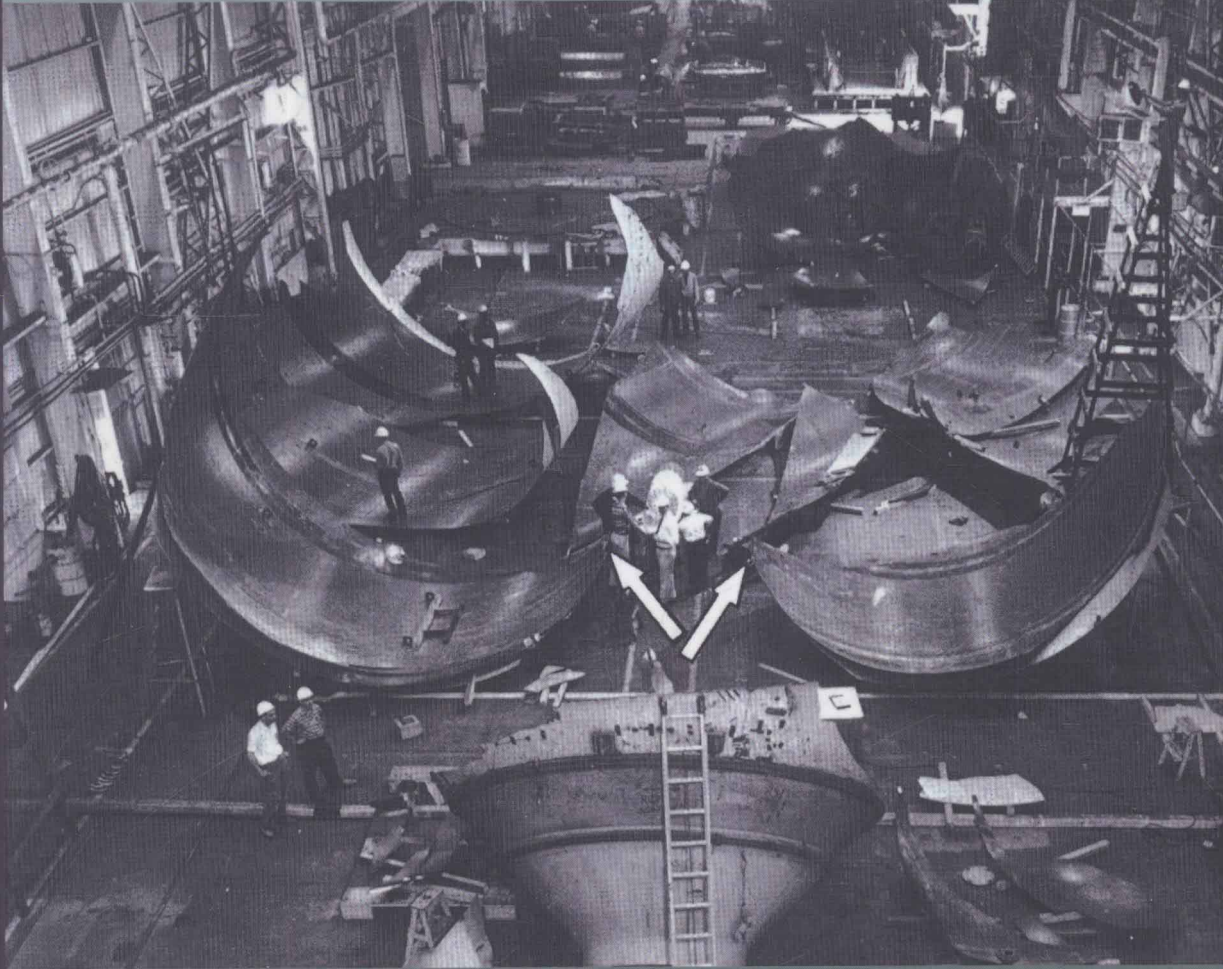


ROBERT P. WEI



# FRACTURE MECHANICS

Integration of  
Mechanics,  
Materials Science,  
and Chemistry

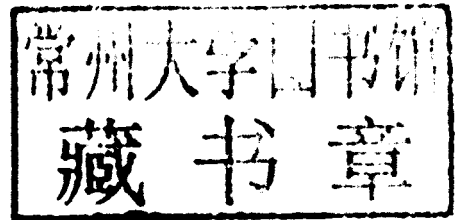
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# Fracture Mechanics

