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MATHEMATICS for ECONOMISTS

EDWARD T. DOWLING

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by

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To my mother, May Finegan Dowling,
and to the memory of my father, Edward T. Dowling, M.D.

EDWARD T. DOWLING is Associate Professor of Economics at Fordham University, where he has been a member of the faculty since 1973. He served as Assistant Chairman of the Economics Department from 1975–1979 and was elected Chairman of the Department in 1979. His major areas of interest are economic development and mathematical economics. Dr. Dowling received his Ph.D. from Cornell University and has spent six years teaching and doing research in Southeast Asia. He has published articles in economic journals and is coauthor with Dominick Salvatore of *Schaum's Outline of Development Economics*. He is also a Jesuit priest, and a member of the Jesuit Community at Fordham.

Schaum's Outline of Theory and Problems of
MATHEMATICS FOR ECONOMISTS

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Preface

The importance of mathematics in the study of economics today requires that the student be familiar with a wide variety of mathematical concepts. *Mathematics for Economists* is designed to fill this need by presenting a thorough, easily understood introduction to differential and integral calculus, matrix algebra, linear programming, differential equations, and difference equations, with applications to economic problems. Since textbooks differ in the order in which differential calculus and linear algebra are presented, Chapters 10 and 11 on linear algebra have been designed for coverage after Chapter 2 if desired, with no loss in continuity.

The theory-and-solved-problem format of each chapter provides concise explanations illustrated by examples, plus numerous problems with fully worked-out solutions. The topics and related problems range in difficulty from simpler mathematical operations to sophisticated applications. No mathematical proficiency beyond the high-school level is assumed. This learning-by-doing pedagogy will enable students to progress at their own rate and adapt the book to their own needs.

Mathematics for Economists is intended primarily as a supplement for undergraduate and graduate students in economics and business. In addition, its comprehensive nature makes it appropriate for use by students of mathematics and the social sciences; it will also serve as a useful guide in preparing for the mathematical proficiency exams.

Mathematics for Economists is the newest title in the Schaum's Outline Series in Economics. The series includes *Microeconomic Theory*, *Macroeconomic Theory*, *Development Economics*, and *International Economics*, as well as the forthcoming *Principles of Economics*.

Because I could not have completed this book alone, I wish to express my deep gratitude to my colleague, Dr. Dominick Salvatore, for his continued support, interest, and availability; and to Sister Mary Immaculate Occhipinti, C.S.A.C., for her patience and diligence in typing the manuscript. I am grateful also to the graduate students at Fordham, especially Mary Acker, Edward Barbour, Evelyn Grossman, Rosemary Thomas, and Ann Waldron, for their helpful contributions throughout the development of the manuscript. Finally, I should like to thank the entire McGraw-Hill staff for their kind assistance.

EDWARD T. DOWLING

Contents

Chapter 1

Terminology, Concepts, and Tools

1.1 CONSTANTS, VARIABLES, PARAMETERS, AND COEFFICIENTS

A typical supply equation takes the form

$$Q_s = -a + bP \tag{1.1}$$

where Q_s = quantity supplied and P = price. A *constant* is a quantity that does not change in a given problem. A *numerical constant* has the same value in all problems; a *symbolic constant* or *parameter*, such as a and b in (*1.1*), has the same value within a given problem but may assume other values in different problems. A *variable* ranges over a set of possible values within a given problem. In (*1.1*), Q_s and P are variables. Since the value of Q_s depends on P, Q_s is called the *dependent variable*, and P is called the *independent variable*. If there is a system of equations, the variables determined within the system are called *endogenous*; those determined outside the system are *exogenous*. A numerical or symbolic constant placed before a variable as a multiplier, such as b in (*1.1*), is called a *coefficient*.

Example 1. Given $C = 50 + 0.85Y$, where C = consumption and Y = income, C and Y are variables because Y can assume any positive value and C will change in the precise way set forth by the equation. 50 and 0.85 are numerical constants. C is the dependent variable and Y is the independent variable, because the value of C depends on Y; 0.85 is the coefficient of Y.

