CONTEMPORARY MATHEMATICS

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Lie Algebras, Vertex Operator Algebras and Their Applications

International Conference in Honor of
James Lepowsky and Robert Wilson
on Their Sixtieth Birthdays
May 17–21, 2005
North Carolina State University
Raleigh, North Carolina

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Photograph of James Lepowsky and Robert Wilson is courtesy of Yi-Zhi Huang.

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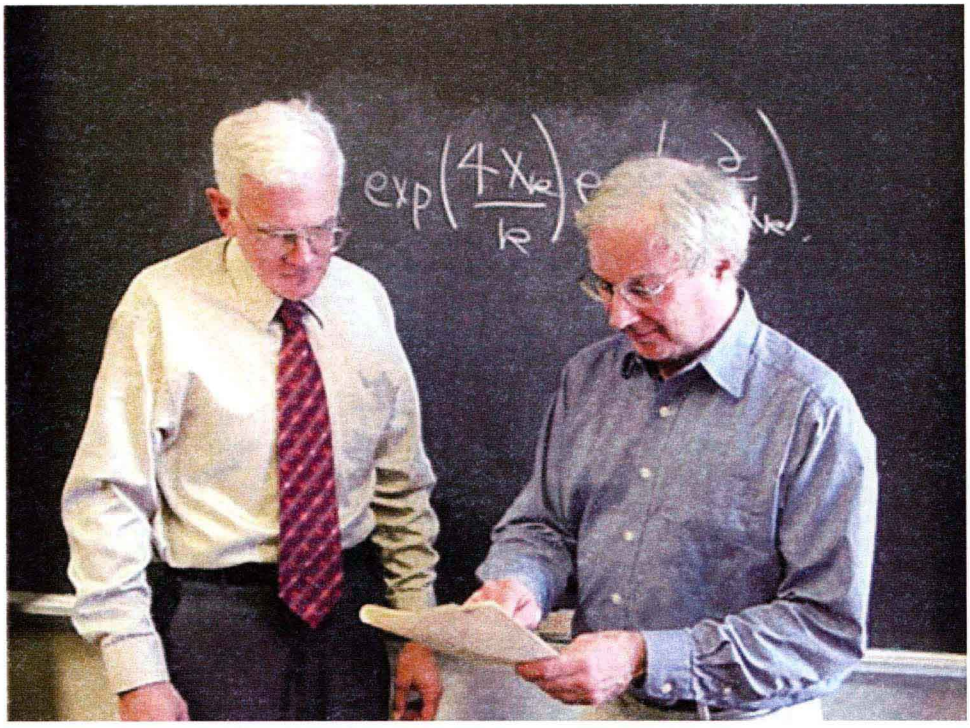

In honor of James Lepowsky and Robert Wilson
on their sixtieth birthdays

Preface

The representation theory of finite- and infinite-dimensional Lie algebras has been an important area of mathematical research with numerous applications in many areas of mathematics and physics. In the last few decades, the classification of the finite-dimensional simple modular Lie algebras has been completed (except for low characteristics) and infinite-dimensional Lie algebras such as Kac-Moody Lie algebras, the Virasoro algebra and their generalizations have been discovered and studied extensively, with exciting connections to many other fields of mathematics, including combinatorics, group theory, number theory, partial differential equations, topology, conformal field theory, statistical mechanics and integrable systems.

The interaction of an important class of infinite-dimensional Lie algebras known as affine Lie algebras with integrable systems led Drinfeld and Jimbo to introduce quantized universal enveloping algebras, also known as quantum groups, associated with symmetrizable Kac-Moody Lie algebras. The representation theory of quantum groups given by Lusztig exhibits similarity with Kac-Moody Lie algebras. In fact Kazhdan and Lusztig showed that the category of modules for a quantum group associated with a finite-dimensional simple Lie algebra at a root of unity is equivalent as a rigid braided tensor category to a suitable category of modules for the corresponding affine Lie algebra. The representation theory of affine Lie algebras together with the theory of the "moonshine module" constructed by Frenkel, Lepowsky and Meurman also led Borcherds to a mathematical definition of a new algebraic structure called a vertex (operator) algebra, which is a mathematically precise algebraic counterpart of the concept of what physicists came to call a "chiral algebra" in two-dimensional conformal field theory as formalized by Belavin, Polyakov and Zamolodchikov. These algebras and their representations play important roles in or have deep connections with a number of areas in mathematics and physics, including, in particular, the representation theory of the Fischer-Griess Monster finite simple group and the phenomena of "monstrous moonshine," the representation theory of the Virasoro algebra and affine Lie algebras, two-dimensional conformal field theory, modular functions, the theory of Riemann surfaces and algebraic curves, the geometric Langlands program, knot invariants and invariants of three-manifolds, quantum groups, monodromy associated with differential equations, mirror symmetry, elliptic genera and elliptic cohomology, topological field theories, and string theory.