**INTEGRATION OF MECHANICS,  
MATERIALS SCIENCE, AND CHEMISTRY**

**Robert P. Wei**

Lehigh University



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## **FRACTURE MECHANICS**

Fracture and “slow” crack growth reflect the response of a material (i.e., its microstructure) to the conjoint actions of mechanical and chemical driving forces and are affected by temperature. Therefore, there is a need for quantitative understanding and modeling of the influences of chemical and thermal environments, and of microstructure, in terms of the key internal and external variables and for their incorporation into design and probabilistic implications. This text, which the author has used in a fracture mechanics course for advanced undergraduate and graduate students, is based on the work of the author’s Lehigh University team whose integrative research combined fracture mechanics, surface and electrochemistry, materials science, and probability and statistics to address a range of fracture safety and durability issues on aluminum, ferrous, nickel, and titanium alloys, and ceramics. Examples from this research are included to highlight the approach and applicability of the findings in practical durability and reliability problems.

Robert P. Wei is the Reinhold Professor of Mechanical Engineering and Mechanics at Lehigh University. His principal research is in fracture mechanics, including chemical, microstructural, and mechanical considerations of stress corrosion cracking, fatigue, and corrosion, and in life-cycle engineering. He is the author of hundreds of refereed research publications. He is a Fellow of the American Society for Testing and Materials; the American Society of Metals International; and the American Institute of Mining, Metallurgical, and Petroleum Engineering and a member of Sigma Xi and the Phi Beta Delta International Honor Societies.

***To Lee***

*For her love, counsel, dedication, and support*

## Preface

Engineering Fracture Mechanics, as a recognized branch of engineering mechanics, had its beginning in the late 1940s and early 1950s, and experienced major growth through the next three decades. The initial efforts were driven primarily by naval and aerospace interests. By the end of the 1980s, most of the readily tractable mechanics problems had been solved, and computational methods have become the norm in solving practical problems in fracture/structural integrity. On the lifeing (“slow” crack growth) side, the predominant emphasis has been on empirical characterization and usage of data for life prediction and reliability assessments.

In reality, fracture and “slow” crack growth reflect the response of a material (*i.e.*, its microstructure) to the conjoint actions of mechanical and chemical driving forces, and are affected by temperature. The need for quantitative understanding and modeling of the influences of chemical and thermal environments and of microstructure (*i.e.*, in terms of the key *internal* and *external* variables), and for their incorporation into design, along with their probabilistic implications, began to be recognized in the mid-1960s.

With support from AFOSR, ALCOA, DARPA, DOE (Basic Energy Sciences), FAA, NSF, ONR, and others, from 1966 to 2008, the group at Lehigh University undertook integrative research that combined fracture mechanics, surface and electrochemistry, materials science, and probability and statistics to address a range of fracture safety and durability issues on aluminum, ferrous, nickel, and titanium alloys and on ceramics. Examples from this research are included to highlight the approach and applicability of the findings in practical problems of durability and reliability. An appended list of publications provides references/sources for more detailed information on research from the overall program.

The title *Fracture Mechanics: Integration of Fracture Mechanics, Materials Science, and Chemistry* gives tribute to those who have shared the vision and have contributed to and supported this long-term, integrative effort, and to those who recognize the need and value for this multidisciplinary team effort.

The author has used the material in this book in a fracture mechanics course for advanced undergraduate and graduate students at Lehigh University. This book should also serve as a reference for the design and management of engineered systems.