1.2 FUNCTIONS

A *function*, such as $y = f(x)$, expresses a relationship between two variables (x, y) such that for each value of x, there exists one and only one value of y, as illustrated in Example 2. The symbol $f(x)$ reads "f of x." y is called the *value* of the function; x is termed the *argument* of the function. x and y are also referred to as the *independent* and *dependent variables*, respectively. Functions can be expressed verbally, algebraically, or graphically (see Example 3).

Example 2. In Fig. 1-1(a), y is a function of x, $y = f(x)$, because for each value of x there exists one and only one value of y (e.g. for $x = 4$, $y = 2$). In Fig. 1-1(b), however, y is not a function of x, $y \neq f(x)$, because there is more than one value of y for a given value of x (e.g. for $x = 4$, $y = 2$ *and* 6).

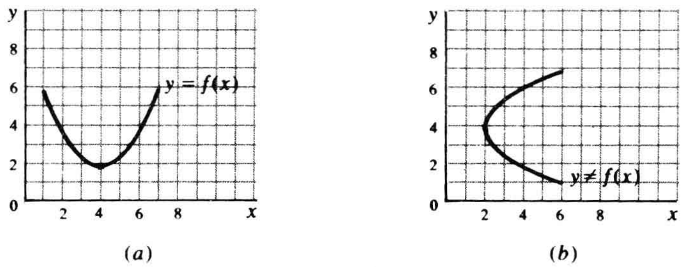

Fig. 1-1

Example 3. Economists frequently refer to the relationship between consumption and income. Since consumption depends on income in a systematic way, consumption is said to be a function of income. The relationship is expressed algebraically as $C = f(Y)$. Note that the symbol $f(Y)$ means that C is a function of Y, and not f multiplied by Y. The graphic representation of this consumption function is shown in Fig. 1-2.

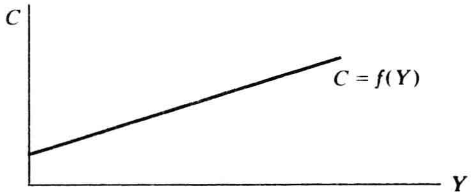

Fig. 1-2

Example 4. Other letters besides f can be used to denote a function. Instead of $y = f(x)$, one may use $y = g(x)$, $y = F(x)$, $y = G(x)$. If y and z are both functions of x, then different notation should be used for each, i.e. $y = f(x)$, $z = g(x)$. The Greek letters ϕ and ψ are also frequently used, as in $y = \phi(x)$ or $z = \psi(x)$. For greater economy and simplicity, economic textbooks frequently eliminate f and g and express functions simply as $y = y(x)$, $z = z(x)$, and $C = C(Y)$.

1.3 GENERAL VS. SPECIFIC FUNCTIONS

A *general function* enumerates the independent variables which influence the dependent variable, but does not enunciate the way in which they influence it. A *specific function* lists the arguments and the way in which they influence the dependent variable. See Example 5.

Example 5. Economists hold that the quantity of a good a consumer will buy depends on its price, the consumer's income, the price of related goods, and taste. This relationship can be expressed by the general function

$$Q_d = f(P, Y, P_r, T) \tag{1.2}$$

where Q_d = quantity demanded, P = price, Y = income, P_r = price of related goods, and T = taste. Equation (*1.2*) lists the arguments but leaves unspecified the way in which demand depends on price or how demand varies with income. The same relationship can be expressed by the specific function

$$Q_d = 250 - 5P + 0.03\,Y + 0.2\,P_r + 0.02\,T \tag{1.3}$$

which clearly enunciates the magnitude and direction of the influence of the independent variables on the dependent variable. Here, for instance, an increase of one unit in P will lead to a decrease of 5 units in Q_d. Parameters can also be used in specific functions. Thus,

$$Q_d = a - bP + cY + dP_r + eT \tag{1.4}$$

In contrast to (*1.2*), (*1.4*) specifies that none of the arguments are raised to a power higher than 1.