During May 17-21, 2005, an international conference on "Lie algebras, vertex operator algebras and their applications" was held in North Carolina State University, in honor of James Lepowsky and Robert Wilson on their sixtieth birthdays. James Lepowsky and Robert Wilson have made enormous contributions, individually and jointly, to the development of both the theory of Lie algebras and the theory

of vertex operator algebras. In 1978, in their joint seminal work on the construction of representations of $\widehat{\mathfrak{sl}(2)}$, they discovered what are now called "twisted vertex operators," which were used in their vertex-operator-theoretic proof of the famous Rogers-Ramanujan identities. Subsequently this led them to the discovery of what they called "Z-algebras," which became an important tool for vertex-operator-theoretic proofs of combinatorial identities. Also, this sequence of ideas played a role in leading Lepowsky, jointly with Frenkel and Meurman, to a construction of the "moonshine module vertex operator algebra," whose automorphism group is the Monster. Most of Lepowsky's recent work involves the representation theory of vertex operator algebras and applications. On the other hand Wilson (largely in joint work with Block) made major contributions towards the classification of the finite-dimensional simple modular Lie algebras. In his most recent work, growing out of the theory of quasideterminants, Wilson has collaborated with Israel Gelfand, Vladimir Retakh and Shirlei Serconek in studying algebras related to polynomial equations and to graphs. In addition to being excellent researchers, Lepowsky and Wilson are also wonderful teachers and mentors. Jointly and individually they have supervised 27 Ph.D. students, and many of these students are now established researchers in their fields. The conference and these proceedings are dedicated to them for their important contributions to these fields and their efforts devoted to the training and mentoring of younger generations of mathematicians.

This conference brought together researchers from all over the world, including some former students and post-doctoral associates of James Lepowsky and Robert Wilson working on various aspects of Lie algebras, vertex operator algebras and their applications. Some of the speakers gave inspiring expository talks on the development and status of their respective research areas. Others outlined and explored challenges as well as future directions of research for the twenty-first century. The focus of the conference was mainly on Lie algebras, quantum groups, vertex operator algebras and their applications to number theory, combinatorics, integrable systems, conformal field theory and statistical mechanics. About one hundred researchers from Argentina, Australia, Brazil, Canada, China, Croatia, France, Germany, India, Israel, Italy, Japan, Korea, Sweden, United Kingdom and USA participated in this conference. The participation of a number of graduate students and junior researchers made the conference very lively and inspiring and the informal interactions among experts, junior researchers and graduate students made the conference a great success. There were about fifty invited and contributed talks given by senior, mid-career and junior researchers working on different aspects of Lie algebras, quantum groups, vertex operator algebras and their applications.

The present volume is the proceedings of this conference. The list of talks given at this conference is included in this volume. All the speakers were invited to contribute to these proceedings and many of them did so. For the convenience of the readers we have grouped these contributions into two broad categories: 1. Lie algebras and related topics. 2. Vertex (operator) algebras and related topics. Of course some of the papers could have been in either category. We hope that the papers in this volume will be beneficial to senior researchers as well as beginners and will inspire more research activities in these directions.

We are very grateful to the National Science Foundation and the College of Physical and Mathematical Sciences and the Mathematics Department at North

Carolina State University for the funding and support of this conference. We appreciate the Mathematics Department staff and the Conference Housing staff at North Carolina State University for their help during the conference. We thank all the participants, the speakers and especially the authors whose papers are included in this volume. Our special thanks to the anonymous referees for their careful reviews of the papers included in this volume.

Yi-Zhi Huang and Kailash C. Misra

Biographies of James Lepowsky and Robert Wilson

James Lepowsky was born on July 5, 1944 in New York City. He attended Stuyvesant High School and then Harvard College, 1961–65, where Shlomo Sternberg introduced him to Lie groups and encouraged him to go to M.I.T. and learn about Lie theory from Bertram Kostant. At M.I.T., Lepowsky decided very soon to study with Kostant, and he received his Ph.D. in 1970 under the joint direction of Sigurdur Helgason and Bertram Kostant. His thesis involved representations of real rank one groups, branching laws and minimal K-types.