## Acknowledgments

The author acknowledges the invaluable contributions and dedication of his colleagues: Dr. Ye T. (Russell) Chou (Materials Science), Dr. Kamil Klier (Surface Chemistry), Dr. Gary Simmons (Surface Chemistry), Dr. D. Gary Harlow (Probability and Statistics/Mechanical Engineering & Mechanics), and Dr. Ming Gao (Materials Science), and the many postdoctoral researchers and graduate students in Mechanical Engineering and Mechanics, Materials Science and Engineering, and Surface Science and Electrochemistry, who made this possible. The author also acknowledges the International Multimedia Resource Center (IMRC) of Lehigh University, under the leadership of Johanna Brams, especially Nyko DePeyer and Dawn Dayawon, for their assistance in graphic arts and manuscript preparation, and Sharon Siegler, Lehigh University librarian, for her counsel and expert assistance.

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# 1 Introduction

Fracture mechanics, or the mechanics of fracture, is a branch of engineering science that addresses the problem of the integrity and durability of materials or structural members containing cracks or cracklike defects. The presence of cracks may be real, having been introduced through the manufacturing processes or during service. On the other hand, their presence may have to be assumed because limitations in the sensitivity of nondestructive inspection procedures preclude full assurance of their absence. A perspective view of fracture mechanics can be gained from the following questions:

- How much load will it carry, with and without cracks? (a question of *structural safety and integrity*).
- How long will it last, with and without cracks? Alternatively, how much longer will it last? (a concern for *durability*).
- Are you sure? (the important issue of *reliability*).
- How sure? (*confidence level*).

The corollary questions are as follows, and will not be addressed here:

- How much will it cost? To buy? (capital or acquisition cost); to run? (operational cost); to get rid of? (disposal/recycling cost)
- Optimize capital (acquisition) costs?
- Optimize overall (life cycle) cost?

These questions appear to be simple, but are in fact profound and difficult to answer. Fracture mechanics attempts to address (or provides the framework for addressing) these questions, where the presence of a crack or cracklike defects is presumed.

The first of the questions deals with the stability of a crack under load. Namely, would it remain stable or grow catastrophically? The second question deals with the issue: “if a crack can grow stably under load, how long would it take before it reaches a length to become unstable, or become unsafe?” The third question, encompassing the first two, has to do with certainty; and the last deals with the confidence in the answers. These questions lead immediately to other questions.

Can the properties that govern crack stability and growth be computed on the basis of first principles, or must they be determined experimentally? How are these properties to be defined, and how well can they be determined? What are the variations in these properties? If the failure load or crack growth life of a material can be measured, what degree of certainty can be attached to the prediction of safe operating load or serviceable life of a structural component made from that material?

### 1.1 Contextual Framework

In-service incidents provide lasting reminders of the “aging” of, or cracking in, engineered systems. Figure 1.1 shows the consequence of an in-flight rupture of an eighteen-foot section of the fuselage of an Aloha Airlines 737 aircraft over the Hawaiian Islands in 1988. The rupture was attributed to the “link up” of extensive fatigue cracking along a riveted longitudinal joint. Fortunately, the pilots were

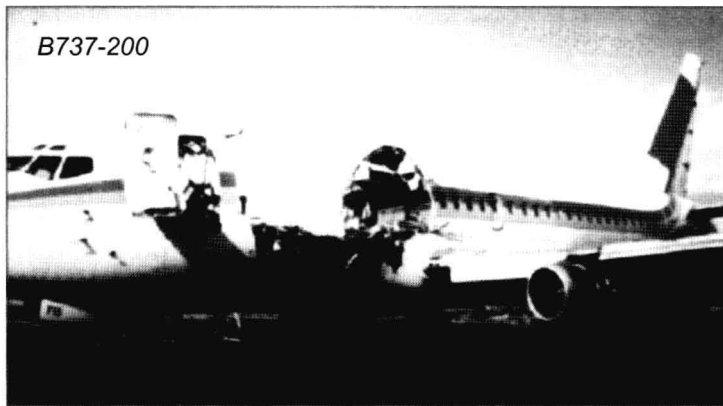


Figure 1.1. In-flight separation of an upper section of the fuselage of a B737-200 aircraft in 1988 attributed to corrosion and fatigue.