1.4 GRAPHS, SLOPES, AND INTERCEPTS

A function of one independent variable, $y = f(x)$, can easily be graphed in a two-dimensional space. The dependent variable y is graphed on the vertical axis; the independent variable x on the horizontal. The *slope* of a line measures the change (Δ) in the value of the variable on the vertical axis divided by the change in the value of the variable on the horizontal axis. Thus, the slope is equal to $\Delta y/\Delta x$. The *vertical intercept* is the point at which the graph crosses the vertical axis. It will be found when the independent variable x equals zero. A simple review of graphs is given in Solved Problems 1.11 and 1.12.

Example 6. The graph of the function $y = 16 - 4x$ is given in Fig. 1-3. The slope $= \Delta y/\Delta x = -4/1 = -4$. The vertical intercept is 16. [When $x = 0$, $y = 16 - 4(0) = 16$.]

The slope of a straight line is constant. A positive slope indicates an upward-sloping line; a negative slope, a downward-sloping line. (Remember that graphs, like the English language, always read from left to right.) Sometimes in economics, however, the direction of the change is taken for granted and only the magnitude of the change is of interest. In this case the *absolute value* of the slope is given (i.e. the value of the slope independent of the sign).

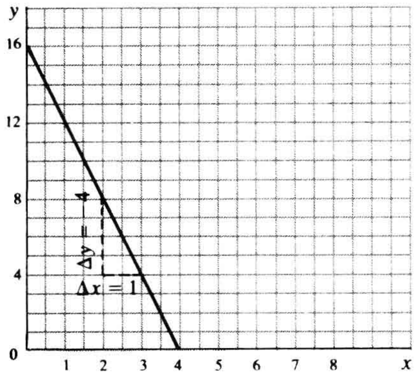

Fig. 1-3

Example 7. Graphs take different forms, depending on the function. Three functions frequently encountered in economics which involve a single independent variable are given below. Their corresponding graphs are shown in Fig. 1-4.

(1) Linear function: $y = a + bx$ No variable is raised to a power higher than 1.

(2) Quadratic function: $y = a + bx + cx^2$ Highest power to which a variable is raised is 2.

(3) Cubic function: $y = a + bx + cx^2 + dx^3$ Highest power to which a variable is raised is 3.

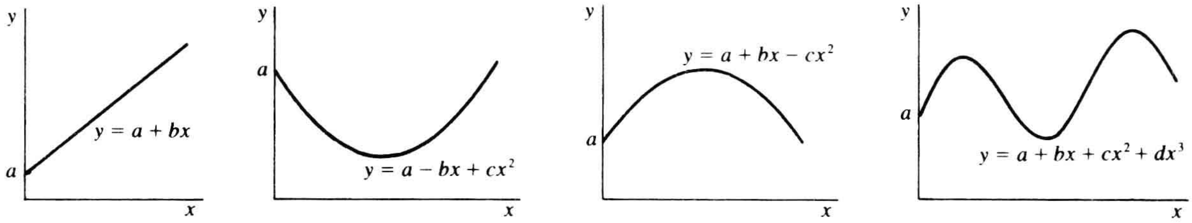

Fig. 1-4

Points to note:

- The graph of a linear function will always be a straight line.
- The graph of a quadratic function with $c > 0$ is an upward-opening parabola; if $c < 0$, it is a downward-opening parabola.
- In all the functions, a gives the value of the vertical intercept.
- For linear functions, b gives the slope of the line. Check Example 6.
- Some of the parameters may at times be zero. For linear functions, a, but not b, may equal zero; for quadratic functions, a and b, but not c, may equal zero; for cubic functions, a, b, and c, but not d, may equal zero. If b and c in (3) equal zero, the cubic function reads $y = a + dx^3$.

1.5 INVERSE FUNCTIONS

Given a function $y = f(x)$, an *inverse function*, $x = f^{-1}(y)$ exists if each value of y yields a unique value of x. See Example 8.

Example 8. In traditional supply and demand analysis, $P = f(Q)$, with P graphed on the vertical axis and Q on the horizontal. In mathematically-oriented texts, $Q = F(P)$, with Q graphed on the vertical axis and P on the horizontal. To convert from one form of expression to the other, the inverse function is needed. Thus, if

$$Q_d = a - bP$$

then $$bP = a - Q_d \quad \text{and} \quad P = \frac{a - Q_d}{b} = \frac{a}{b} - \frac{Q_d}{b}$$

See Solved Problems 1.10 and 2.3.