Lepowsky was a lecturer and research associate at Brandeis University from 1970 to 1972. Around this time, he collaborated with Gerald McCollum and with Nolan Wallach on problems in Lie theory and representation theory. He was an assistant professor at Yale University from 1972 to 1977, including a year as a Yale Junior Faculty Fellow at the Institute for Advanced Study, where he has also been a member during four other periods. At Yale he initiated a study of the structure of "generalized Verma modules" and he developed generalizations of the Bernstein-Gelfand-Gelfand resolution. Howard Garland stimulated Lepowsky's interest in Macdonald's identities and homology. Using the results of collaborations with Garland, Stephen Milne and Alex Feingold at Yale, Lepowsky began collaborating with Robert Wilson, who was on leave from Rutgers in 1976–77, on what became twisted vertex operator realizations of affine Lie algebras and structures they called "Z-algebras," with applications to partition identities.

After lecturing at Université Paris VI in 1978 during the second year of a Sloan Fellowship, Lepowsky began teaching at Rutgers, where he continued collaborating with Wilson, and where, with Arne Meurman, his first Ph.D. student, he began working on what became a collaboration with Igor Frenkel and Meurman on the construction of a "moonshine module" for the Monster group, based on a new algebra of vertex operators. Some of this work was done when Frenkel, Lepowsky and Meurman were at the Mathematical Sciences Research Institute in 1983–84, and their monograph on this work was completed while the authors were at the Institute for Advanced Study in 1987–88, at which time Lepowsky was a Guggenheim Fellow.

Lepowsky has also written monographs with Mirko Primc; with Frenkel and Yi-Zhi Huang, his Rutgers colleague and former student; with Chongying Dong; and with his and Wilson's former student Haisheng Li. Huang and Lepowsky have developed a tensor category theory for suitable classes of modules for a vertex operator algebra. Most of Lepowsky's recent work involves the representation theory of vertex operator algebras and applications.

Robert Wilson was born on January 16, 1946 in Washington, D.C. and attended public schools in Maryland. While he was an undergraduate at The American University in Washington, D.C. from 1962 to 1965, Professor Irving Katz, his professor and mentor, introduced him to abstract algebra and, in particular, to the theory of Lie algebras (by supervising a reading course using Jacobson's recently published *Lie Algebras*). While a graduate student at Yale University from 1965 to 1969, Wilson studied under Nathan Jacobson, George Seligman and Robert Steinberg and was present when Alexei Kostrikin gave lectures in May 1967 describing his seminal work with Igor Shafarevich introducing Lie algebras of Cartan type in prime characteristic. Wilson's thesis, *Nonclassical Simple Lie Algebras*, was written under the direction of George Seligman.

Wilson, a Courant Instructor at New York University from 1969 to 1971, came to Rutgers in 1971 and has remained there since. For two decades he worked extensively on the structure and classification theory of simple Lie algebras in prime characteristic. Large parts of this work, including the classification of the simple restricted Lie algebras over algebraically closed fields of characteristic > 7, were done jointly with Richard Block.

While visiting Yale University (during a sabbatical leave from Rutgers in 1976–77), Wilson was introduced to the theory of affine Lie algebras by Howard Garland, and began a collaboration with James Lepowsky in which they originated the theory of vertex operator representations of affine Lie algebras and used this theory and generalizations to give a vertex-operator theoretic proof of the Rogers-Ramanujan identities and vertex-operator theoretic interpretations of the Gordon-Andrews-Bressoud generalizations of the RR identities. Some of this work was done while Wilson was a Member at the Institute for Advanced Study in fall 1980 and again in 1987–88, and during visits to MSRI during 1983–84. Wilson also collaborated with Earl Taft and David Radford on the construction of certain families of Hopf algebras. In his most recent work, growing out of the theory of quasideterminants, he has collaborated with Israel Gelfand, Vladimir Retakh and Shirlei Serconek in studying algebras related to polynomial equations and to graphs.

Between 1990 and 2003, Wilson devoted much of his time to academic administration at Rutgers, serving as Chair of the Mathematics Department from 1990 to 1993 and then holding several positions (ultimately Vice-Dean) in the office of the Dean of the Faculty of Arts and Sciences.

List of Ph.D. students advised by James Lepowsky and Robert Wilson

Ph.D. students advised by James Lepowsky:

Arne Meurman, 1981
David Mitzman, 1983
Richard Pfister, 1984
Leila Figueiredo, 1986
Haruo Tsukada, 1988 (co-advised with Igor Frenkel)
Cristiano Husu, 1990
Yi-Zhi Huang, 1990
Hong Guo, 1995
Katrina Barron, 1996 (co-advised with Yi-Zhi Huang)
Galin Georgiev, 1996
Antun Milas, 2001
Lin Zhang, 2004
Corina Calinescu, 2006

Ph.D. students advised by Robert Wilson:

