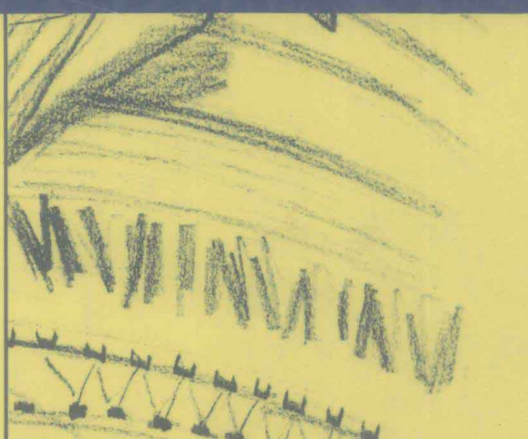
CHAOS

From Theory to Applications



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FROM THEORY TO APPLICATIONS

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Cover illustration: A painting by the renowned Greek painter Takis Alexiou. It depicts the order within strange attractors and the randomness they generate.

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To the smiles of my daughter Michelle

PREFACE

Based on chaos theory two very important points are clear: (1) random-looking aperiodic behavior may be the product of determinism, and (2) nonlinear problems should be treated as nonlinear problems and not as simplified linear problems.

The theoretical aspects of chaos have been presented in great detail in several excellent books published in the last five years or so. However, while the problems associated with applications of the theory—such as dimension and Lyapunov exponents estimation, chaos and nonlinear prediction, and noise reduction—have been discussed in workshops and articles, they have not been presented in book form.

This book has been prepared to fill this gap between theory and applications and to assist students and scientists wishing to apply ideas from the theory of nonlinear dynamical systems to problems from their areas of interest. The book is intended to be used as a text for an upper-level undergraduate or graduate-level course, as well as a reference source for researchers.

My philosophy behind writing this book was to keep it simple and informative without compromising accuracy. I have made an effort to present the concepts by using simple systems and step-by-step derivations. Anyone with an understanding of basic differential equations and matrix theory should follow the text without difficulty. The book was designed to be self-contained. When applicable, examples accompany the theory. The reader will notice, however, that in the later chapters specific examples become less frequent. This is purposely done in the hope that individuals will draw on their own ideas and research projects for examples.

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For me, writing this book was an experience. For the reader, I hope it will be a pleasure.

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CONTENTS

PART I: NOTES

PART II: THEORY

CHAPTER 5

CHAPTER 6

CHAPTER 7

PART III: APPLICATIONS

CHAPTER 8

PART I

NOTES

CHAPTER 1

INTRODUCTION

Simplicity and regularity are associated with predictability. For example, because the orbit of the earth is simple and regular, we can always predict when astronomical winter will come. On the other hand, complexity and irregularity are almost synonymous with unpredictability.

Those who try to explain the world we live in always hope that in the realm of the complexity and irregularity observed in nature, simplicity would be found behind everything and, that, finally, unpredictable events would become predictable. That complexity and irregularity exist in nature is obvious. We need only look around us to realize that practically everything is random in appearance. Or is it? Clouds, like many other structures in nature, come in an infinite number of shapes. Every cloud is different, yet everybody will recognize a cloud. Clouds, though complex and irregular, must on the whole possess a uniqueness that distinguishes them from other structures in nature. The question remains: Is their irregularity completely random, or is there some order behind their irregularity?

Over the last decades physicists, mathematicians, astronomers, biologists, and scientists from many other disciplines have developed a new way of looking at complexity in nature. This way has been termed *chaos* theory. Chaos is mathematically defined as "randomness" generated by simple deterministic systems. This randomness is a result of the sensitivity of chaotic systems to the initial conditions. However, because the systems are deterministic, chaos implies some order. This interesting "mixture" of randomness and order allows us to take a different approach in studying processes that were thought to be completely random. Apparently, the founders of chaos theory had a very good sense of humor, since *chaos* is the Greek word for the complete absence of order.

The mathematical foundations of what is now called chaos were laid down a long time ago by Poincaré in his work on bifurcation theory. However, due to the nonlinear character of the problems involved and the absence of computers, the discovery of chaos did not take place until 1963. That year Edward Lorenz published his monumental work entitled *Deterministic Nonperiodic Flow.* For the first time it was shown that a system of three nonlinear ordinary differential equations exhibits final states that are nonperiodic. Soon after that the theory of chaos developed to what many consider the third most important discovery in the 20th century after relativity and quantum mechanics.

First, the hidden beauty of chaos was revealed by studying simple nonlinear mathematical models such as the logistic equation, the Hénon map, the Lorenz system, and the Rössler system. Beautiful "strange attractors" that described the final states of these systems were produced and studied, and routes that lead a dynamical system to chaos were discovered.

After that the study of chaos moved to the laboratory. Ingenious experiments were set up, and low-dimensional chaotic behavior was observed. These experiments elevated chaos from being just a mathematical curiosity and established it as a physical reality.

The next step was to search for chaos outside the "controlled" laboratory—in nature. This presented an enormous challenge. Now the scientists had to deal with an "uncontrolled" system whose mathematical formulation was not always known accurately. Up to this point, the existence of low-dimensional chaos in physical systems has not been demonstrated beyond any doubt. Many indications have been presented, but a definite answer has not yet emerged. More work is needed in this area.

The acceptance of a new theory depends on its ability to make predictions. For example, the theory of relativity predicted that light must bend in the presence of a strong gravitational field. This prediction (among others) was soon verified, and the theory became widely accepted. Similar comments can be made about quantum mechanics and other accepted theories. Chaos theory tells us that nonlinear deterministic systems are sensitive to initial conditions and because of that their predictive power is lost very quickly. At the same time we have discovered that processes that appear random may be chaotic, and thus they should be treated as deterministic processes. Would it be possible that the underlying determinism of such processes could be used to improve their otherwise limited predictability? Many argued that if chaos was to make an impact it had to be used to obtain improved predictions. Lately, nonlinear prediction has become a major area of research, and some very exciting results have been reported. Other advances in the theory include the use of chaos to reduce noise in the data.

The book is divided into three parts. The first part (Chapters 1–4) reviews concepts from mathematics, physics, and fractal geometry that we will be using in later chapters. These concepts include stability analysis, conservative systems, and ergodic systems. The second part (Chapters 5–7) presents the fundamentals behind the theory of chaos. Chapter 5 introduces the reader to strange attractors and their characteristics. Chapter 6 provides an overview of bifurcation theory and routes to chaos. Chapter 7 is devoted to the existence of chaos in Hamiltonian, quantum, and partial differential equation (PDE) systems. The third part (Chapters 8–11) is dedicated to the applications of chaos theory. Chapter 8 is concerned with reconstructing the dynamics from observables. Here the "burning" question of the necessary number of points is treated in detail. Chapter 9 is devoted to the evidence of chaos in controlled and uncontrolled systems. Chapter 10 introduces the reader to the rapidly growing area of nonlinear prediction. Chapter 11 gives an introduction to two other important research areas, namely shadowing and noise reduction.