Example 9. Economic notation may also be confusing in another way. In mathematics y is typically used to designate the dependent variable. It is graphed on the vertical axis and the slope of the line is $\Delta y/\Delta x$. In economics, however, Y is generally used to designate income. In a typical consumption function such as $C = 25 + 0.75Y$, Y is the independent variable and will be graphed on the horizontal axis, as in Fig. 1-5. Here the slope of the line is $\Delta C/\Delta Y$ (i.e. the change in the dependent variable divided by the change in the independent variable). Be careful, therefore, to think in terms of dependent and independent variables, and not just symbols.

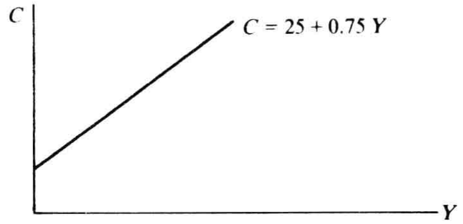

Fig. 1-5

Notice, too, that the consumption function is in the typical linear form $C = a + bY$, where $a = 25 =$ the vertical intercept and $b = 0.75 =$ the slope. Relating economics to math, a represents *autonomous consumption* since it indicates the level of consumption when $Y = 0$ and b equals the *marginal propensity to consume* (MPC), since it measures the change in consumption brought about by a unit change in income, $\Delta C/\Delta Y$.

1.6 SOLUTIONS

A single linear equation with a single unknown can be solved by moving the unknown variable to the left side of the equation and all the constants to the right. A single quadratic equation of the form, $ax^2 + bx + c = 0$, can be solved by factoring or by use of the quadratic formula

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{1.5}$$

(Note that the traditional formula reverses the order of parameters and uses c instead of a for the vertical intercept, etc.)

Example 10. A typical profit function is $\pi = -Q^2 + 11Q - 24$, which from Fig. 1-4(2b) indicates that π will increase over a given range and then decrease. The breakeven point where $\pi = 0$ can be found with the quadratic formula where $a = -1$, $b = 11$, $c = -24$, and $x = Q$. Thus,

$$Q_1, Q_2 = \frac{-11 \pm \sqrt{(11)^2 - 4(-1)(-24)}}{2(-1)} = \frac{-11 \pm \sqrt{121 - 96}}{-2}$$

$$Q_1 = 3 \qquad Q_2 = 8$$

To solve a system of simultaneous equations, the equations must be (1) consistent (noncontradictory), (2) independent (not multiples of each other), and (3) there must be as many consistent and independent equations as variables. Such a system of simultaneous linear equations can be solved by the method of substitution or elimination (see Example 12). The method of determinants, discussed in Chapter 11, can also be used to solve such systems.

Example 11. The two equations below are an obvious example of inconsistency because $x + y$ cannot equal 6 and 10 at the same time.

$$x + y = 6 \qquad x + y = 10$$

The two equations below would not be independent, because the second equation is merely two times the first and so produces no new or independent information. The multiple relationship may not always be as obvious, however. See Problem 1.16.

$$2x + 3y = 13 \qquad 4x + 6y = 26$$

Example 12. The equilibrium conditions for two markets, butter and margarine, where P_b is the price of butter and P_m is the price of margarine, are given in (*1.6*) and (*1.7*) below.

$$8P_b - 3P_m = 7 \tag{1.6}$$

$$-P_b + 7P_m = 19 \tag{1.7}$$

The prices that will bring equilibrium to the model can be found by either the substitution or elimination method for solving simultaneous equations.