David Kopcso, 1974
Mark Hunacek, 1978
Shirlei Serconek, 1980
Kailash Misra, 1982
Marly Mandia, 1986
Shari Prevost, 1989
Yasmine Sanderson, 1995 (co-advised with Olivier Mathieu)
David Nacin, 2005

Ph.D. students co-advised by James Lepowsky and Robert Wilson:

Stefano Capparelli, 1988
Xiaoping Xu, 1992
Elizabeth Jurisich, 1994
Haisheng Li, 1994
Chuanfu Xie, 1994
Wanglai Li, 1997

List of talks

Iana Anguelova
Quantum Vertex Algebras

Katrina Barron
Deformations of the $N = 2$ Neveu-Schwarz algebra and even and odd spectral flow on the worldsheet geometry of $N = 2$ superconformal field theory

Karin Baur
Admissible characters in the sense of Lynch

Georgia Benkart
Perfect crystals

Stephen Berman
Covering Algebras

Richard E. Block
Dual coalgebras and cofree coalgebras

Corina Calinescu
Principal subspaces of representations of affine Lie algebras and vertex operator algebras

Vyjayanthi Chari
Weyl, Demazure and Kirillov-Reshetikhin modules

Chongying Dong
On the uniqueness of the moonshine vertex operator algebra

Rolf Farnsteiner
Affine quivers, polyhedral groups, and representation type

Jörg Feldvoss
Existence of triangular Lie bialgebra structures

Igor Frenkel
Instanton and quiver constructions of affine Lie algebra representations and special bases

Avital Frumkin
A probability approach to Klyachko theorem

Jürgen Fuchs
An analogue of the Verlinde formula for non-rational vertex algebras

Howard Garland
Counting the number of rational points in certain moduli spaces of vector bundles on surfaces

Dimitar Grantcharov
On the automorphisms of Kac-Moody Lie superalgebras

Ayumu Hoshino
Polyhedral realizations of crystal bases for modified quantum algebras of arbitrary rank 2 cases

Keith Hubbard
Vertex operator coalgebras and their relations to VOAs

Dijana Jakelic
Branched crystals and the category $\mathcal{O}$

Seok-Jin Kang
Nakajima's monomials and crystal bases

Rinat Kedem
Kostka polynomials and representations of affine algebras

Apoorva Khare
The BGG category $\mathcal{O}$ over a skew group ring

Alexander Kirillov, Jr.
Wess-Zumino-Witten model as an orbifold theory

Stefan Kolb
Quantum symmetric pairs and the reection equation

Bertram Kostant
Powers of the Dedekind-η function and representation theory

Michael Lau
Bosonic and fermionic representations

James Lepowsky
Some developments in vertex operator algebra theory, old and new

Haisheng Li
Quantum vertex algebras associated with Zamolodchikov-Faddeev algebras

Yiqiang Li
A description of affine canonical bases

Geoffrey Mason
Vertex algebras and partition functions

Olivier Mathieu
Growth for groups and Lie algebras

Arne Meurman
On generators and relations for vertex operator algebras

Antun Milas
Number theoretic properties of Andrews-Gordon series

Tetsuji Miwa
Sklyanin's algebra and trace functional

Adriano Moura
q-characters of fundamental representations of quantum affine algebras

David Nacin
Partially commuting algebras and their connections to Q_n

Toshiki Nakashima
Affine Geometric Crystals and Tropical R

Erhard Neher
Central extensions of Lie tori

Deepak Parashar
Coloured Yang-Baxter operators

Brian Parshall
Some results on Specht modules and algebraic groups

Mirko Primc
Combinatorial bases of Feigin-Stoyanovsky type subspaces and interwining operators

Alexander Retakh
Modular conformal algebras

Vladimir Retakh
Factorizations of polynomials over noncommutative rings and algebras associated with quivers

Eric Rowell
Modular tensor categories: towards classification

Natasha Rozhkovskaya
Central elements of the universal enveloping algebra of $\mathfrak{gl}(n, \mathbb{C})$ and Yangians

Alistair Savage
Branching rules and quiver varieties

Christoph Schweigert
Twining characters and Picard groups in rational conformal field theories

Shirlei Serconek
Distributivity of lattices of defining relations

Bin Shu
Cartan invariants and blocks for Zassenhaus algebras

Helmut Strade
The classification of the simple Lie algebras over fields of positive characteristic: history, state of art, outlook

David Taylor
Trace functions of integrable modules over classical subalgebras of $\widehat{\mathfrak{gl}}_\infty$

Imre Tuba
Reconstructing braided semisimple tensor categories

Monica Vazirani
Vanishing integrals of Macdonald polynomials

Weiqiang Wang
Hilbert schemes, vertex operators, and integrable hierarchies

Robert Wilson
Modules and combinatorial identities

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