of the absolute-value symbol. An important change occurs in this edition with our introduction of set notation, which we employ in Chapter 2 on functions and functional notation. It is important for the student to understand and develop facility in the use of set notation. We use it in those situations where genuine ambiguity could result from the employment of classical notation. However, there are times when the use of set notation is cumbersome and does not add to the understanding of the subject matter. On such occasions, and when the additional precision of set notation is not needed, we continue to use standard (traditional) notations.

Chapter 3 contains an intuitive introduction to limit and the development of the derivative. In Chapter 4 we give a precise treatment of limit and we state without proof the most useful basic properties. The intuitive material on integration, previously in Chapter 3, has been moved to the end of Chapter 6, immediately before the chapter devoted to the definition and calculation of integrals. These improvements in order and articulation result from the classroom experience of our colleagues and many other users of the text.

Chapters 5 and 6 give a thorough development of the differentiation of algebraic functions and applications to problems of maxima and minima, related rates, and approximation.

Chapter 7 begins with a careful definition of area (Jordan content). This leads to the definition of integral and to two forms of the Fundamental Theorem of Calculus. There are applications to problems of liquid pressure and work, but these may be omitted without loss of continuity in the presentaion.

The natural logarithm is defined by the integral, and the exponential function is defined as its inverse. These topics, as well as the differentiation and integration of trigonometric and inverse trigonometric functions, are taken up in Chapters 8 and 9. The sections on relations and inverse functions have been completely rewritten to obtain both greater precision and improved exposition.

Vectors in the plane are the subject of Chapter 12. Certain logical difficulties are avoided by defining a vector as an equivalence class of directed line segments. Furthermore, the discussion of equivalence classes puts this abstract concept in a natural setting. The statements and proofs of the theorems in this chapter have been revised extensively in this edition. For those who wish to study vectors early in the course, the material in Chapter 12 could easily be inserted after Chapter 4.

Chapters 13 and 14 discuss techniques in integration and their applications.

Chapter 15 is devoted to solid analytic geometry, with coordinates used throughout. Once the student has mastered this material, the applications using vectors in three dimensions, taken up in Chapter 16, may be attacked with confidence.

The study of infinite series, the subject of Chapter 17, completes the customary course in the calculus of functions of one variable. Chapters 18 and 19 are devoted to the initial topics in the calculus of functions of several variables. Partial differentiation, line integrals, and applications are taken up in Chapter 18. A definition of volume (Jordan content), analogous to that given for area in Chapter 7, is discussed in Chapter 19. The elements of multiple integration with applications to area, volume, and mass are treated. In addition there are a number of physical applications to problems concerning center of mass, moment of inertia, and so forth.

The last chapter contains an elementary study of linear algebra. This chapter replaces the unit on differential equations which formerly appeared at the end of most texts on calculus. The presentation here is intended as a beginning study. Additional material on linear algebra, as well as a broad selection of topics usually given in advanced calculus courses, may be found in our text *Modern Mathematical Analysis* (Addison-Wesley, 1964). Chapters 6 through 17 of that book cover topics in infinite series (including Fourier series), Green's and Stokes' theorems, linear transformations and their representations, and ordinary differential equations.

An important feature of the second edition is the addition of a large number of challenging exercises, which have been inserted at the end of various sections. Also, in order to increase the variety, many of the regular exercises have been changed.

The presentation of hyperbolic functions has been changed in this edition. All the material formerly in the body of the text has been placed in a special appendix (Appendix 3). In this way, the treatment of hyperbolic functions can be emphasized for those students in engineering and technology who wish it, while the subject can be skipped or assigned as outside reading to those students for whom the subject is of only marginal interest.