Substitution method

(1) Solve one of the equations for one variable in terms of the other. Solving (*1.7*),

$$-P_b + 7P_m = 19$$
$$P_b = 7P_m - 19$$

(2) Substitute the value of that term in (*1.6*).

$$8P_b - 3P_m = 7$$
$$8(7P_m - 19) - 3P_m = 7$$
$$56P_m - 152 - 3P_m = 7$$
$$53P_m = 159 \qquad P_m = 3$$

(3) Substitute $P_m = 3$ in either (*1.6*) or (*1.7*) to find P_b.

$$8P_b - 3P_m = 7$$
$$8P_b - 3(3) = 7$$
$$8P_b = 16 \qquad P_b = 2$$

Elimination method

(1) Multiply (*1.7*) by the absolute value of the coefficient of P_b (or P_m) in (*1.6*) and (*1.6*) by the absolute value of the coefficient of P_b (or P_m) in (*1.7*). If P_m is selected,

$$7(8P_b - 3P_m = 7) \qquad 56P_b - 21P_m = 49 \qquad (1.8)$$
$$3(-P_b + 7P_m = 19) \qquad -3P_b + 21P_m = 57 \qquad (1.9)$$

(2) Add or subtract (*1.8*) and (*1.9*), whichever is needed to eliminate the selected variable. Adding here,

$$53P_b = 106 \qquad P_b = 2$$

(3) Substitute $P_b = 2$ in either (*1.6*) or (*1.7*) to find P_m, as in Step 3 above.

Solved Problems

FUNCTIONAL RELATIONSHIPS

1.1. Given the two sets of equations

$$(1)\quad Q_d = a + bP \qquad (2)\quad S = -50 + 0.3Y$$
$$Q_s = c + dP \qquad I = 250 - 0.2i$$

where S = savings, I = investment, i = interest rate, and all the other variables are familiar, identify (*a*) the constants, as numerical and parametric; (*b*) the variables, as independent and dependent; and (*c*) the coefficients.

(*a*) In set (1), the constants are a, b, c, d; they are all parametric constants or parameters. In set (2), -50, 0.3, 250, and -0.2 are all numerical constants.

(*b*) In set (1), the variables are Q_d, Q_s, P, where Q_d and Q_s are dependent because their value is determined by P, and P is independent. In set (2), S, Y, I, and i are variables; S and I are the dependent variables, Y and i are the independent variables.

(*c*) In set (1), b and d are coefficients, each being a multiplicative constant of the variable P. In set (2), 0.3 and -0.2 are coefficients.

1.2. (*a*) Define the term function. (*b*) Which of the graphs in Fig. 1-6 does not represent y as a function of x? Why?

(*a*) A function, $y = f(x)$, indicates a relationship between y and x in which y depends on x and for each value of x there is a unique value for y.

(*b*) Figure 1-6(*a*) does not represent y as a function of x because for any given value of x, there is more than one value of y. For x_1, for example, there are three values of y: y_1, y_2, y_3. Figure 1-6(*b*), on the other hand, does indicate y is a function of x. For any given value of x, there is one and *only* one value of y. At x_2, $y = y_4$.

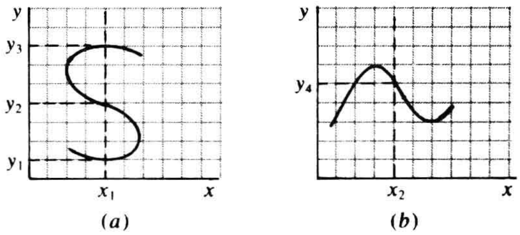

Fig. 1-6

1.3. Express each of the following statements in functional notation, giving first the general function, then the specific function. Practice the use of symbols other than $f(\ )$.

(*a*) The area (A) of a circle as a function of the radius (r).

(*b*) The perimeter (P) of a rectangle as a function of its length (l) and width (w).

(*c*) Total factor costs (TFC) as a function of the amount of labor (L) hired and the amount of capital (K) hired, when the price of labor is \$3 and the price of capital is \$5.

(*d*) Total revenue (TR) as a function of output (Q), when $P_Q = 5$.

(*e*) The daily wage bill (W) for labor (L) as a function of L, when $P_L = 42.50$ a day.