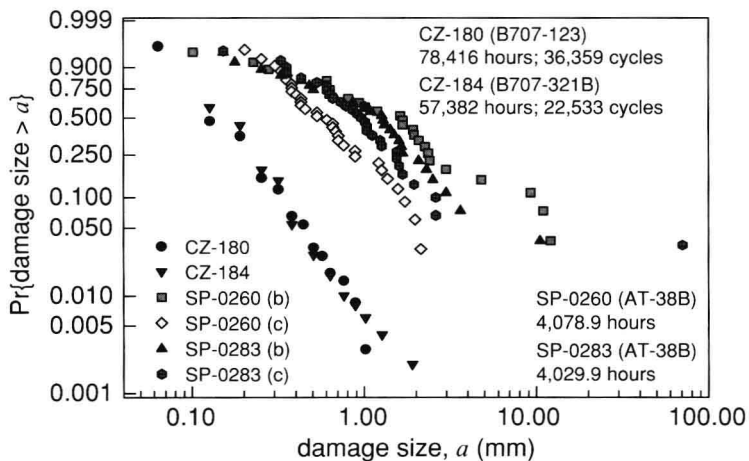


Figure 1.2. Damage distribution in aged B707 (CZ-180 and CZ184) after more than twenty years of service, and AT-38B aircraft after more than 4,000 hours of service [3].

able to land the aircraft safely, with the loss of only one flight attendant who was serving in the cabin. Tear-down inspection data on retired commercial transport and military aircraft [1, 2] (Fig. 1.2), provide some sense of the damage that can accrue in engineered structures, and of the need for robust design, inspection, and maintenance.

On the other end of the spectrum, so to speak, the author encountered a fatigue failure in the “Agraph” of a chamber grand piano (Figs. 1.3 and 1.4). An Agraph is typically a bronze piece that supports the keyboard end of piano strings (wires). It

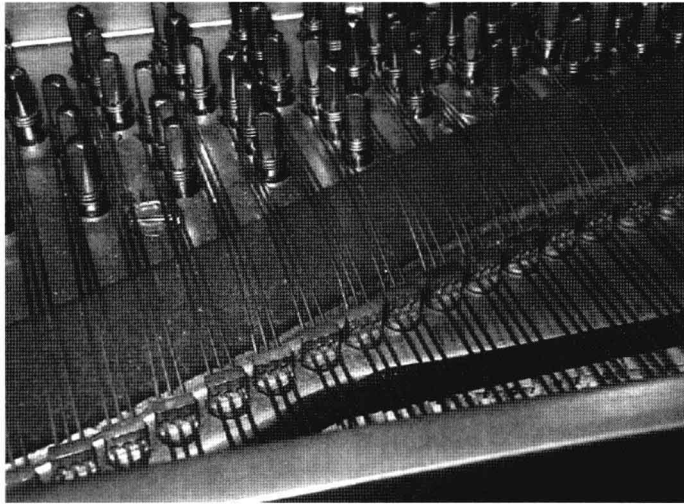


Figure 1.3. Interior of a chamber grand piano showing a row of Agraphs aligned just in front of the red velvet cushion.

