



Engineering Mechanics *Dynamics*

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Engineering Mechanics

DYNAMICS

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Preface

Mechanics Reform

In writing this text, we first and foremost believe that the fundamental principles of engineering mechanics have not changed. We have not tried to revolutionize statics or dynamics. However, the practice of engineering has evolved largely because of the power that computers and software have brought to engineering. In particular, computational software, such as MATLAB®, Mathcad®, Maple®, and Mathematica®, is now a significant tool in the curriculum of most engineering schools, and has changed the way an engineer handles calculations as the slide rule and calculator once did. We wrote this text with the idea that a new undergraduate text in mechanics should reflect this changing aspect of engineering education. We hope to reform the presentation of mechanics, and bring the material up to date with modern engineering practice.

Our presentation focuses on understanding mechanics conceptually as opposed to rote memorization of formulae or duplication of a particular sample problem, and we use computational software as a tool to gain this insight. We believe traditional texts rely too heavily on homework problems that specify geometry and loading and miss the important transition to design concepts. By using computational software, students are free to explore the parameters of a problem or an example, explorations which truly form the basis of engineering design. In our own courses, we have found that students gain a greater insight into mechanics by solving a problem for every angle or dimension, and then graphing the solution to see the effects of parametric changes. First, we focus our students on modeling, using free-body diagrams, and then writing the equations of motion and the constraint equations. Then, students use computational software to help solve numerical problems, and most importantly, investigate design parameters. By using software, we have found that students learn more about both fundamental mechanics and design.

MATLAB, MathCAD, Maple, And Mathematica Supplements

In the original draft of this text, we integrated applications from particular computational software programs into the text itself, that is, where

appropriate, the sample problem in the book included instruction in the use of a certain computational package. But we decided that this made the text too restrictive, and instead created an approach where integrated, software-specific supplements work hand in hand with the text. Students study the basic mechanics in the text, and refer to a software specific supplement that shows how using either MATLAB, Mathcad, Maple, or Mathematica can enrich and expand their understanding of the concept. These softbound supplements are packaged with the text for free. Professors choose the appropriate text/supplement bundle depending on what software package is emphasized in their department. The choices include the following.

Supplements to Accompany Engineering Mechanics: Statics:

- MATLAB Supplement for Statics (ISBN 0-13-794116-1)
- Mathcad Supplement for Statics (ISBN 0-13-794124-2)
- Mathematica Supplement for Statics (ISBN 0-13-794132-3)
- Maple Supplement for Statics (ISBN 0-13-011427-8)

Supplements to Accompany Engineering Mechanics: Dynamics:



- MATLAB Supplement for Dynamics (ISBN 0-13-794157-9)
- Mathcad Supplement for Dynamics (ISBN 0-13-794165-X)
- Mathematica Supplement for Dynamics (ISBN 0-13-794173-0)
- Maple Supplement for Dynamics (ISBN 0-13-011428-6)

Teaching Dynamics With This Text The study of dynamics of particles and rigid bodies involve three major parts:

- the modeling of the body through the use of a free-body diagram,
- the formulation of the equations of motion and the constraint equations,
- and the solution of the differential equations to determine the motion of the body.

Traditional texts do an excellent job on the first two aspects, but solve only the simplest differential equations. In the past, the inability to solve nonlinear differential equations restricted many solutions to a quasi-static approach, where the acceleration was determined at the “instant of release” or “at a particular position or time.” This has always restricted the student from fully understanding the motion of the object, the true beauty of dynamics. All computational software packages contain methods for the solution of an initial value differential equation, linear or nonlinear. Since the solution of the differential equations of motion is essential to the full use of dynamics, we have devoted considerable time to the differential equations in Chapter 1. We have stressed analytical solutions of the differential equations where such solutions can be obtained. Although a Runge-Kutta numerical solution is much more accurate than the simple Euler or

tangent line method, most solutions in the computational supplements and in the solutions manual are solved using the Euler method. We have found that it is easy for the students to understand and can be presented with minimum loss of class time. Since all computational software has a Runge-Kutta function, this method can be introduced to the student at any time and we have included its use in the computational supplements.

Problem Sets We have included a wealth of problems in this text that test a student's understanding of the basic principles of engineering. We have also included many motivating engineering applications in these problems that give students an understanding of "real-world" mechanics. Many of the problems are given with general geometries and loadings and we encourage the students to obtain the differential equations of motion and the constraint equations in terms of these parameters. If there is an analytical solution to the differential equation, we ask that the students determine the general solution of the problem. When computational software is necessary to solve the problem, we give the student numerical values of the parameters or ask the student to choose parameters such that a particular motion will result. In this manner, we are able to introduce design concepts in the course. We encourage students to use both traditional and computational approaches highlighted in the companion manuals to solve these problems. We have marked some problems with a single computer icon () where using software is particularly useful. Other problems that require the use of computational software are noted with a double computer icon ()

A Tested Approach Adopting a new mechanics text can seem like a risky proposition. However, we have used this approach to teach Mechanics courses at our schools over the last four years, and in doing so, have thoroughly refined this book so it is ready for publication. In addition, this manuscript also went through an initial preliminary edition to guarantee accuracy. Working with the publisher, we set actual text pages and produced a softbound version of this book that we used for a year at our schools and at several other institutions. Only after thoroughly reviewing and analyzing our results, did we produce this final product.

Dictionary There is a dictionary of terms at the end of the text with an index where terms occur in the text. We feel this will make the text a more valuable reference for students in other courses.

Supplements Instructors solutions manuals for the text are available through Prentice Hall for both *Statics* and *Dynamics*. They contain worked out solutions to all the problems as well as Software solutions in Mathcad

and MATLAB to those homework problems marked with computer icons. They are:

- Engineering Mechanics: Statics Solutions Manual CD (0-13-794108-0)
- Engineering Mechanics: Dynamics Solutions Manual CD (0-13-794140-4)

The CD can be used to copy and print transparencies of the solutions for classroom presentations. We are also developing a web page located at www.prenhall.com/soutas to further support this text, and answer your questions and comments. We, of course, will post any errata that we find in the first printing at this site.

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We welcome comments, corrections, and suggestions. We believe mechanics education is evolving, and hope our text will too.

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