(*a*) $A = f(r)$, $A = \pi r^2$

(*b*) $P = g(l, w)$, $P = 2l + 2w$

(*c*) $\text{TFC} = \phi(K, L,)$, $\text{TFC} = 3L + 5K$

(*d*) $\text{TR} = \text{TR}(Q)$, $\text{TR} = 5Q$

(*e*) $W = W(L)$, $W = 42.50L$

1.4. A club agrees to serve 100 members at a price of \$15 each and any additional guests at a price of \$20 each. (*a*) Express the cost c of the banquet as a function of guests (G). (*b*) Identify the dependent and independent variables. (*c*) Draw the function as a graph.

(*a*) Since 100 members are coming at a cost of \$15 each, \$1500 is a fixed cost that will not vary with the number of guests invited. Costs will increase above \$1500 by \$20 for every guest invited. Thus, $c = 1500 + 20G$.

(*b*) c is the dependent variable since it depends on the number of guests invited. G is the independent variable because it is determined independently of the equation.

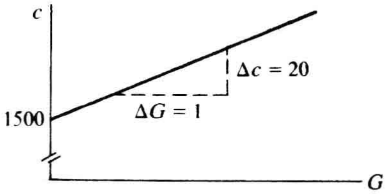

Fig. 1-7

(*c*) Since cost is the dependent variable, it is graphed on the vertical axis; the number of guests on the horizontal. The function is in the linear form of $y = a + bx$; 1500 is the vertical intercept and the slope is 20. See Fig. 1-7.

1.5. A firm's fixed costs (FC) are \$600 regardless of output; variable costs (VC) are \$5 per unit of output (Q). Total costs (TC) = FC + VC. The selling price of the good is \$10 per unit. State (*a*) the fixed cost function, (*b*) the variable cost function, (*c*) the total cost function, and (*d*) the total revenue function. (*e*) Find the breakeven point algebraically, and (*f*) graphically.

(*a*) Since the fixed cost is independent of output (Q), the fixed cost function is simply $\text{FC} = 600$.

(*b*) Variable costs are \$5 per unit of output, so $\text{VC} = 5Q$.

(*c*) Since $\text{TC} = \text{FC} + \text{VC}$, $\text{TC} = 600 + 5Q$.

(*d*) $\text{TR} = PQ$. Substituting $P = 10$, $\text{TR} = 10Q$.

(*e*) At the breakeven point, TR = TC. Substituting from (*c*) and (*d*) above,

$$\text{TR} = \text{TC}$$
$$10Q = 600 + 5Q$$
$$5Q = 600 \qquad Q = 120 \qquad \text{the breakeven point}$$

(*f*) See Fig. 1-8.

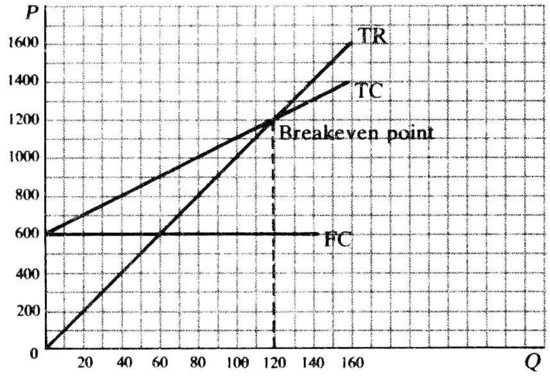

Fig. 1-8

1.6. From Table 1, (*a*) express savings (S) as a linear function of income. (*b*) Graph the function.

Table 1

Y	0	1000	2000	3000	4000
S	−400	−200	0	200	400

(*a*) As a linear function, the equation will take the form $S = a + bY$, where a is the vertical intercept and b is the slope. The vertical intercept is defined as the point where the independent variable (here Y) is zero. Hence, $a = -400$. In the savings function this point is also called *autonomous savings* since it represents a level of saving that takes place independent of income, or when income equals zero. The slope of the line (b) measures the change in the dependent variable (S) brought about by a change in the independent variable. Since saving increases by 200 for every 1000 increase in income, the slope $\Delta S/\Delta Y = \frac{200}{1000} = 0.2$. In economic terms, the slope $\Delta S/\Delta Y$ is also the *marginal propensity to save* (MPS), or the change in savings related to a unit change in income. The equation therefore reads

$$S = -400 + 0.2\,Y$$

(*b*) The function is graphed with savings (the dependent variable) on the vertical axis and income (the independent variable) on the horizontal. The graph may be plotted directly from the data in Table 1 or more simply by making use of the equation derived in (*a*). It will start at the vertical intercept (−400) and proceed upward at a rate (slope) of 0.2. See Fig. 1-9.