This edition contains several appendices which we hope will add to the versatility of the text. Appendix 1 discusses the axioms of algebra and number systems. This appendix not only is useful as additional reading

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1

INEQUALITIES

1. INEQUALITIES*

Almost all high school students learn plane geometry as a single logical development in which theorems are proved on the basis of a system of axioms or postulates. Unlike plane geometry, however, algebra has traditionally been taught in high school without the aid of a formal logical system. In this method the student simply learns a few rules—or many—for manipulating algebraic quantities; these rules lead to success in solving problems but do not shed any light on the structure of algebra. In recent years, however, mathematicians have developed a number of new experimental high school programs which present algebra in a logical manner analogous to the one first developed by Euclid for plane geometry.

The usual rules of algebra are logical consequences of the system of axioms known as the Axioms of Algebra. To prove the rules we use for manipulating algebraic expressions directly from the Axioms, as in Euclidean geometry, would be cumbersome and unwieldy. Therefore we shall assume that the reader is familiar with the usual laws of algebra and begin with a discussion of inequalities. The Axioms of Algebra are given in Appendix 4 at the end of the book, and we recommend their study to students unfamiliar with them.

In elementary algebra and geometry we study equalities almost exclusively. The solution of linear and quadratic algebraic equations, the congruence of geometric figures, and relationships among various trigonometric functions are topics concerned with equality. As we progress in the development of mathematical ideas—especially in that branch of mathematics of which calculus is a part—we shall see that the study of inequalities is both interesting and useful. An inequality is involved when we are more concerned with the approximate size of a quantity than we are with its true value. Since the proofs of some of the most important theorems in calculus depend on certain approximations, it is essential that we develop a facility for working with inequalities.

We shall be concerned with inequalities among real numbers, and we begin by recalling some familiar relationships. Given that a and b are any two real numbers, the symbol

$$a < b$$

means that a **is less than** b.† We may also write the same inequality in the *opposite direction*,

$$b > a,$$

which is read b **is greater than** a.

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* This chapter and Chapter 2 consist of review material for many students of calculus. Students who do not have a thorough working knowledge of inequalities should begin here. Readers familiar with inequalities may start with Chapter 2.

† Which is true if and only if $b - a$ is positive (see Appendix 1, §2).

The rules for handling inequalities can be proved on the basis of the Axioms for Algebra. The rules themselves are only slightly more complicated than the ones we learned in algebra for equalities. However, the differences are so important that we state them as four Theorems about Inequalities, and they must be learned carefully.

Theorem 1. *If $a < b$ and $b < c$, then $a < c$. In words: if a is less than b and b is less than c, then a is less than c.*

Theorem 2. *If c is any number and $a < b$, then it is also true that $a + c < b + c$ and $a - c < b - c$. In words: if the same number is added to or subtracted from each side of an inequality, the result is an inequality in the same direction.*

Theorem 3. *If $a < b$ and $c < d$ then $a + c < b + d$. That is, inequalities in the same direction may be added.*

It is important to note that in general inequalities may not be subtracted. For example, $2 < 5$ and $1 < 7$. We can say, by addition, that $3 < 12$, but note that subtraction would state the absurdity that 1 is less than -2.

Theorem 4. *If $a < b$ and c is any positive number, then*

$$ac < bc,$$

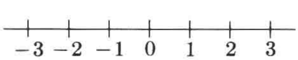
while if c is a negative number, then

$$ac > bc.$$

In words: multiplication of both sides of an inequality by the same positive number preserves the direction, while multiplication by a negative number reverses the direction of the inequality.

Since dividing an inequality by a number d is the same as multiplying it by $1/d$, we see that Theorem 4 applies for division as well as for multiplication.

From the geometric point of view we associate a horizontal axis with the totality of real numbers. The origin may be selected at any convenient point, with positive numbers to the right and negative numbers to the left (Fig. 1–1).



1–1

1–2

For every real number there will be a corresponding point on the line and, conversely, every point will represent a real number. Then the inequality $a < b$ may be read: *a is to the left of b.* This geometric way of looking at inequalities is frequently of help in solving problems. It is also helpful to introduce the notion of an *interval of numbers* or *points*. If a and b are numbers (as shown in Fig. 1–2), then the **open interval** from a to b is the collection