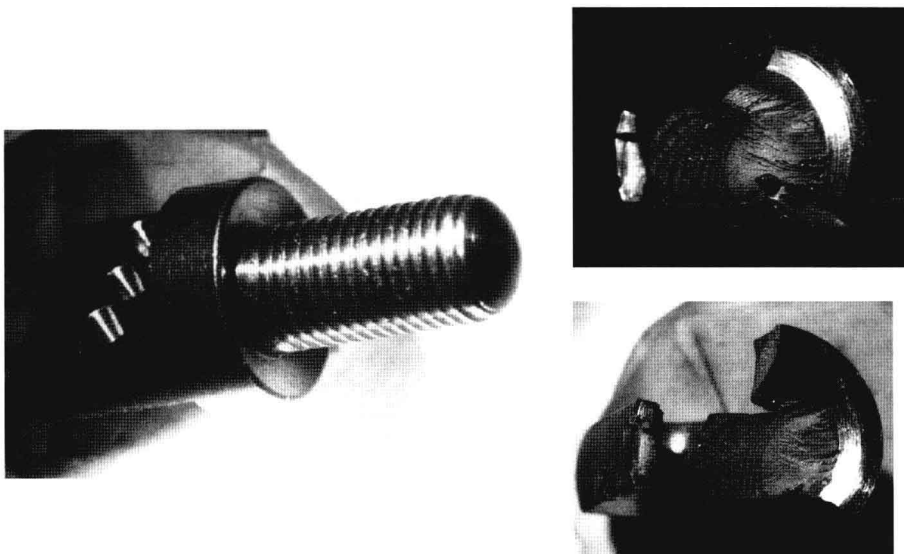


Figure 1.4. (left) Photograph of a new Agraph from a chamber grand piano, and (right) scanning electron micrographs of the mating halves of a fractured Agraph showing fatigue markings and final fracture.



sets the effective length of the strings and carries the effect of tension in the strings that ensures proper tuning. As such, it carries substantial static (from tuning tension) and vibratory loads (when the string is struck) and undergoes fatigue.

## 1.2 Lessons Learned and Contextual Framework

Key lessons learned from aging aircraft and other research over the past four decades showed that:

- Empirically based, discipline-specific methodologies for design and management of engineered systems are not adequate.
- Design and management methodologies need to be science-based, much more holistic, and better integrated.

Tear-down inspections of B-707 and AT-38B aircraft [1, 2] showed:

- The significance of localized corrosion on the evolution and distribution of fatigue damage was not fully appreciated.
- Its impact could not have been predicted by the then existing and current technologies.

As such, transformation in thinking and approach is needed.

Fracture mechanics need to be considered in the context of a modern design paradigm. Such a contextual framework and simplified flow chart is given in Fig. 1.5. The paradigm needs to address the following:

- Optimization of life-cycle cost (*i.e.*, cost of ownership)
- System/structural integrity, performance, safety, durability, reliability, etc.
- Enterprise planning
- Societal issues (*e.g.*, environmental impact)

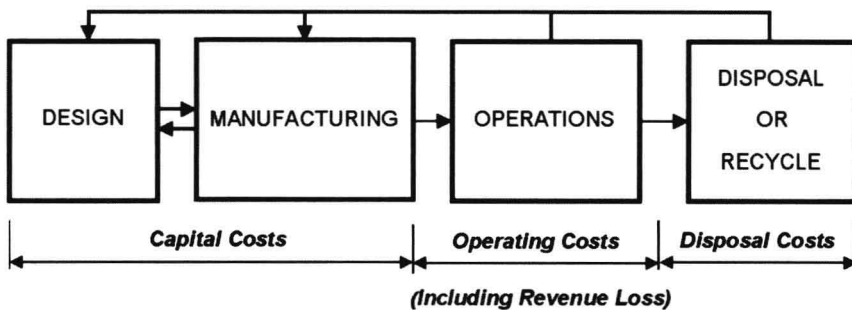


Figure 1.5. Contextual framework and simplified flow diagram for the design and management of engineered systems.

A schematic flow diagram that underlies the processes of reliability and safety assessments is depicted in Fig. 1.6. The results should be used at different levels to aid in operational and strategic planning.