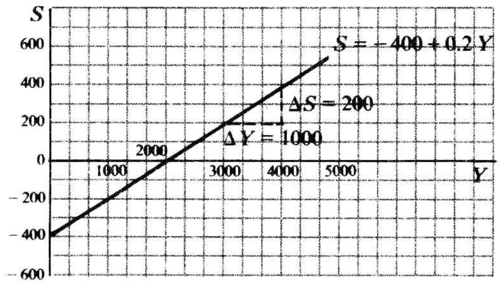

Fig. 1-9

1.7. Three separate economies are represented by the consumption functions (*a*) $C_1 = 1600 + 0.75\,Y$, (*b*) $C_2 = 750 + 0.9\,Y$, and (*c*) $C_3 = 1200 + 0.8\,Y$. Derive their respective savings functions.

(*a*) In a linear consumption function of the form $C = a + bY$, a = autonomous consumption and b = MPC (see Example 9). Here, therefore, 1600 is consumed independently of income. For this to be possible people must borrow 1600 or dissave that amount by dipping into their previous savings. Consequently, the level of autonomous saving is -1600. With $b = 0.75$ = MPC, the MPS = 0.25 since MPC + MPS = 1. The savings function, therefore, is $S_1 = -1600 + 0.25\,Y$. This answer can also be found by using $S = Y - C$.

(*b*) $S_2 = -750 + 0.1\,Y$

(*c*) $S_3 = -1200 + 0.2\,Y$

1.8. Given $C = 25 + 0.6\,Yd$, $Yd = Y - T$, and $T = 5$, where Yd = disposable income and T = tax, express C as a function of Y, not Yd.

Substituting $Yd = Y - T$, $\quad C = 25 + 0.6\,Yd$

Substituting $T = 5$, $\quad C = 25 + 0.6(Y - T)$

$$C = 25 + 0.6(Y - 5) = 25 + 0.6\,Y - 3 = 22 + 0.6\,Y$$

1.9. Identify (*a*) $Y = f(C, I)$, (*b*) $Y = C_0 + bY + I_0$, and (*c*) $\pi = -Q^2 + 13Q - 42$ as a general or specific function. Explain.

Function (*a*) is general. It indicates a relationship between Y and C, I, but does not detail the precise way in which they are related.

Function (*b*) is specific, because it stipulates a linear relationship.

Function (*c*) is also specific. It is a quadratic function delineating the exact way in which Q influences π.

1.10. Find the inverse function for (*a*) $I = 130 + 0.25\,Y$ and (*b*) $Q_d = 75 - 15P$.

(*a*) Since I is a function of Y, the inverse function is found simply by making Y a function of I by solving for Y in terms of I. Thus,

$$I = 130 + 0.25\,Y$$
$$0.25\,Y = I - 130$$
$$Y = 4I - 520$$

(*b*) Similarly,

$$Q_d = 75 - 15P$$
$$15P = 75 - Q_d$$
$$P = 5 - \tfrac{1}{15}Q_d$$

GRAPHS

1.11. Graph the following equations and give the slopes and vertical intercepts:

(*a*) $3y - 6x = 3$ (*b*) $2y - \frac{2}{3}x = 6$ (*c*) $y + 5x - 20 = 0$ (*d*) $x + 2y - 4 = 0$

To graph an equation, first solve for one variable in terms of the other, usually the dependent in terms of the independent, or y in terms of x. Thus,

(*a*) $3y - 6x = 3$, $\quad 3y = 3 + 6x$, $\quad y = 1 + 2x$

(*b*) $2y - \frac{2}{3}x = 6$, $\quad 2y = 6 + \frac{2}{3}x$, $\quad y = 3 + \frac{1}{3}x$

(*c*) $y + 5x - 20 = 0$, $\quad y = 20 - 5x$

(*d*) $x + 2y - 4 = 0$, $\quad y = 2 - \frac{1}{2}x$

Then construct a schedule of the different combinations of points that fit the equation and plot the points. See Fig. 1-10. For a linear function you need only draw a straight line between any two points, or you can use your knowledge of a linear equation $y = a + bx$ to construct the line directly by starting from the vertical intercept a and proceeding with the proper slope b.

Equation (*a*):

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{+2}{+1} = +2$$

For every positive one unit change in x, y increases by 2.

$$\text{Vertical intercept} = 1$$

When $x = 0$, $y = 1 + 2(0) = 1$.

x	y
0	1
1	3
2	5
3	7
4	9
5	11

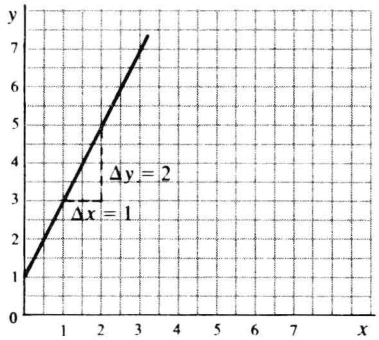

Fig. 1-10 (*a*)

Equation (*b*):

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{+1}{+3} = +\frac{1}{3}$$

For every positive one unit change in x, y increases by $\frac{1}{3}$.

$$\text{Vertical intercept} = 3$$

When $x = 0$, $y = 3 + \frac{1}{3}(0) = 3$.

x	y
0	3
1	$3\frac{1}{3}$
2	$3\frac{2}{3}$
3	4
4	$4\frac{1}{3}$
5	$4\frac{2}{3}$
6	5

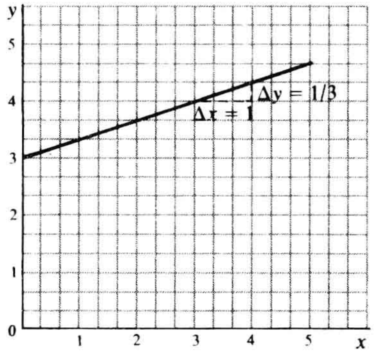

Fig. 1-10 (*b*)

Equation (*c*):

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{-5}{+1} = -5$$

For every positive one unit change in x, y decreases by 5.

$$\text{Vertical intercept} = 20$$

When $x = 0$, $y = 20 - 5(0) = 20$.

x	y
0	20
1	15
2	10
3	5
4	0

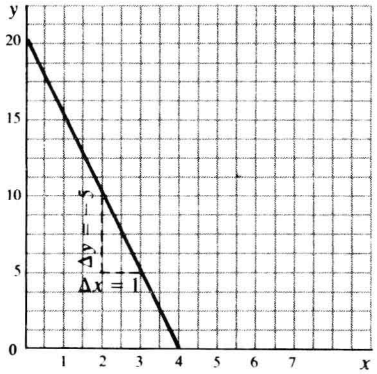

Fig. 1-10 (*c*)

Equation (*d*):

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{-1}{+2} = -\frac{1}{2}$$

For every positive one unit change in x, y falls by $\frac{1}{2}$.

$$\text{Vertical intercept} = 2$$

When $x = 0$, $y = 2 - \frac{1}{2}(0) = 2$.

x	y
0	2
1	$1\frac{1}{2}$
2	1
3	$\frac{1}{2}$
4	0

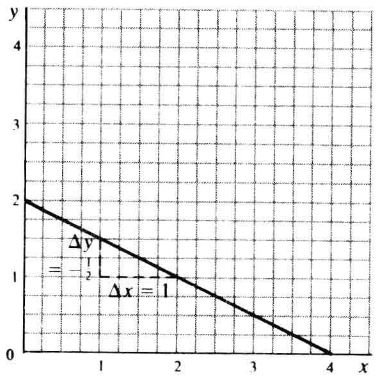

Fig. 1-10 (*d*)